Exercise 6G

1 **a** **i** Maximum value of $\cos x$ is 1.
This occurs when $x = 0°$.

ii Minimum value is –1, which occurs when $x = 180°$.

b **i** Maximum value of $\sin x$ is 1, so the maximum value of $4 \sin x$ is 4.
This occurs when $x = 90°$.

ii Minimum value of $4 \sin x$ is = –4.
This occurs when $x = 270°$.

c The graph of $\cos(-x)$ is a reflection of the graph of $\cos x$ in the y-axis. This is the same curve; $\cos(-x) = \cos x$.

i Maximum value of $\cos(-x)$ is 1.
This occurs when $x = 0°$.

ii Minimum value of $\cos(-x)$ is –1.
This occurs when $x = 180°$.

d The graph of $3 + \sin x$ is the graph of $\sin x$ translated by +3 vertically.

i Maximum is 4 when $x = 90°$.
ii Minimum is 2 when $x = 270°$.

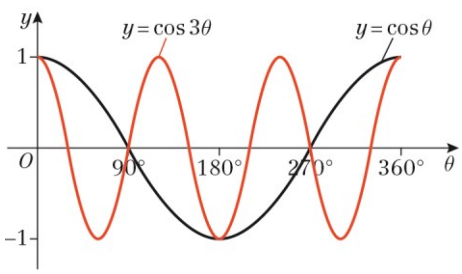
e The graph of $-\sin x$ is the reflection of the graph of $\sin x$ in the x-axis.

i Maximum is 1 when $x = 270°$.
ii Minimum is –1 when $x = 90°$.

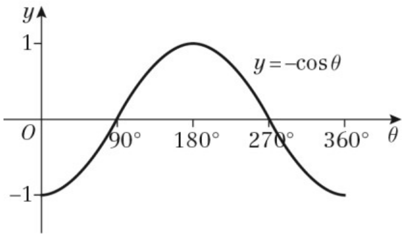
f The graph of $\sin 3x$ is the graph of $\sin x$ stretched by $\frac{1}{3}$ in the x direction.

i Maximum is 1 when $x = 30°$.
ii Minimum is –1 when $x = 90°$.

2

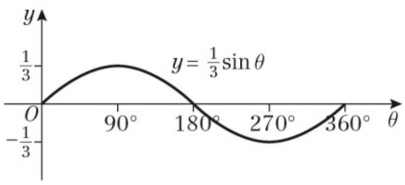


3 **a** The graph of $y = -\cos\theta$ is the graph of $y = \cos\theta$ reflected in the θ-axis.



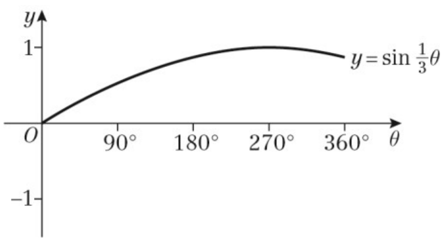
The graph:
meets the θ-axis at (90°, 0), (270°, 0)
meets the y-axis at (0°, –1)
has a maximum at (180°, 1)
has minima at (0°, –1) and (360°, –1).

b The graph of $y = \frac{1}{3}\sin\theta$ is the graph of $y = \sin\theta$ stretched by scale factor $\frac{1}{3}$ in the y direction.



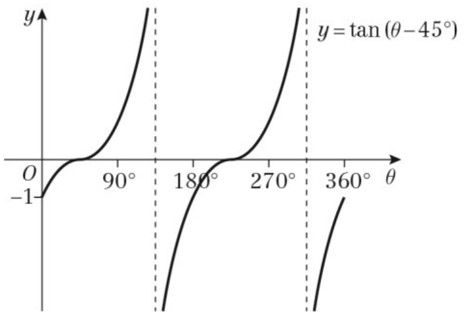
The graph:
meets the θ-axis at (0°, 0), (180°, 0), (360°, 0)
meets the y-axis at (0°, 0)
has a maximum at $\left(90°, \frac{1}{3}\right)$
has a minimum at $\left(270°, -\frac{1}{3}\right)$.

c The graph of $y = \sin\frac{1}{3}\theta$ is the graph of $y = \sin\theta$ stretched by scale factor 3 in the θ direction.



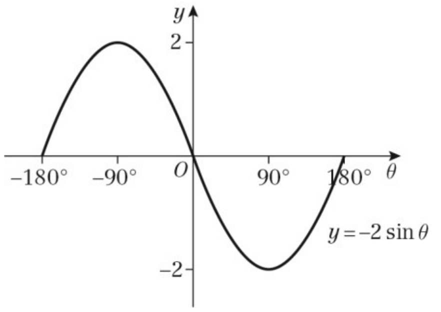
The graph:
only meets the axes at the origin,
has a maximum at (270°, 1).

3 d The graph of $y = \tan(\theta - 45°)$ is the graph of $\tan\theta$ translated by 45° to the right.



