

Exercise 8C

1 a $f(x) = x^7$
 $f'(x) = 7x^6$

b $f(x) = x^8$
 $f'(x) = 8x^7$

c $f(x) = x^4$
 $f'(x) = 4x^3$

d $f(x) = x^{\frac{1}{3}}$
 $f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

e $f(x) = x^{\frac{1}{4}}$
 $f'(x) = \frac{1}{4}x^{\frac{1}{4}-1} = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$

f $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$
 $f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

g $f(x) = x^{-3}$
 $f'(x) = -3x^{-3-1} = -3x^{-4}$

h $f(x) = x^{-4}$
 $f'(x) = -4x^{-4-1} = -4x^{-5}$

i $f(x) = \frac{1}{x^2} = x^{-2}$
 $f'(x) = -2x^{-2-1} = -2x^{-3} = -\frac{2}{x^3}$

j $f(x) = \frac{1}{x^5} = x^{-5}$
 $f'(x) = -5x^{-5-1} = -5x^{-6} = -\frac{5}{x^6}$

k $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$
 $f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x^{\frac{3}{2}}}$

1 l $f(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$
 $f'(x) = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{1}{3}x^{-\frac{4}{3}} = -\frac{1}{3x^{\frac{4}{3}}}$

m $f(x) = x^3 \times x^6 = x^{3+6} = x^9$
 $f'(x) = 9x^8$

n $f(x) = x^2 \times x^3 = x^5$
 $f'(x) = 5x^4$

o $f(x) = x \times x^2 = x^3$
 $f'(x) = 3x^2$

p $f(x) = \frac{x^2}{x^4} = x^{-2}$
 $f'(x) = -2x^{-2-1} = -2x^{-3} = -\frac{2}{x^3}$

q $f(x) = \frac{x^3}{x^2} = x$
 $f'(x) = 1x^0 = 1$

r $f(x) = \frac{x^6}{x^3} = x^3$
 $f'(x) = 3x^2$

2 a $y = 3x^2$
 $\frac{dy}{dx} = 2 \times 3x^{2-1} = 6x$

b $y = 6x^9$
 $\frac{dy}{dx} = 9 \times 6x^{9-1} = 54x^8$

c $y = \frac{1}{2}x^4$
 $\frac{dy}{dx} = 4 \times \frac{1}{2}x^{4-1} = 2x^3$

d $y = 20x^{\frac{1}{4}}$
 $\frac{dy}{dx} = \frac{1}{4} \times 20x^{\frac{1}{4}-1} = 5x^{-\frac{3}{4}} = \frac{5}{x^{\frac{3}{4}}}$

$$2 \text{ e } y = 6x^{\frac{5}{4}}$$

$$\frac{dy}{dx} = \frac{5}{4} \times 6x^{\frac{5}{4}-1} = \frac{15}{2}x^{\frac{1}{4}}$$

$$f \quad y = 10x^{-1}$$

$$\frac{dy}{dx} = -1 \times 10x^{-1-1} = -10x^{-2}$$

$$g \quad y = \frac{4x^6}{2x^3} = 2x^3$$

$$\frac{dy}{dx} = 3 \times 2x^{3-1} = 6x^2$$

$$h \quad y = \frac{x}{8x^5} = \frac{1}{8}x^{-4}$$

$$\frac{dy}{dx} = -4 \times \frac{1}{8}x^{-4-1} = -\frac{1}{2}x^{-5} = -\frac{1}{2x^5}$$

$$i \quad y = -\frac{2}{\sqrt{x}} = -2x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \left(-\frac{1}{2}\right) \times (-2)x^{-\frac{1}{2}-1} = x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}}$$

$$j \quad y = \sqrt{\frac{5x^4 \times 10x}{2x^2}} = 5x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2} \times 5x^{\frac{3}{2}-1} = \frac{15}{2}x^{\frac{1}{2}} = \frac{15\sqrt{x}}{2}$$

$$3 \text{ a } y = 3\sqrt{x} = 3x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} \times 3x^{\frac{1}{2}-1} = \frac{3}{2}x^{-\frac{1}{2}} = \frac{3}{2\sqrt{x}}$$

$$\text{When } x = 4, \frac{dy}{dx} = \frac{3}{2\sqrt{4}} = \frac{3}{4}$$

$$b \quad \text{When } x = 9, \frac{dy}{dx} = \frac{3}{2\sqrt{9}} = \frac{3}{6} = \frac{1}{2}$$

$$c \quad \text{When } x = \frac{1}{4}, \frac{dy}{dx} = \frac{3}{2\sqrt{\frac{1}{4}}} = \frac{3}{1} = 3$$

$$d \quad \text{When } x = \frac{9}{16}, \frac{dy}{dx} = \frac{3}{2\sqrt{\frac{9}{16}}} = \frac{3}{\frac{3}{2}} = 2$$

$$4 \quad \begin{aligned} 2y^2 - x^3 &= 0 \\ 2y^2 &= x^3 \\ y^2 &= \frac{1}{2}x^3 \\ y &= \frac{1}{\sqrt{2}}x^{\frac{3}{2}} \end{aligned}$$

$$\frac{dy}{dx} = \frac{3}{2} \times \frac{1}{\sqrt{2}}x^{\frac{3}{2}-1} = \frac{3}{2\sqrt{2}}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{\frac{x}{2}}$$