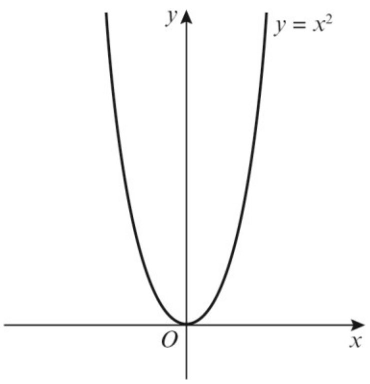
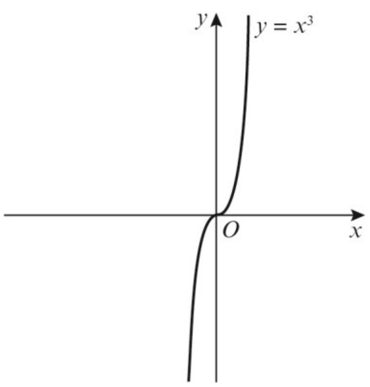
Exercise 4D

1 Sketches of original graphs:

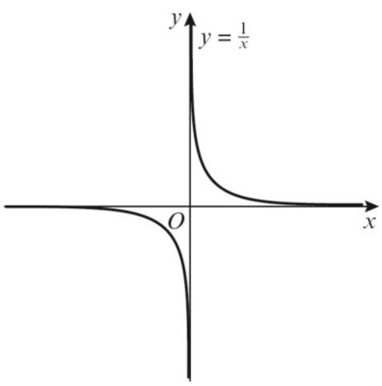
$f(x) = x^2$



$f(x) = x^3$

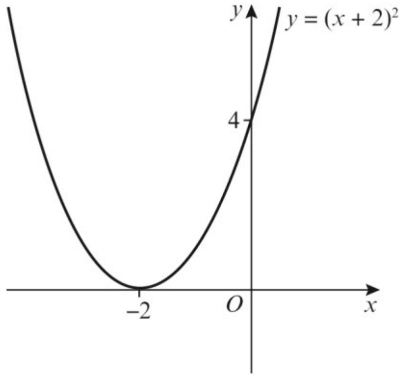


$f(x) = \dfrac{1}{x}$



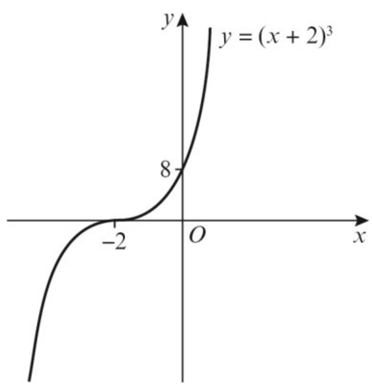
a $f(x + 2)$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, or two units to the left.

a i $f(x) = x^2$, $f(x + 2) = (x + 2)^2$



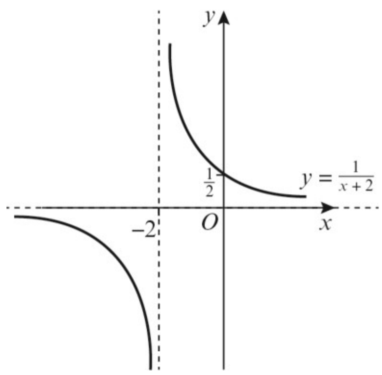
The curve touches the x-axis at $(-2, 0)$ and crosses the y-axis at $(0, 4)$.

ii $f(x) = x^3$, $f(x + 2) = (x + 2)^3$



The curve crosses the x-axis at $(-2, 0)$ and crosses the y-axis at $(0, 8)$.

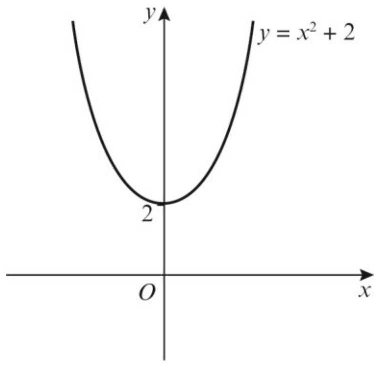
iii $f(x) = \dfrac{1}{x}$, $f(x + 2) = \dfrac{1}{x+2}$



The curve crosses the y-axis at $(0, \frac{1}{2})$.
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = -2$.

