Core Mathematics C1 Paper D

1. Solve the equation

$$x^2 - 4x - 8 = 0,$$

giving your answers in the form $a + b\sqrt{3}$ where a and b are integers. [3]

2. The curve C has the equation

$$y = x^2 + ax + b,$$

where a and b are constants.

Given that the minimum point of C has coordinates $(-2, 5)$, find the values of a and b. [4]

3. *(i)* Solve the simultaneous equations

$$y = x^2 - 6x + 7$$

$$y = 2x - 9$$ [4]

(ii) Hence, describe the geometrical relationship between the curve $y = x^2 - 6x + 7$ and the straight line $y = 2x - 9$. [1]

4. *(i)* Evaluate

$$(36^{\frac{1}{2}} + 16^{\frac{1}{4}})^{\frac{1}{3}}.$$ [3]

(ii) Solve the equation

$$3x^{-\frac{1}{2}} - 4 = 0.$$ [3]

5. *(i)* Sketch on the same diagram the curve with equation $y = (x - 2)^2$ and the straight line with equation $y = 2x - 1$.

Label on your sketch the coordinates of any points where each graph meets the coordinate axes. [4]

(ii) Find the set of values of x for which

$$(x - 2)^2 > 2x - 1.$$ [3]

6. *(i)* Given that $y = x^{\frac{1}{3}}$, show that the equation

$$2x^{\frac{1}{3}} + 3x^{-\frac{1}{3}} = 7$$

can be rewritten as

$$2y^2 - 7y + 3 = 0.$$ [3]

(ii) Hence, solve the equation

$$2x^{\frac{1}{3}} + 3x^{-\frac{1}{3}} = 7.$$ [4]

7. Given that

$$y = \sqrt{x} - \frac{4}{\sqrt{x}},$$

(i) find $\frac{dy}{dx}$, [3]

(ii) find $\frac{d^2y}{dx^2}$, [2]

(iii) show that

$$4x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - y = 0.$$ [3]

8. $$f(x) = 2 + 6x^2 - x^3.$$

(i) Find the coordinates of the stationary points of the curve $y = f(x)$. [4]

(ii) Determine whether each stationary point is a maximum or minimum point. [3]

(iii) Sketch the curve $y = f(x)$. [2]

(iv) State the set of values of k for which the equation $f(x) = k$ has three solutions. [1]

Turn over

9. The points P and Q have coordinates (7, 4) and (9, 7) respectively.

(i) Find an equation for the straight line l which passes through P and Q. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

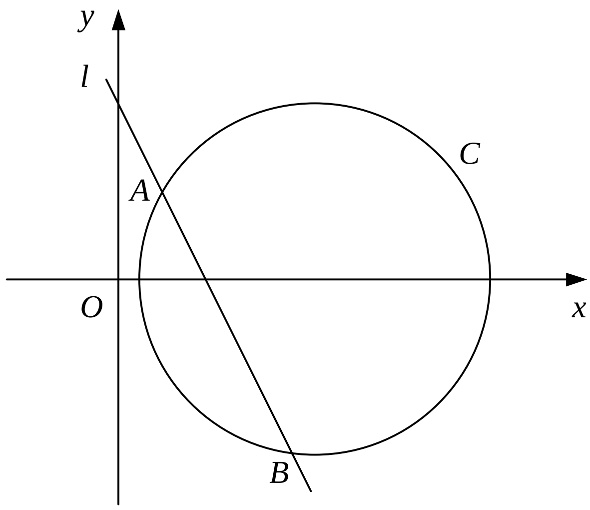
The straight line m has gradient 8 and passes through the origin, O.

(ii) Write down an equation for m. [1]

The lines l and m intersect at the point R.

(iii) Show that $OP = OR$. [5]

10.



The diagram shows the circle C and the straight line l.

The centre of C lies on the x-axis and l intersects C at the points A (2, 4) and B (8, –8).

(i) Find the gradient of l. [2]

(ii) Find the coordinates of the mid-point of AB. [2]

(iii) Find the coordinates of the centre of C. [5]

(iv) Show that C has the equation

$$x^2 + y^2 - 18x + 16 = 0.$$ [3]