Practice exam paper

1 a $\left(2a\sqrt{7b}\right)^2 = 4a^2 \times 7b$
$= 28a^2b$

b $\left(2a^2\sqrt[3]{6b}\right)^3 = \left(2a^2\right)^3 \times \left(\sqrt[3]{6b}\right)^3$
$= 8a^6 \times 6b$
$= 48a^6b$

c $\frac{1-\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{3+\sqrt{7}-3\sqrt{7}-7}{9-7}$
$= \frac{-4-2\sqrt{7}}{2}$
$= -2-\sqrt{7}$

2 $y+3x+1=0$ **(1)**
$y^2+11x^2+3x=0$ **(2)**
Rearrange equation **(1)** to get
$y=-3x-1$ **(3)**
Substitute equation **(3)** into equation **(2)** to get
$(-3x-1)^2+11x^2+3x=0$
$9x^2+6x+1+11x^2+3x=0$
$20x^2+9x+1=0$
$(4x+1)(5x+1)=0$
$x=-\frac{1}{4}$ or $x=-\frac{1}{5}$
Now substitute these values into equation **(3)** to find y.
When $x=-\frac{1}{4}$, $y=-\frac{1}{4}$
When $x=-\frac{1}{5}$, $y=-\frac{2}{5}$

3 $f'(x)=\sqrt{x}-\frac{1}{\sqrt{x}}$
$= x^{\frac{1}{2}} - x^{-\frac{1}{2}}$
$f(x)=\int\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$
$=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+c$
Since the point (9, 5) lies on f(x)
$5=\frac{2}{3}(9)^{\frac{3}{2}}-2(9)^{\frac{1}{2}}+c$
$=\frac{2}{3}(27)-2(3)+c$
$=18-6+c$
$=12+c$
$c=-7$
Therefore
$f(x)=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}-7$

4 a $\frac{x^2-10}{5\sqrt{x}}=\frac{x^2-10}{5x^{\frac{1}{2}}}$
$=\frac{x^2}{5x^{\frac{1}{2}}}-\frac{10}{5x^{\frac{1}{2}}}$
$=\frac{1}{5}x^{\frac{3}{2}}-2x^{-\frac{1}{2}}$
So $A=\frac{1}{5}$, $B=-2$, $p=\frac{3}{2}$ and $q=-\frac{1}{2}$

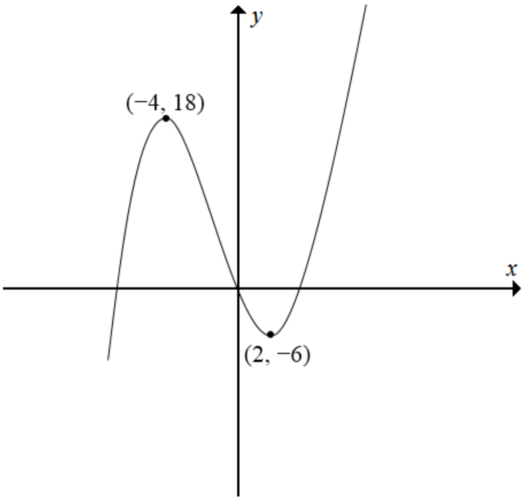
b $y=\frac{x^2-10}{5\sqrt{x}}$
$=\frac{1}{5}x^{\frac{3}{2}}-2x^{-\frac{1}{2}}$
$\frac{dy}{dx}=\frac{3}{10}x^{\frac{1}{2}}+x^{-\frac{3}{2}}$

4 c $y = \dfrac{x^2 - 10}{5\sqrt{x}}$

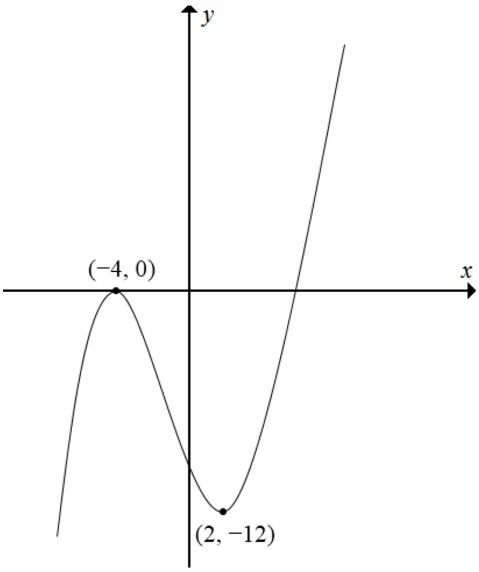
$$\int y \, \mathrm{d}x = \int \left(\frac{1}{5} x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} \right) \mathrm{d}x$$

$$= \frac{2}{25} x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$$

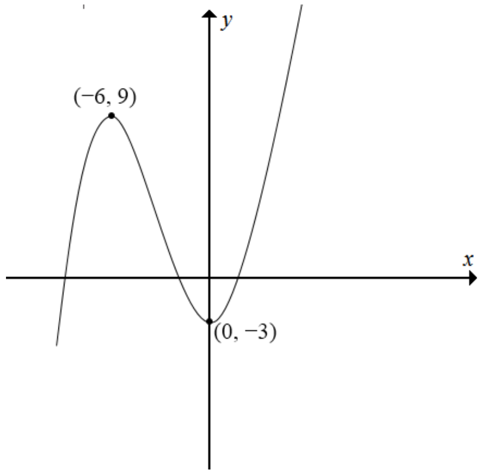
5 a $y = 2\mathrm{f}(x)$ is a vertical stretch of scale factor 2.