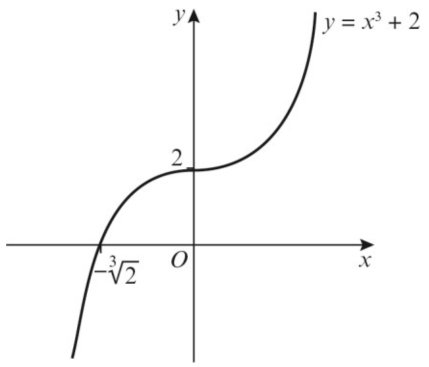
1 b f(x) + 2 is a translation of the graph of f(x) by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, or two units up.

i $f(x) = x^2$, $f(x) + 2 = x^2 + 2$



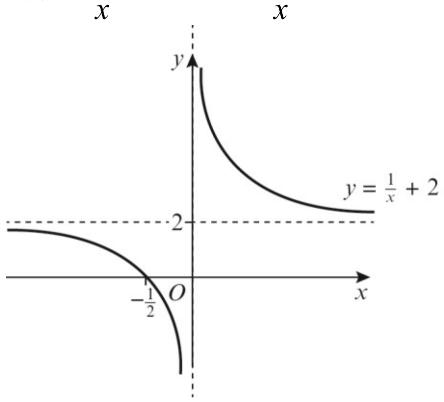
The curve crosses the y-axis at (0, 2).

ii $f(x) = x^3$, $f(x) + 2 = x^3 + 2$



The curve crosses the x-axis at $\left(-\sqrt[3]{2}, 0\right)$ and crosses the y-axis at (0, 2).

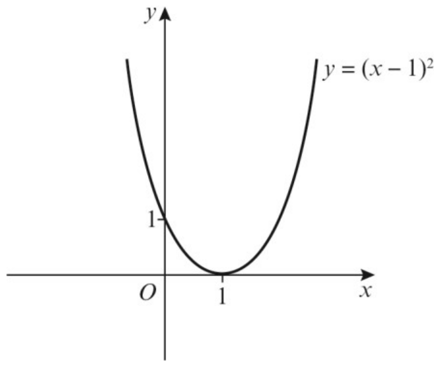
iii $f(x) = \dfrac{1}{x}$, $f(x) + 2 = \dfrac{1}{x} + 2$



The horizontal asymptote is $y = 2$.
The vertical asymptote is $x = 0$.

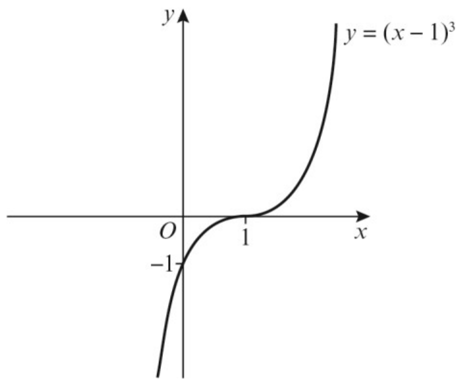
c f(x − 1) is a translation of the graph of f(x) by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, or one unit to the right.

i $f(x) = x^2$, $f(x - 1) = (x - 1)^2$



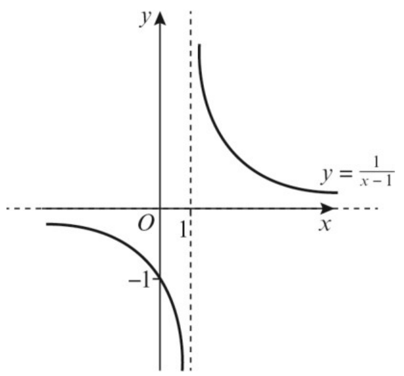
The curve touches the x-axis at (1, 0) and crosses the y-axis at (0, 1).

ii $f(x) = x^3$, $f(x - 1) = (x - 1)^3$



The curve crosses the x-axis at (1, 0) and crosses the y-axis at (0, −1).

