Exercise 5F

1 a The gradients of the lines are 4 and $-\frac{1}{4}$.
The product of the gradients is
$4\times-\frac{1}{4}=-1$.
The lines are perpendicular.

b The gradients of the lines are $\frac{2}{3}$ and $\frac{2}{3}$,
i.e. they have the same gradient.
The lines are parallel.

c The gradients of the lines are $\frac{1}{5}$ and 5.
The product of the gradients is $\frac{1}{5}\times5=1$.
The lines are neither perpendicular nor parallel.

d The gradients of the lines are -3 and $\frac{1}{3}$.
The product of the gradients is
$-3\times\frac{1}{3}=-1$
The lines are perpendicular.

e The gradients of the lines are $\frac{3}{5}$ and $-\frac{5}{3}$.
The product of the gradients is
$\frac{3}{5}\times-\frac{5}{3}=-1$.
The lines are perpendicular.

f The gradients of the lines are $\frac{5}{7}$ and $\frac{5}{7}$,
i.e. they have the same gradient.
The lines are parallel.

g The gradient of $y = 5x - 3$ is 5.
$5x-y+4=0$
$5x+4=y$
$y=5x+4$
The gradient of $5x - y + 4 = 0$ is 5.
The lines have the same gradient.
The lines are parallel.

h $5x-y-1=0$
$5x-1=y$
$y=5x-1$
The gradient of $5x - y - 1 = 0$ is 5.
The gradient of $y=-\frac{1}{5}x$ is $-\frac{1}{5}x$.
The product of the gradients is
$5\times-\frac{1}{5}x=-1$.
The lines are perpendicular.

i The gradient of $y=-\frac{3}{2}x+8$ is $-\frac{3}{2}$.
$2x-3y-9=0$
$2x-9=3y$
$3y=2x-9$
$y=\frac{2}{3}x-3$
The gradient of $2x - 3y - 9 = 0$ is $\frac{2}{3}$.
The product of the gradients is
$\frac{2}{3}\times-\frac{3}{2}=-1$.
The lines are perpendicular.

j $4x-5y+1=0$
$4x+1=5y$
$5y=4x+1$
$y=\frac{4}{5}x+\frac{1}{5}$
The gradient of $4x - 5y + 1 = 0$ is $\frac{4}{5}$.
$8x-10y-2=0$
$8x-2=10y$
$10y=8x-2$
$y=\frac{8}{10}x-\frac{2}{10}$
$y=\frac{4}{5}x-\frac{1}{5}$
The gradient of $8x - 10y - 2 = 0$ is $\frac{4}{5}$.
The lines have the same gradient, so they are parallel.

1 k $3x+2y-12=0$

$3x+2y=12$

$2y=-3x+12$

$y=-\frac{3}{2}x+6$

The gradient of $3x + 2y - 12 = 0$ is $-\frac{3}{2}$.

$2x+3y-6=0$

$2x+3y=6$

$3y=-2x+6$

$y=-\frac{2}{3}x+2$

The gradient of $2x + 3y - 6 = 0$ is $-\frac{2}{3}$.

The product of the gradients is $-\frac{3}{2}\times-\frac{2}{3}=1$.

So the lines are neither parallel nor perpendicular.

l $5x-y+2=0$

$5x+2=y$

$y=5x+2$

The gradient of $5x - y + 2 = 0$ is 5.

$2x+10y-4=0$

$2x+10y=4$

$10y=-2x+4$

$y=-\frac{2}{10}x+\frac{4}{10}$

$y=-\frac{1}{5}x+\frac{2}{5}$

The gradient of $2x + 10y - 4 = 0$ is $-\frac{1}{5}$.

The product of the gradients is $5\times-\frac{1}{5}=-1$.

The lines are perpendicular.

2 The gradient of $y = 6x - 9$ is 6.

So the gradient of the perpendicular line is $-\frac{1}{6}$.

The line goes through the point (0, 1).

$y-y_1=m(x-x_1)$

$y-1=-\frac{1}{6}(x-0)$

$y=-\frac{1}{6}x+1$

3 Rearrange $3x + 8y - 11 = 0$:

$8y = -3x + 11$

$y=-\frac{3}{8}x+\frac{11}{8}$, and the gradient is $-\frac{3}{8}$.

So the gradient of the perpendicular line is $\frac{8}{3}$.

The line goes through the point (0, −8).

$y-y_1=m(x-x_1)$

$y-(-8)=\frac{8}{3}(x-0)$

$y=\frac{8}{3}x-8$

4 The gradient of $y = 3x + 5$ is 3.

The gradient of a line perpendicular to $y = 3x + 5$ is $-\frac{1}{3}$.

The line goes through the point (6, −2).

$y-y_1=m(x-x_1)$

$y-(-2)=-\frac{1}{3}(x-6)$

$y+2=-\frac{1}{3}x+2$

$y=-\frac{1}{3}x$

The equation of the line is $y=-\frac{1}{3}x$.

5 The gradient of a line perpendicular to $y = 3x + 6$ is $-\frac{1}{3}$.

The line goes through the point (−2, 5).