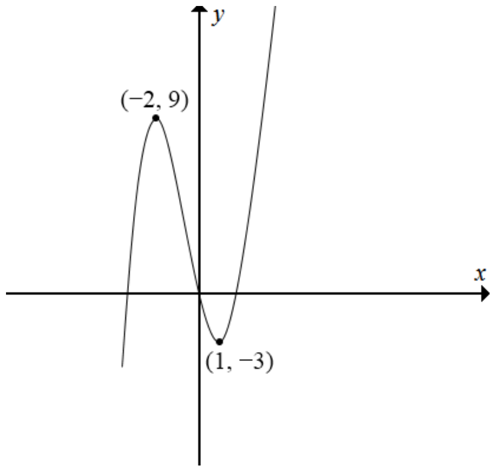
b $y = \mathrm{f}(x) - 9$ translates all points 9 units down.



5 c $y = \mathrm{f}(x + 2)$ translates all points 2 units to the left.



d $y = \mathrm{f}(2x)$ is a horizontal compression of scale factor 2.



6 $y = 2x^2 + 9\sqrt[3]{x} + \dfrac{x^3 - 6}{4\sqrt{x}}$

$$= 2x^2 + 9x^{\frac{1}{3}} + \frac{1}{4} x^{\frac{5}{2}} - \frac{3}{2} x^{-\frac{1}{2}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 3x^{-\frac{2}{3}} + \frac{5}{8} x^{\frac{3}{2}} + \frac{3}{4} x^{-\frac{3}{2}}$$

7 a $x^2 + kx + (k + 4) = 0$

Since the equation has distinct real roots

$b^2 - 4ac > 0$

$k^2 - 4(1)(k + 4) > 0$

$k^2 - 4k - 16 > 0$ as required

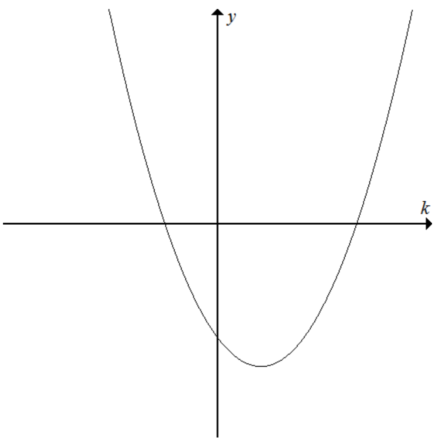
7 b $k^2 - 4k - 16 > 0$

$(k-2)^2 - 20 > 0$

$k - 2 > \pm\sqrt{20}$

$k > 2 \pm 2\sqrt{5}$

Now sketch the graph of $k^2 - 4k - 16 > 0$



$k^2 - 4k - 16 > 0$ when $k < 2 - 2\sqrt{5}$ and $k > 2 + 2\sqrt{5}$

8 $x - 2py = -3$ **(1)**

$3x - 2y + q = -4$ **(2)**

The solution to the equations are $x = 1$ and $y = q$.

Substituting these values into equations **(1)** and **(2)** gives

$1 - 2pq = -3$ **(3)**

$3 - q = -4$ **(4)**

Rearranging equation **(4)** gives

$q = 7$

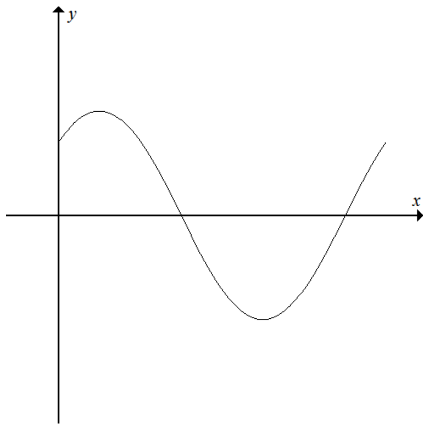
Substituting $q = 7$ into equation **(3)** gives

$1 - 14p = -3$

$14p = 4$

$p = \frac{2}{7}$

9 a



b The curve crosses the y-axis at $x = 0$, so

$y = \cos\left(0 - \frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2}$

Therefore, the curve crosses the y-axis at at $\left(0, \frac{\sqrt{2}}{2}\right)$.

The curve crosses the x-axis at $y = 0$, so

$\cos\left(x - \frac{\pi}{4}\right) = 0$

$x - \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

Therefore, the curve crosses the x-axis at at $\left(\frac{3\pi}{4}, 0\right)$.

The curve crosses the x-axis again after another π at $\left(\frac{7\pi}{4}, 0\right)$.

10 The line has gradient 3 and passes through the point (2, 1).
Substitute into $y - y_1 = m(x - x_1)$ to get

$$y - 1 = 3(x - 2)$$
$$y = 3x - 5$$

This meets the line $2x - 3y + 6 = 0$ at the point P.
Substituting $y = 3x - 5$ into $2x - 3y + 6 = 0$ gives

$$2x - 3(3x - 5) + 6 = 0$$
$$2x - 9x + 15 + 6 = 0$$
$$-7x + 21 = 0$$
$$x = 3$$

When $x = 3$,
$y = 3(3) - 5$
$= 4$
So P is the point (3, 4).