Exercise 2G

1 a i $f(x) = x^2 + 8x + 3$
$b^2 - 4ac = 8^2 - 4(1)(3)$
$= 64 - 12$
$= 52$

ii $g(x) = 2x^2 - 3x + 4$
$b^2 - 4ac = (-3)^2 - 4(2)(4)$
$= 9 - 32$
$= -23$

iii $h(x) = -x^2 + 7x - 3$
$b^2 - 4ac = 7^2 - 4(-1)(-3)$
$= 49 - 12$
$= 37$

iv $j(x) = x^2 - 8x + 16$
$b^2 - 4ac = (-8)^2 - 4(1)(16)$
$= 64 - 64$
$= 0$

v $k(x) = 2x - 3x^2 - 4$
$= -3x^2 + 2x - 4$
$b^2 - 4ac = (2)^2 - 4(-3)(-4)$
$= 4 - 48$
$= -44$

b i This graph has two distinct real roots and has a maximum, so $a < 0$: h(x).

ii This graph has two distinct real roots and has a minimum, so $a > 0$: f(x).

iii This graph has no real roots and has a maximum, so $a < 0$: k(x).

iv This graph has one repeated root and has a minimum, so $a > 0$: j(x).

v This graph has no real roots and has a minimum, so $a > 0$: g(x).

2 $x^2 + 6x + k = 0$
$a = 1$, $b = 6$ and $c = k$
For two real solutions, $b^2 - 4ac > 0$
$6^2 - 4 \times 1 \times k > 0$
$36 - 4k > 0$
$36 > 4k$
$9 > k$
So $k < 9$

3 $2x^2 - 3x + t = 0$
$a = 2$, $b = -3$ and $c = t$
For exactly one solution, $b^2 - 4ac = 0$
$(-3)^2 - 4 \times 2 \times t = 0$
$9 - 8t = 0$
So $t = \frac{9}{8}$

4 $f(x) = sx^2 + 8x + s$
$a = s$, $b = 8$ and $c = s$
For equal roots, $b^2 - 4ac = 0$
$8^2 - 4 \times s \times s = 0$
$64 - 4s^2 = 0$
$64 = 4s^2$
$16 = s^2$
So $s = \pm 4$
The positive solution is $s = 4$.

5 $3x^2 - 4x + k = 0$
$a = 3$, $b = -4$ and $c = k$
For no real solutions, $b^2 - 4ac < 0$
$(-4)^2 - 4 \times 3 \times k < 0$
$16 - 12k < 0$
$16 < 12k$
$4 < 3k$
So $k > \frac{4}{3}$

6 a $g(x) = x^2 + 3px + (14p - 3)$
$a = 1$, $b = 3p$ and $c = (14p - 3)$
For two equal roots, $b^2 - 4ac = 0$
$(3p)^2 - 4 \times 1 \times (14p - 3) = 0$
$9p^2 - 56p + 12 = 0$
$(p - 6)(9p - 2) = 0$
$p = 6$ or $p = \frac{2}{9}$
p is an integer, so $p = 6$

6 b When $p = 6$,
$x^2 + 3px + (14p - 3)$
$= x^2 + 3(6)x + (14(6) - 3)$
$= x^2 + 18x + 81$
$x^2 + 18x + 81 = 0$
$(x + 9)(x + 9) = 0$
So $x = -9$

7 a $h(x) = 2x^2 + (k + 4)x + k$
$a = 2$, $b = (k + 4)$ and $c = k$
$b^2 - 4ac = (k + 4)^2 - 4 \times 2 \times k$
$= k^2 + 8k + 16 - 8k = k^2 + 16$

b $k^2 \geqslant 0$, therefore $k^2 + 16$ is also > 0.
If $b^2 - 4ac > 0$, then $h(x)$ has two distinct real roots.

Challenge

a For distinct real roots, $b^2 - 4ac > 0$.
Therefore $b^2 > 4ac$
If $a > 0$ and $c > 0$, or $a < 0$ and $c < 0$, choose b such that $b > \sqrt{4ac}$.
If $a > 0$ and $c < 0$, or $a < 0$ and $c > 0$, $4ac < 0$, therefore $4ac < b^2$ for all b.

b For equal roots, $b^2 - 4ac = 0$.
Therefore $b^2 = 4ac$.
If $4ac < 0$, then there is no value for b to satisfy $b^2 = 4ac$ as b^2 is always positive.