Chapter review 9

1 a $\int (x+1)(2x-5)\ \mathrm{d}x = \int (2x^2-3x-5)\ \mathrm{d}x$
$= \frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x$

b $\int \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right) \mathrm{d}x = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$
$= \frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} + c$

2 $\mathrm{f}'(x) = x^2 - 3x - \frac{2}{x^2} = x^2 - 3x - 2x^{-2}$
So $\mathrm{f}(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + c$
$\mathrm{f}(1) = 1 \Rightarrow \frac{1}{3} - \frac{3}{2} + 2 + c = 1$
So $c = \frac{1}{6}$
The equation is $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + \frac{1}{6}$

3 a $\int (8x^3 - 6x^2 + 5)\ \mathrm{d}x = 8\frac{x^4}{4} - 6\frac{x^3}{3} + 5x + c$
$= 2x^4 - 2x^3 + 5x + c$

b $\int (5x+2)x^{\frac{1}{2}}\ \mathrm{d}x = \int \left(5x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\right) \mathrm{d}x$
$= 5\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$
$= 2x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

4 $y = \frac{(x+1)(2x-3)}{\sqrt{x}}$
$= (2x^2 - x - 3)x^{-\frac{1}{2}}$
$= 2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$
$\int y\ \mathrm{d}x = \int \left(2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}\right) \mathrm{d}x$
$= 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$
$= \frac{4}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$

5 $\frac{\mathrm{d}x}{\mathrm{d}t} = (t+1)^2 = t^2 + 2t + 1$
$\Rightarrow x = \frac{1}{3}t^3 + t^2 + t + c$
$x = 0$ when $t = 2$.
So $0 = \frac{8}{3} + 4 + 2 + c$
$\Rightarrow c = -\frac{26}{3}$
So $x = \frac{1}{3}t^3 + t^2 + t - \frac{26}{3}$
When $t = 3$, $x = \frac{27}{3} + 9 + 3 - \frac{26}{3}$
So $x = \frac{37}{3}$ or $12\frac{1}{3}$

6 a $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$
$y = \left(x^{\frac{1}{3}} + 3\right)^2$
$= \left(x^{\frac{1}{3}}\right)^2 + 6x^{\frac{1}{3}} + 9$
$= x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9$
$(A = 6, B = 9)$

b $\int y\ \mathrm{d}x = \int \left(x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9\right) \mathrm{d}x$
$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 6\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 9x + c$
$= \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$

7 a $y^{\frac{1}{2}} = 3x^{\frac{1}{4}} - 4x^{-\frac{1}{4}}$
$y = (3x^{\frac{1}{4}} - 4x^{-\frac{1}{4}})^2$
$= 9x^{\frac{1}{2}} - 24 + 16x^{-\frac{1}{2}}$
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9}{2}x^{-\frac{1}{2}} - 8x^{-\frac{3}{2}}$

b $\int \left(9x^{\frac{1}{2}} - 24 + 16x^{-\frac{1}{2}}\right) \mathrm{d}x$
$= \frac{9x^{\frac{3}{2}}}{\frac{3}{2}} - 24x + \frac{16x^{\frac{1}{2}}}{\frac{1}{2}} + c$
$= 6x^{\frac{3}{2}} - 24x + 32x^{\frac{1}{2}} + c$

8 $\int\left(\frac{a}{3x^3}-ab\right)\mathrm{d}x=\int\left(\frac{a}{3}x^{-3}-ab\right)\mathrm{d}x$

$$=\frac{a}{3}\times\frac{x^{-2}}{-2}-abx+c$$
$$=-\frac{a}{6x^2}-abx+c$$
$$=-\frac{2}{3x^2}+14x+c$$

Equating coefficients $-\frac{a}{6}=-\frac{2}{3}$

and $-ab=14$

$a=4$, $b=-3.5$

9 $f'(t)=-9.8t$

$$f(t)=-\frac{9.8t^2}{2}+c$$
$$=-4.9t^2+c$$

$$f(0)=-4.9(0)^2+c$$
$$=70$$
$$c=70$$

$f(t)=-4.9t^2+70$

$$f(3)=-4.9(3)^2+70$$
$$=25.9$$

The height of the rock above the ground after 3 seconds is 25.9 m.