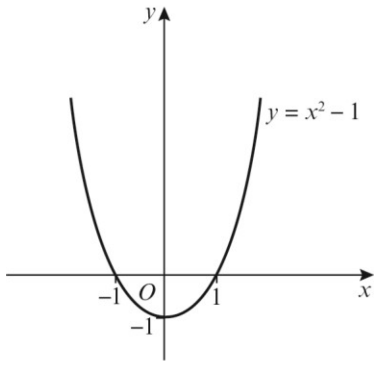
iii $f(x) = \dfrac{1}{x}$, $f(x - 1) = \dfrac{1}{x-1}$



The curve crosses the y-axis at (0, −1).
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = 1$.

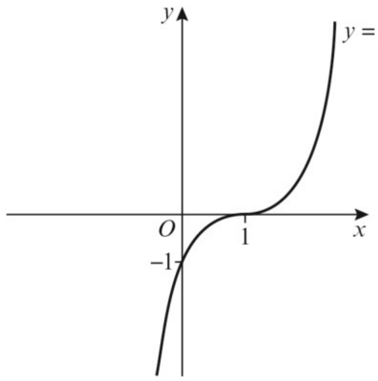
1 d f(x) − 1 is a translation of the graph of f(x) by $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, or one unit down.

i $f(x) = x^2$, $f(x) - 1 = x^2 - 1$



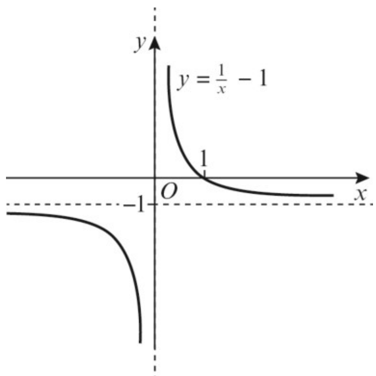
The curve crosses the x-axis at (−1, 0) and (1, 0) and crosses the y-axis at (0, −1).

ii $f(x) = x^3$, $f(x) - 1 = x^3 - 1$



The curve crosses the x-axis at (1, 0) and crosses the y-axis at (0, −1).

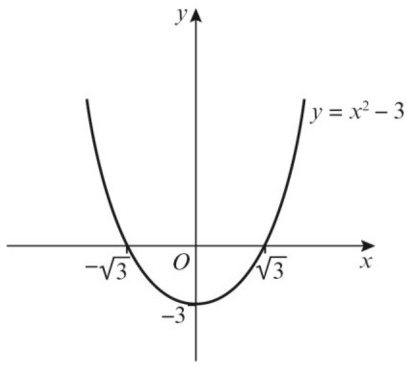
iii $f(x) = \dfrac{1}{x}$, $f(x) - 1 = \dfrac{1}{x} - 1$



1 d iii The curve crosses the x-axis at (1, 0).
The horizontal asymptote is $y = -1$.
The vertical asymptote is $x = 0$.

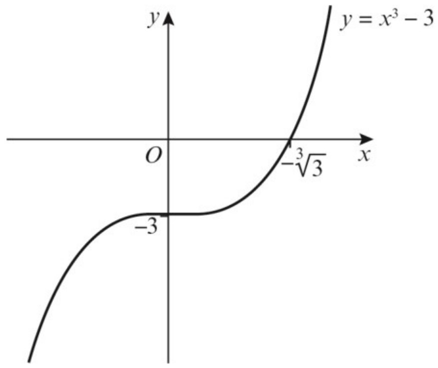
e f(x) − 3 is a translation of the graph of f(x) by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$, or three units down.