$y-y_1=m(x-x_1)$

$y-5=-\frac{1}{3}(x-(-2))$

$y-5=-\frac{1}{3}(x+2)$

$y-5=-\frac{1}{3}x-\frac{2}{3}$

$y=-\frac{1}{3}x+\frac{13}{3}$

6 The gradient of the line $4x - 6y + 7 = 0$ is $\frac{2}{3}$.
The gradient of a line perpendicular to $4x - 6y + 7 = 0$ is $-\frac{3}{2}$.
The line goes through the point (3, 4).

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\tfrac{3}{2}(x - 3)$$
$$y - 4 = -\tfrac{3}{2}x + \tfrac{9}{2}$$
$$y = -\tfrac{3}{2}x + \tfrac{17}{2}$$

7 The gradient of a line perpendicular to $y = \frac{2}{3}x + 5$ is $-\dfrac{1}{\frac{2}{3}} = -\dfrac{3}{2}$.
The line goes through the point (5, −5).

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = -\tfrac{3}{2}(x - 5)$$
$$y + 5 = -\tfrac{3}{2}(x - 5)$$

Multiply each term by 2:

$$2y + 10 = -3(x - 5)$$
$$2y + 10 = -3x + 15$$
$$3x + 2y + 10 = 15$$
$$3x + 2y - 5 = 0$$

The equation of the line is $3x + 2y - 5 = 0$.

8 The gradient of a line perpendicular to $y = -\frac{4}{7}x + 5$ is $-\dfrac{1}{-\frac{4}{7}} = \dfrac{7}{4}$.
The line goes through the point (−2, −3).

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = \tfrac{7}{4}(x - (-2))$$
$$y + 3 = \tfrac{7}{4}(x + 2)$$

Multiply each term by 4:

$$4y + 12 = 7(x + 2)$$
$$4y + 12 = 7x + 14$$
$$4y = 7x + 2$$
$$0 = 7x + 2 - 4y$$
$$7x - 4y + 2 = 0$$

The equation of the line is $7x - 4y + 2 = 0$.

9 $(x_1, y_1) = (-3, 0)$, $(x_2, y_2) = (3, -2)$
The gradient of l is:

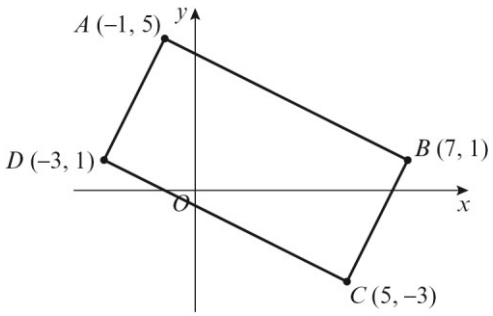
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{3 - (-3)}$$
$$= -\frac{1}{3}$$

$(x_1, y_1) = (1, 8)$, $(x_2, y_2) = (-1, 2)$
The gradient of n is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{-1 - 1}$$
$$= \frac{-6}{-2}$$
$$= 3$$

The product of the gradients is $-\frac{1}{3} \times 3 = -1$
The lines are perpendicular.

10



The gradient of AB is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-1-7}$$
$$= \frac{4}{-8}$$
$$= -\frac{1}{2}$$

The gradient of DC is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-1}{5-(-3)}$$
$$= \frac{-4}{8}$$
$$= -\frac{1}{2}$$

The gradient of AB is the same as the gradient of DC, so the lines are parallel.

The gradient of AD is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-1-(-3)}$$
$$= -\frac{4}{-1+3}$$
$$= \frac{4}{2}$$
$$= 2$$

The gradient of BC is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-1}{5-7}$$
$$= \frac{-4}{-2}$$
$$= 2$$

The gradient of AD is the same as the gradient of BC, so the lines are parallel. The line AD is perpendicular to the line AB, since $2 \times -\frac{1}{2} = -1$.

So $ABCD$ is a rectangle.

11 a The line l_1, $5x + 11y - 7 = 0$, crosses the x-axis when $y = 0$, so:

$5x + 11(0) - 7 = 0$

$x = \frac{7}{5}$

The point A is $(\frac{7}{5}, 0)$.

b Rearranging $5x + 11y - 7 = 0$ to find the gradient gives:

$11y = -5x + 7$

$y = -\frac{5}{11}x + \frac{7}{11}$

The gradient is $-\frac{5}{11}$.

The gradient of the perpendicular line is $\frac{11}{5}$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \tfrac{11}{5}(x - \tfrac{7}{5})$$
$$y = \tfrac{11}{5}x - \tfrac{77}{25}$$

l_2: $55x - 25y - 77 = 0$

12 The gradient of line AB is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{-3-0}$$

$$= \frac{-4}{-3}$$

$$= \frac{4}{3}$$

The gradient of BC is $-\frac{3}{4}$.

The gradient of the line BC is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{c-0}{0-(-3)}$$

$$= \frac{c}{3}$$

$$\frac{c}{3} = -\frac{3}{4}$$

$$c = -\frac{9}{4}$$