Exercise 2E

1 a $f(1) = 5(1) + 3$
$= 5 + 3$
$= 8$

b $g(3) = 3^2 - 2$
$= 9 - 2$
$= 7$

c $h(8) = \sqrt{8+1}$
$= \sqrt{9}$
$= 3$

d $f(1.5) = 5(1.5) + 3$
$= 7.5 + 3$
$= 10.5$

e $g\left(\sqrt{2}\right) = \left(\sqrt{2}\right)^2 - 2$
$= 2 - 2$
$= 0$

f $h(-1) = \sqrt{-1+1}$
$= 0$

g $f(4) + g(2) = 5(4) + 3 + 2^2 - 2$
$= 20 + 3 + 4 - 2$
$= 25$

h $f(0) + g(0) + h(0) = 5(0) + 3 + 0^2 - 2 + \sqrt{0+1}$
$= 0 + 3 + 0 - 2 + 1$
$= 2$

i $\dfrac{g(4)}{h(3)} = \dfrac{4^2 - 2}{\sqrt{3+1}}$
$= \dfrac{16-2}{\sqrt{4}}$
$= \dfrac{14}{2}$
$= 7$

2 $f(a) = a^2 - 2a = 8$
$a^2 - 2a - 8 = 0$
$(a - 4)(a + 2) = 0$
So $a = 4$ or $a = -2$

3 a $f(x) = 0$
$10 - 15x = 0$
$5(2 - 3x) = 0$
$x = \frac{2}{3}$
The root of $f(x)$ is $\frac{2}{3}$.

b $g(x) = 0$
$(x + 9)(x - 2) = 0$
$x = -9$ or $x = 2$
The roots of $g(x)$ are -9 and 2.

c $h(x) = 0$
$x^2 + 6x - 40 = 0$
$(x + 10)(x - 4) = 0$
$x = -10$ or $x = 4$
The roots of $h(x)$ are -10 and 4.

d $j(x) = 0$
$144 - x^2 = 0$
$(12 + x)(12 - x) = 0$
$x = -12$ or 12
The roots of $j(x)$ are 12 and -12.

e $k(x) = 0$
$x(x + 5)(x + 7) = 0$
$x = 0, x = -5$ or $x = -7$
The roots of $k(x)$ are 0, -5 and -7.

f $m(x) = 0$
$x^3 + 5x^2 - 24x = 0$
$x(x^2 + 5x - 24) = 0$
$x(x + 8)(x - 3) = 0$
$x = 0, x = -8$ or $x = 3$
The roots of $m(x)$ are 0, -8 and 3.

4 $p(x) = q(x)$
$x^2 - 3x = 2x - 6$
$x^2 - 5x + 6 = 0$
$(x - 3)(x - 2) = 0$
So $x = 3$ or $x = 2$

5 $f(x) = g(x)$
$2x^3 + 30x = 17x^2$
$2x^3 - 17x^2 + 30x = 0$
$x(2x^2 - 17x + 30) = 0$
$x(2x - 5)(x - 6) = 0$
So $x = 0, x = \frac{5}{2}$ or $x = 6$

6 a $f(x) = x^2 - 2x + 2$
$= (x - 1)^2 - 1^2 + 2$
$= (x - 1)^2 + 1$
$p = -1$ and $q = 1$

b $(x - 1)^2$ is a squared term so is always ≥ 0.
Therefore, the minimum value of
$f(x) = 0 + 1 = 1$, so $f(x) > 0$.

7 a
$$f(x) = 0$$
$$x^6 + 9x^3 + 8 = 0$$
$$(x^3)^2 + 9(x^3) + 8 = 0$$
$$(x^3 + 1)(x^3 + 8) = 0$$
So $x^3 = -1$ or $x^3 = -8$
$x^3 = -1 \Rightarrow x = -1$
$x^3 = -8 \Rightarrow x = -2$
The roots of $f(x)$ are -1 and -2.

b
$$g(x) = 0$$
$$x^4 - 12x^2 + 32 = 0$$
$$(x^2)^2 - 12(x^2) + 32 = 0$$
$$(x^2 - 4)(x^2 - 8) = 0$$
So $x^2 = 4$ or $x^2 = 8$
$x^2 = 4 \Rightarrow x = \pm 2$
$x^2 = 8 \Rightarrow x = \pm\sqrt{8} = \pm\sqrt{4 \times 2} = \pm 2\sqrt{2}$
The roots of $g(x)$ are -2, 2, $-2\sqrt{2}$ and $2\sqrt{2}$.

c
$$h(x) = 0$$
$$27x^6 + 26x^3 - 1 = 0$$
$$27(x^3)^2 + 26(x^3) - 1 = 0$$
$$(27x^3 - 1)(x^3 + 1) = 0$$
$x^3 = \frac{1}{27} \Rightarrow x = \frac{1}{3}$
$x^3 = -1 \Rightarrow x = -1$
The roots of $h(x)$ are -1 and $\frac{1}{3}$.

d
$$j(x) = 0$$
$$32x^{10} - 33x^5 + 1 = 0$$
$$32(x^5)^2 - 33(x^5) + 1 = 0$$
$$(32x^5 - 1)(x^5 - 1) = 0$$
So $x^5 = \frac{1}{32}$ or $x^5 = 1$
$x^5 = \frac{1}{32} \Rightarrow x = \frac{1}{2}$
$x^5 = 1 \Rightarrow x = 1$
The roots of $j(x)$ are $\frac{1}{2}$ and 1.

7 e
$$k(x) = 0$$
$$x - 7\sqrt{x} + 10 = 0$$
$$\left(x^{\frac{1}{2}}\right)^2 - 7\left(x^{\frac{1}{2}}\right) + 10 = 0$$
$$\left(x^{\frac{1}{2}} - 2\right)\left(x^{\frac{1}{2}} - 5\right) = 0$$
So $x^{\frac{1}{2}} = 2$ or $x^{\frac{1}{2}} = 5$
$x^{\frac{1}{2}} = 2 \Rightarrow x = 4\frac{n!}{r!(n-r)!}$
$x^{\frac{1}{2}} = 5 \Rightarrow x = 25$
The roots of $k(x)$ are 4 and 25.

f
$$m(x) = 0$$
$$2x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 12 = 0$$
$$\left(x^{\frac{1}{3}}\right)^2 + \left(x^{\frac{1}{3}}\right) - 6 = 0$$
$$\left(x^{\frac{1}{3}} - 2\right)\left(x^{\frac{1}{3}} + 3\right) = 0$$
So $x^{\frac{1}{3}} = 2$ or $x^{\frac{1}{3}} = -3$
$x^{\frac{1}{3}} = 2 \Rightarrow x = 8$
$x^{\frac{1}{3}} = -3 \Rightarrow x = -27$
The roots of $m(x)$ are 8 and -27.

8 a $3^{2x} - 28(3^x) + 27 = (3^x)^2 - 28(3^x) + 27$
$= (3^x - 27)(3^x - 1)$

b
$$f(x) = 0$$
$$(3^x - 27)(3^x - 1) = 0$$
$3^x = 27 \Rightarrow x = 3$
$3^x = 1 \Rightarrow x = 0$
The roots of $f(x)$ are 0 and 3.