Exercise 5D

1 $y = 4x - 8$
Substitute $y = 0$:
$4x - 8 = 0$
$4x = 8$
$x = 2$
So A has coordinates (2, 0).
For a line through A with gradient 3:
$y - y_1 = m(x - x_1)$
$y - 0 = 3(x - 2)$
$y = 3x - 6$
The equation of the line is $y = 3x - 6$.

2 $y = -2x + 8$
Substitute $x = 0$:
$y = -2(0) + 8$
$y = 8$
So B has coordinates (0, 8).

We can substitute $m = 2$ and y-intercept 8 into $y = mx + c$.
Or we can calculate using the formula.
For a line through B with gradient 2:
$y - y_1 = m(x - x_1)$
$y - 8 = 2(x - 0)$
$y - 8 = 2x$
$y = 2x + 8$
The equation of the line is $y = 2x + 8$.

3 To find where the line $y = \frac{1}{2}x + 6$ crosses the x-axis, substitute $y = 0$:
$\frac{1}{2}x + 6 = 0$
$\frac{1}{2}x = -6$
$x = -12$
So C has coordinates (−12, 0).
$y - y_1 = m(x - x_1)$
$y - 0 = \frac{2}{3}(x - (-12))$
$y = \frac{2}{3}(x + 12)$
$y = \frac{2}{3}x + 8$
Multiply each term by 3:
$3y = 2x + 24$
$0 = 2x + 24 - 3y$

3 $2x - 3y + 24 = 0$
The equation of the line is $2x - 3y + 24 = 0$.

4 To find where the line $y = \frac{1}{4}x + 2$ crosses the y-axis, substitute $x = 0$:
$y = \frac{1}{4}(0) + 2$
$y = 2$
So B has coordinates (0, 2).
C has coordinates (−5, 3).
To find the gradient of BC:
$$\text{The gradient } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{-5-0} = -\frac{1}{5}$$
The gradient of the line joining B and C is $-\frac{1}{5}$.

5 To find the equation of the line passing through (2, −5) and (−7, 4):
$$\text{The gradient } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-5)}{-7-2} = -1$$
The equation is $y - y_1 = m(x - x_1)$
$y - (-5) = -1(x - 2)$
$y + 5 = -x + 2$
$y = -x - 3$
Substitute $y = 0$:
$0 = -x - 3$
$x = -3$
The line meets the x-axis at $P(-3, 0)$.

6 To find the equation of the line passing through (−3, −5) and (4, 9):

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{9-(-5)}{4-(-3)}$

$= \dfrac{14}{7}$

$= 2$

The equation is $y - y_1 = m(x - x_1)$

$y - (-5) = 2(x - (-3))$

$y + 5 = 2(x + 3)$

$y + 5 = 2x + 6$

$y = 2x + 1$

For point G, substitute $x = 0$:

$y = 2(0) + 1 = 1$

The coordinates of G are (0, 1).

7 To find the equation of the line passing through $(3, 2\frac{1}{2})$ and $(-1\frac{1}{2}, 4)$:

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{4-2\frac{1}{2}}{-1\frac{1}{2}-3}$

$= \dfrac{1\frac{1}{2}}{-4\frac{1}{2}}$

$= -\dfrac{1}{3}$

The equation is $y - y_1 = m(x - x_1)$

$y - 2\frac{1}{2} = -\frac{1}{3}(x - 3)$

Multiply through by 6.

$6y - 15 = -2(x - 3)$

$6y - 15 = -2x + 6$

$6y = -2x + 21$

Substitute $x = 0$:

$6y = -2(0) + 21$

The coordinates of J are $(0, \frac{7}{2})$.

8 Substitute $y = x$ in the equation $y = 2x - 5$:

$x = 2x - 5$

$0 = x - 5$

$x = 5$

Substitute $x = 5$ in the equation $y = x$:

$y = 5$

The coordinates of A are (5, 5).

To find the equation of the line through A, with gradient $\frac{2}{5}$:

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{2}{5}(x - 5)$

$y - 5 = \frac{2}{5}x - 2$

$y = \frac{2}{5}x + 3$

The equation of the line is $y = \frac{2}{5}x + 3$.

9 Substitute $y = x - 1$ in the equation $y = 4x - 10$:

$x - 1 = 4x - 10$

$-1 = 3x - 10$

$9 = 3x$

$x = 3$

Now substitute $x = 3$ into the equation $y = x - 1$:

$y = 3 - 1$

$y = 2$

The coordinates of T are (3, 2).

To find the equation of the line through T with gradient $-\frac{2}{3}$:

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{2}{3}(x - 3)$

$y - 2 = -\frac{2}{3}x + 2$

$\frac{2}{3}x + y - 2 = 2$

$\frac{2}{3}x + y - 4 = 0$

$2x + 3y - 12 = 0$

The equation of the line is $2x + 3y - 12 = 0$.

10 The equation of the line p is:

$y - (-12) = \frac{2}{3}(x - 6)$

$y + 12 = \frac{2}{3}x - 4$

$y = \frac{2}{3}x - 16$

The equation of the line q is:

$y - 5 = -1(x - 5)$

$y - 5 = -x + 5$

$y = -x + 10$

For the coordinates of A, substitute $x = 0$ into the equation $y = \frac{2}{3}x - 16$.