i $f(x) = x^2$, $f(x) - 3 = x^2 - 3$



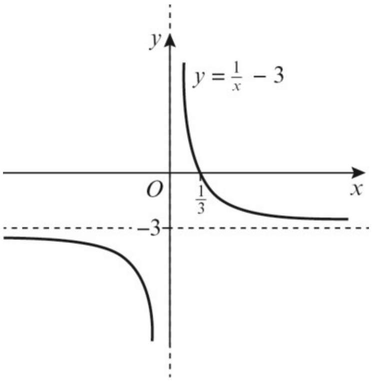
The curve crosses the x-axis at $\left(-\sqrt{3}, 0\right)$ and $\left(\sqrt{3}, 0\right)$ and crosses the y-axis at (0, −3).

ii $f(x) = x^3$, $f(x) - 3 = x^3 - 3$



The curve crosses the x-axis at $\left(-\sqrt[3]{3}, 0\right)$ and crosses the y-axis at (0, −3).

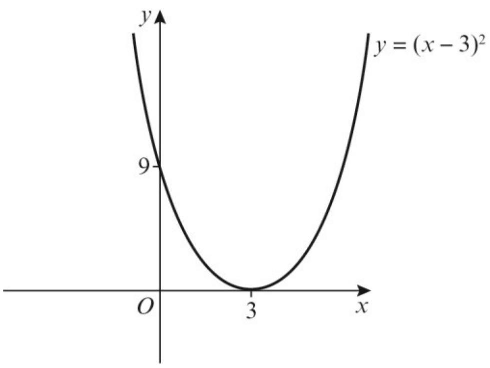
1 e iii $f(x) = \dfrac{1}{x}$, $f(x) - 3 = \dfrac{1}{x} - 3$



The curve crosses the x-axis at $\left(\frac{1}{3}, 0\right)$.
The horizontal asymptote is $y = -3$.
The vertical asymptote is $x = 0$.

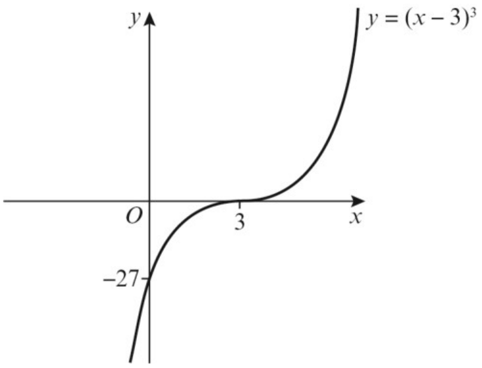
f $f(x - 3)$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$, or three units to the right.

i $f(x) = x^2$, $f(x - 3) = (x - 3)^2$



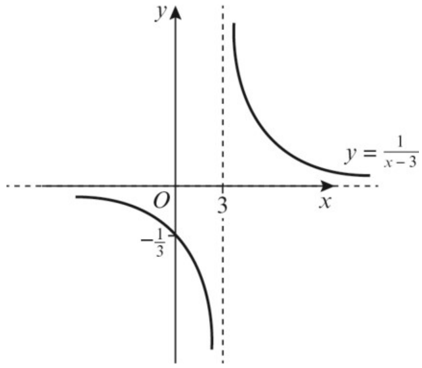
The curve touches the x-axis at (3, 0) and crosses the y-axis at (0, 9).

ii $f(x) = x^3$, $f(x - 3) = (x - 3)^3$



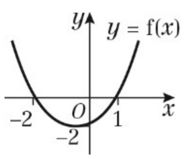
f ii The curve crosses the x-axis at $\left(\sqrt[3]{2}, 0\right)$ and crosses the y-axis at $(0, -27)$.

iii $f(x) = \dfrac{1}{x}$, $f(x - 3) = \dfrac{1}{x - 3}$

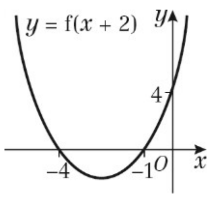


The curve crosses the y-axis at $\left(0, -\frac{1}{3}\right)$.
The horizontal asymptote is $y = 0$.
The vertical asymptote is $x = 3$.