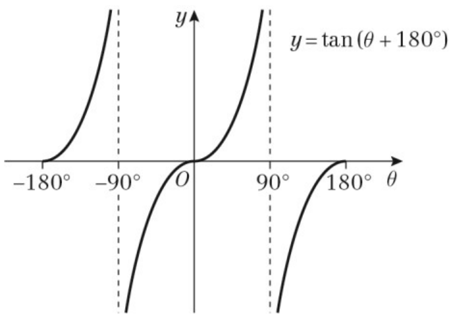
The graph:
meets the θ-axis at (45°, 0), (225°, 0)
meets the y-axis at (0°, –1)
has asymptotes at $\theta = 135°$ and $\theta = 315°$.

4 a This is the graph of $y = \sin\theta$ stretched by scale factor –2 in the y-direction (i.e. reflected in the θ-axis and scaled by 2 in the y-direction).



The graph:
meets the θ-axis at (–180°, 0), (0°, 0), (180°, 0)
has a maximum at (–90°, 2)
has a minimum at (90°, –2).

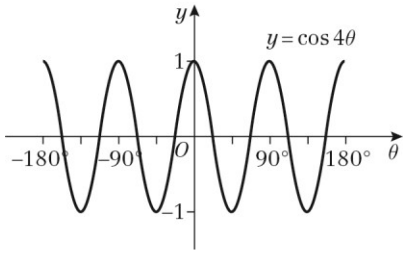
b This is the graph of $y = \tan\theta$ translated by 180° to the left.



As $\tan\theta$ has a period of 180°,
$\tan(\theta + 180)° = \tan\theta$

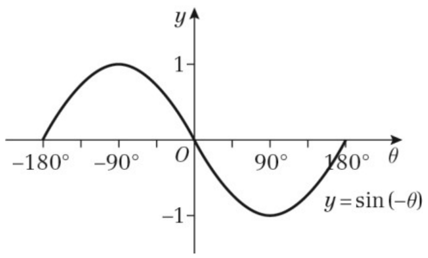
4 b The graph meets the θ-axis at (–180°, 0), (0°, 0), (180°, 0)

c This is the graph of $y = \cos\theta$ stretched by scale factor $\frac{1}{4}$ horizontally.



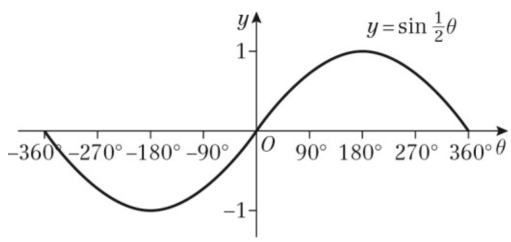
The graph:
meets the θ-axis at $\left(-157\frac{1}{2}°, 0\right)$, $\left(-112\frac{1}{2}°, 0\right)$, $\left(-67\frac{1}{2}°, 0\right)$, $\left(-22\frac{1}{2}°, 0\right)$, $\left(22\frac{1}{2}°, 0\right)$, $\left(67\frac{1}{2}°, 0\right)$, $\left(112\frac{1}{2}°, 0\right)$, $\left(157\frac{1}{2}°, 0\right)$
meets the y-axis at (0°, 1)
has maxima at (–180°, 1), (–90°, 1), (0°, 1), (90°, 1), (180°, 1)
has minima at (–135°, –1), (–45°, –1), (45°, –1), (135°, –1).

d This is the graph of $y = \sin\theta$ reflected in the y-axis.
(This is the same as $y = -\sin\theta$.)

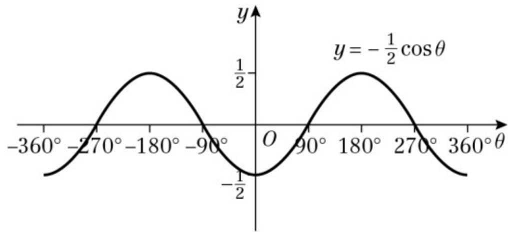


The graph:
meets the θ-axis at (–180°, 0), (0°, 0), (180°, 0)
has a maximum at (–90°, 1)
has a minimum at (90°, –1).