$y = \frac{2}{3}(0) - 16$

$y = -16$

The required coordinates are $A(0, -16)$.

For the coordinates of B, substitute $y = 0$ into the equation $y = -x + 10$.

$0 = -x + 10$

$x = 10$

The required coordinates are $B(10, 0)$.

The gradient of AB is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-16)}{10-0} = \frac{16}{10} = \frac{8}{5}$$

The gradient of the line joining A and B is $\frac{8}{5}$.

11 To find where the line $y = -2x + 6$ crosses the x-axis, substitute $y = 0$:

$0 = -2x + 6$

$2x = 6$

$x = 3$

The point P has coordinates $(3, 0)$.

$y = \frac{3}{2}x - 4$

To find where the line crosses the y-axis, substitute $x = 0$:

$y = \frac{3}{2}(0) - 4$

$y = -4$

The point Q has coordinates $(0, -4)$.

11 The gradient of PQ is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-4)}{3-0} = \frac{4}{3}$$

The equation of PQ is:

$y - y_1 = m(x - x_1)$

Substitute $(3, 0)$:

$y - 0 = \frac{4}{3}(x - 3)$

$y = \frac{4}{3}x - 4$

The equation of the line through P and Q is $y = \frac{4}{3}x - 4$.

12 To find where the line $y = 3x - 5$ crosses the x-axis, substitute $y = 0$:

$3x - 5 = 0$

$3x = 5$

$x = \frac{5}{3}$

M has coordinates $(\frac{5}{3}, 0)$.

$y = -\frac{2}{3}x + \frac{2}{3}$

Substitute $x = 0$:

$y = -\frac{2}{3}(0) + \frac{2}{3}$

$y = \frac{2}{3}$

N has coordinates $(0, \frac{2}{3})$.

The gradient of MN is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-\frac{2}{3}}{\frac{5}{3}-0} = \frac{-\frac{2}{3}}{\frac{5}{3}} = -\frac{2}{5}$$

The equation of MN is:

$y - y_1 = m(x - x_1)$

Substitute $(\frac{5}{3}, 0)$:

$y - 0 = -\frac{2}{5}\left(x - \frac{5}{3}\right)$

$y = -\frac{2}{5}x + \frac{2}{3}$

Multiply each term by 15:

$15y = -6x + 10$

$6x + 15y = 10$

$6x + 15y - 10 = 0$

13 To find where the line $y = 2x - 10$ crosses the x-axis, substitute $y = 0$:
$2x - 10 = 0$
$x = 5$
The coordinates of A are (5, 0).
Substitute $x = 0$ into $y = -2x + 4$:
$y = -2(0) + 4 = 4$
The coordinates of B are (0, 4).
To find the equation of AB:

$$\text{The gradient } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-5} = -\frac{4}{5}$$

The equation is $y - y_1 = m(x - x_1)$
$y - 0 = -\frac{4}{5}(x - 5)$
Multiply through by 5.
$5y = -4(x - 5)$
$y = -\frac{4}{5}x + 4$

14 To find where the line $y = 4x + 5$ crosses the y-axis, substitute $x = 0$:
$y = 4(0) + 5 = 5$
The coordinates of C are (0, 5).
Substitute $y = 0$ in the equation $y = -3x - 15$:
$0 = -3x - 15$
$3x = -15$
$x = -5$
The coordinates of D are (−5, 0).
To find the equation of CD:

$$\text{The gradient } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{-5-0} = 1$$

The equation is $y - y_1 = m(x - x_1)$
$y - 5 = 1(x - 0)$
$y = x + 5$
$x - y + 5 = 0$

15 $y = 3x - 13$
$y = x - 5$
So $3x - 13 = x - 5$
$3x = x + 8$
$2x = 8$
$x = 4$
When $x = 4$, $y = 4 - 5 = -1$
The coordinates of S are (4, −1).
To find the equation of ST:

$$\text{The gradient } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-1)}{-4-4} = -\frac{3}{8}$$

The equation is $y - y_1 = m(x - x_1)$
$y - (-1) = -\frac{3}{8}(x - 4)$
Multiply through by 8.
$8y + 8 = -3(x - 4)$
$8y + 8 = -3x + 12$
$8y = -3x + 4$
$y = -\frac{3}{8}x + \frac{1}{2}$

16 $y = x + 7$
$y = -2x + 1$
So $x + 7 = -2x + 1$
$3x + 7 = 1$
$3x = -6$
$x = -2$
When $x = -2$, $y = (-2) + 7 = 5$
The coordinates of L are (−2, 5).
To find the equation of LM:

$$\text{The gradient } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-5}{-3-(-2)} = 4$$

The equation is $y - y_1 = m(x - x_1)$
$M = (-3, 1)$
$y - 1 = 4(x - (-3))$
$y - 1 = 4(x + 3)$
$y - 1 = 4x + 12$
$y = 4x + 13$