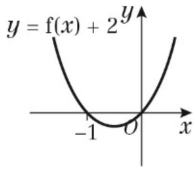
2 a $y = (x - 1)(x + 2)$
As $a = 1$ is positive, the graph has a $\bigcup$ shape and a minimum point.
$0 = (x - 1)(x + 2)$
So $x = 1$ or $x = -2$
The curve crosses the x-axis at (1, 0) and (−2, 0).
When $x = 0$, $y = (-1) \times 2 = -2$
The curve crosses the y-axis at (0, −2).



b i $f(x + 2)$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, or two units to the left.



2 b ii $f(x) + 2$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, or two units up.

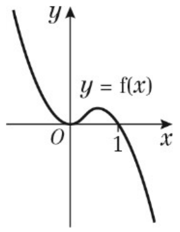


Since the axis of symmetry of $f(x)$ is at $x = -\frac{1}{2}$, the same axis of symmetry applies to $f(x) + 2$.
Since one root is at $x = 0$, the other must be symmetric at $x = -1$.

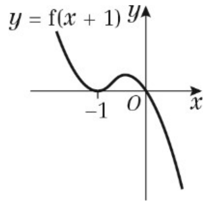
c i $y = f(x + 2)$ is
$y = (x + 2 - 1)(x + 2 + 2)$
$= (x + 1)(x + 4)$
When $x = 0$, $y = 4$

ii $y = f(x) + 2$ is
$y = (x - 1)(x + 2) + 2$
$= x^2 + x - 2 + 2$
$= x^2 + x$
When $x = 0$, $y = 0$

3 a $y = x^2(1 - x)$
$0 = x^2(1 - x)$
So $x = 0$ or $x = 1$
The curve crosses the x-axis at $(1, 0)$ and touches it at $(0, 0)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$

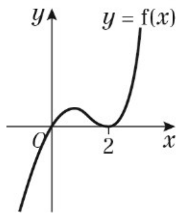


3 b $f(x + 1)$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, or one unit to the left.

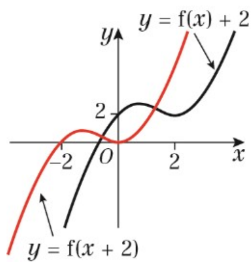


c $f(x + 1) = (x + 1)^2(1 - (x + 1))$
$= -x(x + 1)^2$
When $x = 0$, $y = 0$; the curve passes through $(0, 0)$.

4 a $y = x(x - 2)^2$
$0 = x(x - 2)^2$
So $x = 0$ or $x = 2$
The curve crosses the x-axis at $(0, 0)$ and touches it at $(2, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



b $f(x) + 2$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, or two units up.
$f(x + 2)$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, or two units to the left.



c $f(x + 2) = (x + 2)((x + 2) - 2)^2$
$= (x + 2)x^2$
$(x + 2)(x)^2 = 0$
So $x = 0$ and $x = -2$
The graph crosses the axes at $(0, 0)$ and $(-2, 0)$.

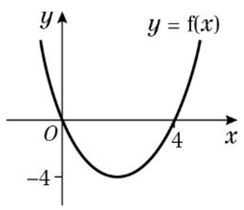
5 a $y = x(x - 4)$

As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

$0 = x(x - 4)$

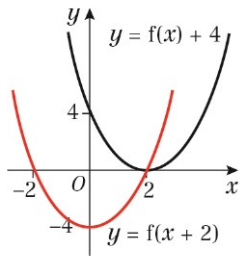
So $x = 0$ or $x = 4$

The curve crosses the x-axis at $(0, 0)$ and $(4, 0)$.



b $f(x + 2)$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, or two units to the left.

$f(x) + 4$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$, or four units up.



c $f(x + 2) = (x + 2)((x + 2) - 4)$
$= (x + 2)(x - 2)$

$0 = (x + 2)(x - 2)$

So $x = -2$ or $x = 2$

When $x = 0$, $y = 2 \times (-2) = -4$

So $f(x + 2)$ crosses the x-axis at $(-2, 0)$ and $(2, 0)$ and the y-axis at $(0, -4)$.

$f(x) + 4 = x(x - 4) + 4$
$= x^2 - 4x + 4$
$= (x - 2)^2$

$0 = (x - 2)^2$

So $x = 2$

When $x = 0$, $y = (-2)^2 = 4$

So $f(x) + 4$ touches the x-axis at $(2, 0)$ and crosses the y-axis at $(0, 4)$.

