Core Mathematics C1 Paper H

1. $f(x) = (\sqrt{x} + 3)^2 + (1 - 3\sqrt{x})^2$.

Show that $f(x)$ can be written in the form $ax + b$ where a and b are integers to be found. [3]

2. Find in exact form the real solutions of the equation

$$x^4 = 5x^2 + 14.$$ [4]

3. $f(x) = x^3 + 4x^2 - 3x + 7$.

Find the set of values of x for which $f(x)$ is increasing. [5]

4. Express each of the following in the form $p + q\sqrt{2}$ where p and q are rational.

(i) $(4 - 3\sqrt{2})^2$ [2]

(ii) $\dfrac{1}{2+\sqrt{2}}$ [3]

5. Given that the equation

$$x^2 + 4kx - k = 0$$

has no real roots,

(i) show that

$$4k^2 + k < 0,$$ [3]

(ii) find the set of possible values of k. [3]

6. The curve with equation $y = x^2 + 2x$ passes through the origin, O.

(i) Find an equation for the normal to the curve at O. [4]

(ii) Find the coordinates of the point where the normal to the curve at O intersects the curve again. [3]

7. A circle has centre (5, 2) and passes through the point (7, 3).

(i) Find the length of the diameter of the circle. [2]

(ii) Find an equation for the circle. [2]

(iii) Show that the line $y = 2x - 3$ is a tangent to the circle and find the coordinates of the point of contact. [5]

8. *(i)* Sketch the graphs of $y = 2x^4$ and $y = 2\sqrt{x}$, $x \geq 0$ on the same diagram and write down the coordinates of the point where they intersect. [4]

(ii) Describe fully the transformation that maps the graph of $y = 2\sqrt{x}$ onto the graph of $y = 2\sqrt{x-3}$. [2]

(iii) Find and simplify the equation of the graph obtained when the graph of $y = 2x^4$ is stretched by a factor of 2 in the x-direction, about the y-axis. [3]

9. The straight line l_1 passes through the point A (–2, 5) and the point B (4, 1).

(i) Find an equation for l_1 in the form $ax + by = c$, where a, b and c are integers. [4]

The straight line l_2 passes through B and is perpendicular to l_1.

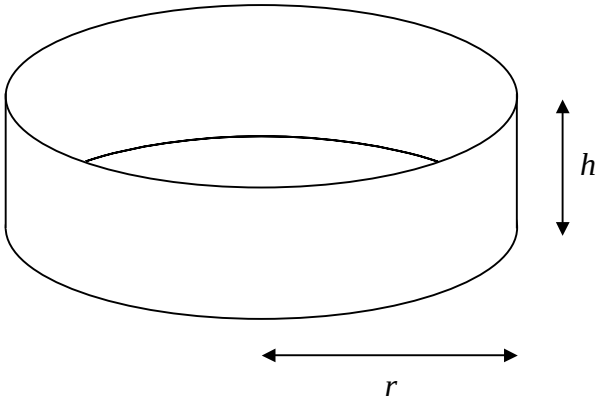
(ii) Find an equation for l_2. [3]

Given that l_2 meets the y-axis at the point C,

(iii) show that triangle ABC is isosceles. [4]

Turn over

10.



The diagram shows an open-topped cylindrical container made from cardboard. The cylinder is of height h cm and base radius r cm.

Given that the area of card used to make the container is 192π cm^2,

(i) show that the capacity of the container, V cm^3, is given by

$$V = 96\pi r - \tfrac{1}{2}\pi r^3.$$ [5]

(ii) Find the value of r for which V is stationary. [4]

(iii) Find the corresponding value of V in terms of π. [2]

(iv) Determine whether this is a maximum or a minimum value of V. [2]