10 a $f(t)=\int(5+2t)\,\mathrm{d}t$

$$=5t+t^2+c$$

As $f(0)=0$, $5(0)+0^2+c=0$

$c=0$

$f(t)=5t+t^2$

b When $f(t)=100$, $5t+t^2=100$

$t^2+5t-100=0$

Using the quadratic formula:

$$t=\frac{-5\pm\sqrt{5^2-4(1)(-100)}}{2(1)}$$
$$=\frac{-5\pm\sqrt{425}}{2}$$

$t=7.8$ or $t=-12.8$

As $t>0$, $t=7.8$ seconds

11 a $y=3x^{\frac{1}{2}}-4x^{-\frac{1}{2}}$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\tfrac{3}{2}x^{-\frac{1}{2}}+\tfrac{1}{2}\times 4x^{-\frac{3}{2}}$$
$$=\tfrac{3}{2}x^{-\frac{1}{2}}+2x^{-\frac{3}{2}}$$

b $\int y\,\mathrm{d}x=\int\left(3x^{\frac{1}{2}}-4x^{-\frac{1}{2}}\right)\mathrm{d}x$

$$=\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{4x^{\frac{1}{2}}}{\frac{1}{2}}+c$$
$$=2x^{\frac{3}{2}}-8x^{\frac{1}{2}}+c$$

12 a $y=12x^{\frac{1}{2}}-x^{\frac{3}{2}}$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=6x^{-\frac{1}{2}}-\tfrac{3}{2}x^{\frac{1}{2}}$$
$$=\tfrac{3}{2}x^{-\frac{1}{2}}(4-x)$$

b $\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow x=4,\ y=12\times2-2^3=16$

So B is the point $(4,16)$.

13 $\int\left(\frac{9}{x^2}-8\sqrt{x}+4x-5\right)\mathrm{d}x$

$$=\int(9x^{-2}-8x^{\frac{1}{2}}+4x-5)\,\mathrm{d}x$$
$$=\frac{9x^{-1}}{-1}-\frac{8x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{4x^2}{2}-5x+c$$
$$=-\frac{9}{x}-\frac{16x^{\frac{3}{2}}}{3}+2x^2-5x+c$$

14 a $f'(x)=\frac{(2-x^2)^3}{x^2}$

$$=\frac{(2-x^2)(2-x^2)(2-x^2)}{x^2}$$
$$=\frac{(4-4x^2+x^4)(2-x^2)}{x^2}$$
$$=x^{-2}(8-12x^2+6x^4-x^6)$$
$$=8x^{-2}-12+6x^2-x^4$$

So $A=6$ and $B=-1$

b $f(x)=\int(8x^{-2}-12+6x^2-x^4)\,\mathrm{d}x$

$$=\frac{8x^{-1}}{-1}-12x+\frac{6x^3}{3}-\frac{x^5}{5}+c$$
$$=-\frac{8}{x}-12x+2x^3-\frac{x^5}{5}+c$$

14 b When $x = -2$ and $y = 9$

$$-\frac{8}{(-2)} - 12(-2) + 2(-2)^3 - \frac{(-2)^5}{5} + c = 9$$

$$4 + 24 - 16 + \tfrac{32}{5} + c = 9$$

$$c = -\tfrac{47}{5}$$

$$f(x) = -\frac{8}{x} - 12x + 2x^3 - \frac{x^5}{5} - \frac{47}{5}$$

Challenge

a $\frac{dy}{dx} = 6x^2 - 6x + k$

$y = 2x^3 - 3x^2 + kx + c$

When $x = 1, y = 4 \Rightarrow 4 = 2 - 3 + k + c$

So $c = 5 - k$

When $x = 2, y = 12 \Rightarrow 12 = 16 - 12 + 2k + c$

So $c = 8 - 2k$

Then $5 - k = 8 - 2k$

So $k = 3$

b $y = 2x^3 - 3x^2 + 3x + c$

The curve passes through the two points given, so choose either one and solve for c. Here, the point (1, 4) has been used:

$4 = 2(1)^3 - 3(1)^2 + 3(1) + c$

$4 = 2 - 3 + 3 + c$

$\Rightarrow c = 2$

So, the curve's equation is

$y = 2x^3 - 3x^2 + 3x + 2$