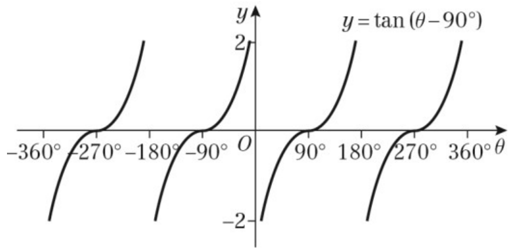
5 a Period = 720°



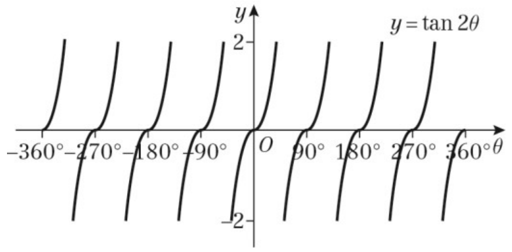
5 b Period = 360°



c Period = 180°

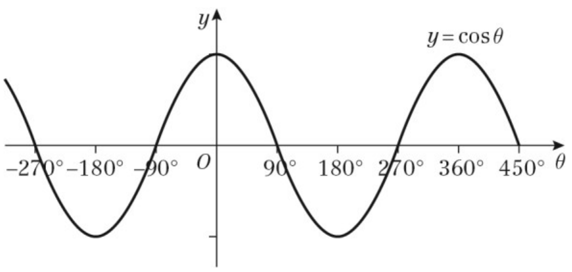


d Period = 90°

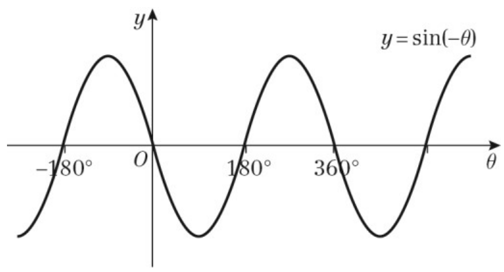


6 a i $y = \cos(-\theta)$ is a reflection of

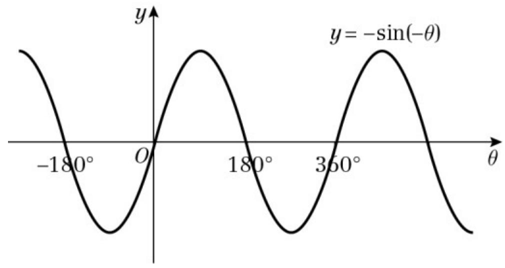
$y = \cos\theta$ in the y-axis, which is the same curve, so $\cos\theta = \cos(-\theta)$.



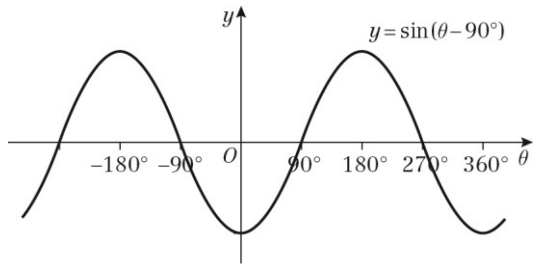
ii $y = \sin(-\theta)$ is a reflection of $y = \sin\theta$ in the y-axis.



6 a ii $y = -\sin(-\theta)$ is a reflection of $y = \sin(-\theta)$ in the θ-axis, which is the graph of $y = \sin\theta$, so $-\sin(-\theta) = \sin\theta$.



iii $y = \sin(\theta - 90°)$ is the graph of $y = \sin\theta$ translated by 90° to the right, which is the graph of $y = -\cos\theta$.
So $\sin(\theta - 90°) = -\cos\theta$.



b Using **a ii**

$$\sin(90° - \theta) = -\sin(-(90° - \theta))$$
$$= -\sin(\theta - 90°)$$

Using **a iii**

$$-\sin(\theta - 90°) = -(-\cos\theta)$$
$$= \cos\theta$$

So $\sin(90° - \theta) = \cos\theta$.

c Using **a i**

$$\cos(90° - \theta) = \cos(\theta - 90°)$$
$$= \sin\theta$$

So $\cos(90° - \theta) = \sin\theta$.

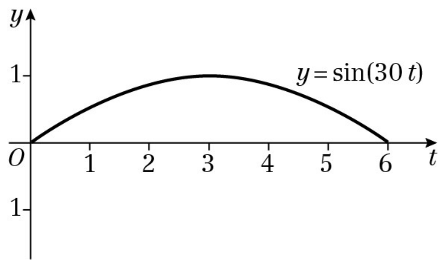
7 a The curve crosses the x-axis at −270° − 30°, −90° − 30°, 90° − 30° and 270° − 30°; θ = −300°, −120°, 60° and 240°.
Coordinates are (−300°, 0), (−120°, 0), (60°, 0) and (240°, 0)

b $\cos 30° = \frac{\sqrt{3}}{2}$; $\left(0, \frac{\sqrt{3}}{2}\right)$

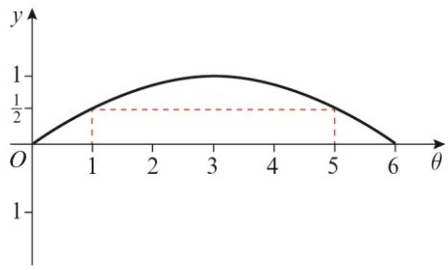
8 **a** The graph is a translation left 60° of the sine graph.
Therefore, $y = \sin(x + 60°)$
$k = 60°$

b Yes, the graph could be a translation right 300°, so $y = \sin(x - 300°)$

9 **a**



9 **b**



Between 1 p.m. and 5 p.m.