6 a $y = f(x - 2)$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, or two units to the right.

So P translates to $(6, -1)$.

b $y = f(x) + 3$ is a translation of the graph of $f(x)$ by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$, or three units up.

So P translates to $(4, 2)$.

7 $y = f(x)$ has asymptotes at $x = 0$ and $y = 0$. Asymptotes after the translation are at $x = 4$ and $y = 0$, therefore the graph has been translated four units to the right.

$f(x) = \frac{1}{x}$, $f(x - 4) = \frac{1}{x - 4}$

$y = \frac{1}{x - 4}$

8 a $y = x^3 - 5x^2 + 6x$
$= x(x^2 - 5x + 6)$
$= x(x - 2)(x - 3)$

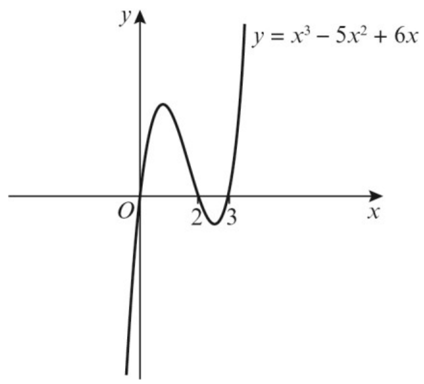
$0 = x(x - 2)(x - 3)$

So $x = 0$, $x = 2$ or $x = 3$

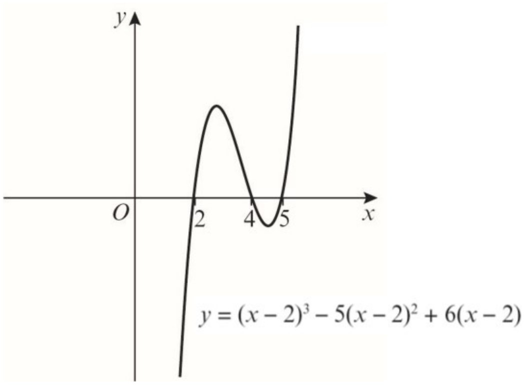
The curve crosses the x-axis at $(0, 0)$, $(2, 0)$ and $(3, 0)$.

$x \to \infty$, $y \to \infty$

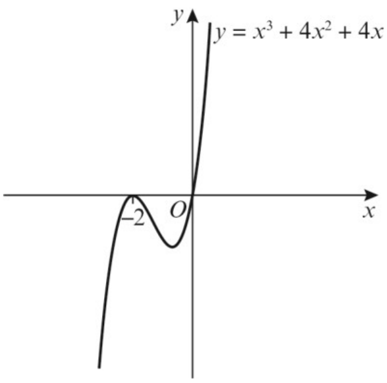
$x \to -\infty$, $y \to -\infty$



8 b Let $f(x) = x^3 - 5x^2 + 6x$
$(x-2)^3 - 5(x-2)^2 + 6(x-2)$ is $f(x-2)$, which is a translation of two units to the right.



9 a $y = x^3 + 4x^2 + 4x$
$= x(x^2 + 4x + 4)$
$= x(x+2)^2$
So $x = 0$ or $x = -2$
The curve crosses the x-axis at (0, 0) and touches it at (−2, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



b $y = (x+a)^3 + 4(x+a)^2 + 4(x+a)$
$y = x^3 + 4x^2 + 4x$ crosses the x-axis at (0, 0) and (−2, 0).
So for the point (−1, 0) to lie on the curve, the graph must be translated either one unit to the left or one unit to the right.
$a = -1$ or $a = 1$

Challenge

1 a $y = f(x+2) - 5$ is a translation by $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$,
or two units to the left and five units down.
So the point $Q(-5, -7)$ is transformed to the point $(-7, -12)$.

b The coordinates of the point $Q(-5, -7)$ are transformed to the point $(-3, -6)$.
This is a translation of two units to the right and one unit up.
So $y = f(x-2) + 1$