

Exercise 8D

1 a $y = 2x^2 - 6x + 3$

$$\frac{dy}{dx} = 2(2x) - 6(1) + 0 = 4x - 6$$

b $y = \frac{1}{2}x^2 + 12x$

$$\frac{dy}{dx} = \frac{1}{2}(2x) + 12(1) = x + 12$$

c $y = 4x^2 - 6$

$$\frac{dy}{dx} = 4(2x) - 0 = 8x$$

d $y = 8x^2 + 7x + 12$

$$\frac{dy}{dx} = 8(2x) + 7(1) + 0 = 16x + 7$$

e $y = 5 + 4x - 5x^2$

$$\frac{dy}{dx} = 0 + 4(1) - 5(2x) = 4 - 10x$$

2 a $y = 3x^2$

$$\frac{dy}{dx} = 6x$$

At the point (2, 12), $x = 2$

Substituting $x = 2$ into $\frac{dy}{dx} = 6x$ gives:

$$\text{Gradient} = 6 \times 2 = 12$$

b $y = x^2 + 4x$

$$\frac{dy}{dx} = 2x + 4$$

At the point (1, 5), $x = 1$

Substituting $x = 1$ into $\frac{dy}{dx} = 2x + 4$ gives:

$$\text{Gradient} = 2 \times 1 + 4 = 6$$

c $y = 2x^2 - x - 1$

$$\frac{dy}{dx} = 4x - 1$$

At the point (2, 5), $x = 2$

Substituting $x = 2$ into $\frac{dy}{dx} = 4x - 1$ gives:

$$\text{Gradient} = 4 \times 2 - 1 = 7$$

2 d $y = \frac{1}{2}x^2 + \frac{3}{2}x$

$$\frac{dy}{dx} = x + \frac{3}{2}$$

At the point (1, 2), $x = 1$

Substituting $x = 1$ into $\frac{dy}{dx} = x + \frac{3}{2}$ gives:

$$\text{Gradient} = 1 + \frac{3}{2} = 2\frac{1}{2}$$

e $y = 3 - x^2$

$$\frac{dy}{dx} = -2x$$

At the point (1, 2), $x = 1$

Substituting $x = 1$ into $\frac{dy}{dx} = -2x$ gives:

$$\text{Gradient} = -2 \times 1 = -2$$

f $y = 4 - 2x^2$

$$\frac{dy}{dx} = -4x$$

At the point (-1, 2), $x = -1$

Substituting $x = -1$ into $\frac{dy}{dx} = -4x$ gives:

$$\text{Gradient} = -4 \times -1 = 4$$

3 $y = 3 + 2x - x^2$

When $x = 1$, $y = 3 + 2 - 1$

$$\Rightarrow y = 4 \text{ when } x = 1$$

$$\frac{dy}{dx} = 2 - 2x$$

When $x = 1$, $\frac{dy}{dx} = 2 - 2$

$$\Rightarrow \frac{dy}{dx} = 0 \text{ when } x = 1$$

Therefore, the y -coordinate is 4 and the gradient is 0 when the x -coordinate is 1 on the given curve.

4 $y = x^2 + 5x - 4$

$$\frac{dy}{dx} = 2x + 5$$

$$2x + 5 = 3$$

$$2x = -2$$

$$x = -1$$

- 4 Substituting $x = -1$ into $y = x^2 + 5x - 4$:
 $y = (-1)^2 + 5(-1) - 4 = 1 - 5 - 4 = -8$
 So $(-1, -8)$ is the point where the gradient is 3.

- 5 The curve $y = x^2 - 5x + 10$ meets the line $y = 4$ when:
 $x^2 - 5x + 10 = 4$
 $x^2 - 5x + 6 = 0$
 $(x - 3)(x - 2) = 0$
 $x = 3$ or $x = 2$

$$\text{Gradient of curve} = \frac{dy}{dx} = 2x - 5$$

$$\text{When } x = 3, \frac{dy}{dx} = 2 \times 3 - 5 = 1$$

$$\text{When } x = 2, \frac{dy}{dx} = 2 \times 2 - 5 = -1$$

So the gradient is -1 at $(2, 4)$
 and 1 at $(3, 4)$.

- 6 The curve $y = 2x^2$ meets the line $y = x + 3$ when:
 $2x^2 = x + 3$
 $2x^2 - x - 3 = 0$
 $(2x - 3)(x + 1) = 0$
 $x = 1.5$ or -1

$$\text{Gradient of curve} = \frac{dy}{dx} = 4x$$

$$\text{When } x = -1, \frac{dy}{dx} = 4 \times -1 = -4$$

$$\text{When } x = 1.5, \frac{dy}{dx} = 4 \times 1.5 = 6$$

So the gradient is -4 at $(-1, 2)$
 and 6 at $(1.5, 4.5)$.