C1 Paper H – Marking Guide

1. $f(x) = x + 6\sqrt{x} + 9 + 1 - 6\sqrt{x} + 9x$ — M1 A1
$= 10x + 10, \quad a = 10,\ b = 10$ — A1 **(3)**

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2. $x^4 - 5x^2 - 14 = 0$
$(x^2 + 2)(x^2 - 7) = 0$ — M2
$x^2 = -2$ (no solutions) or 7 — A1
$x = \pm\sqrt{7}$ — A1 **(4)**

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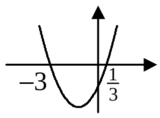
3. $f'(x) = 3x^2 + 8x - 3$ — M1 A1
increasing when $3x^2 + 8x - 3 \geq 0$ — M1
$(3x - 1)(x + 3) \geq 0$ — M1



$x \leq -3$ and $x \geq \frac{1}{3}$ — A1 **(5)**

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4. *(i)* $= 16 - 24\sqrt{2} + 18 = 34 - 24\sqrt{2}$ — M1 A1
(ii) $= \frac{1}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{2-\sqrt{2}}{4-2} = 1 - \frac{1}{2}\sqrt{2}$ — M2 A1 **(5)**

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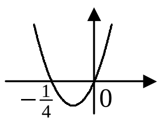
5. *(i)* no real roots $\therefore\ b^2 - 4ac < 0$
$(4k)^2 - [4 \times 1 \times (-k)] < 0$ — M2
$16k^2 + 4k < 0$
$4k^2 + k < 0$ — A1
(ii) $k(4k + 1) < 0$, critical values: $-\frac{1}{4}, 0$ — M1



— M1
$\therefore\ -\frac{1}{4} < k < 0$ — A1 **(6)**

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6. *(i)* $\frac{dy}{dx} = 2x + 2$ — M1
grad of tangent = 2 — A1
grad of normal = $\frac{-1}{2} = -\frac{1}{2}$ — M1
$\therefore\ y = -\frac{1}{2}x$ — A1
(ii) $x^2 + 2x = -\frac{1}{2}x$
$2x^2 + 5x = 0$
$x(2x + 5) = 0$ — M1
$x = 0$ (at O), $-\frac{5}{2}$ — A1
$\therefore\ (-\frac{5}{2}, \frac{5}{4})$ — A1 **(7)**

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7. *(i)* $= 2 \times \sqrt{4+1} = 2\sqrt{5}$ — M1 A1
(ii) $(x - 5)^2 + (y - 2)^2 = (\sqrt{5})^2$ — M1
$(x - 5)^2 + (y - 2)^2 = 5$ — A1
(iii) sub. $y = 2x - 3$ into eqn of C:
$(x - 5)^2 + [(2x - 3) - 2]^2 = 5$ — M1
$(x - 5)^2 + (2x - 5)^2 = 5$
$x^2 - 6x + 9 = 0$ — A1
$(x - 3)^2 = 0$ — M1
repeated root $\therefore$ tangent — A1
point of contact (3, 3) — A1 **(9)**

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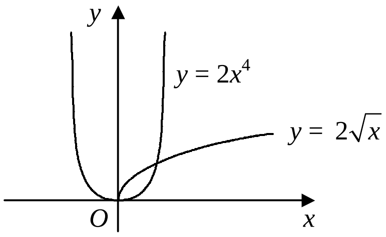
8. *(i)*



B1 B1

intersect at (1, 2) — B2

(ii) translation by 3 units in the positive x-direction — B2

(iii) $y = 2(\frac{x}{2})^4 = \frac{1}{8}x^4$ — M1 A2 **(9)**

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9. *(i)* grad $= \frac{1-5}{4-(-2)} = -\frac{2}{3}$ — M1 A1

$\therefore\ y - 5 = -\frac{2}{3}(x + 2)$ — M1

$3y - 15 = -2x - 4$

$2x + 3y = 11$ — A1

(ii) grad $l_2 = \frac{-1}{-\frac{2}{3}} = \frac{3}{2}$ — M1 A1

$\therefore\ y - 1 = \frac{3}{2}(x - 4)$ [$3x - 2y = 10$] — A1

(iii) at C, $x = 0$ $\therefore y = -5$ $\Rightarrow$ $C(0, -5)$ — B1

$AB = \sqrt{(4+2)^2 + (1-5)^2} = \sqrt{36+16} = \sqrt{52}$ — M1 A1

$BC = \sqrt{(0-4)^2 + (-5-1)^2} = \sqrt{16+36} = \sqrt{52}$

$AB = BC$ $\therefore$ triangle ABC is isosceles — A1 **(11)**

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10. *(i)* $A = \pi r^2 + 2\pi rh = 192\pi$ — M1

$\therefore\ h = \frac{192 - r^2}{2r} = \frac{96}{r} - \frac{r}{2}$ — M1 A1

$V = \pi r^2 h = \pi r^2(\frac{96}{r} - \frac{r}{2}) = 96\pi r - \frac{1}{2}\pi r^3$ — M1 A1

(ii) $\frac{dV}{dr} = 96\pi - \frac{3}{2}\pi r^2$ — M1 A1

for SP, $96\pi - \frac{3}{2}\pi r^2 = 0$ — M1

$r^2 = 64$

$r = 8$ — A1

(iii) $= (96\pi \times 8) - (\frac{1}{2}\pi \times 8^3) = 512\pi$ — M1 A1

(iv) $\frac{d^2V}{dr^2} = -3\pi r$ — M1

when $r = 8$, $\frac{d^2V}{dr^2} = -24\pi$, $\frac{d^2V}{dr^2} < 0$ $\therefore$ maximum — A1 **(13)**

Total **(72)**