Oxford Cambridge and RSA

GCE

Mathematics

Unit **4721**: Core Mathematics 1

Advanced Subsidiary GCE

Mark Scheme for June 2016

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals , Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Here are the subject specific instructions for this question paper
3.

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

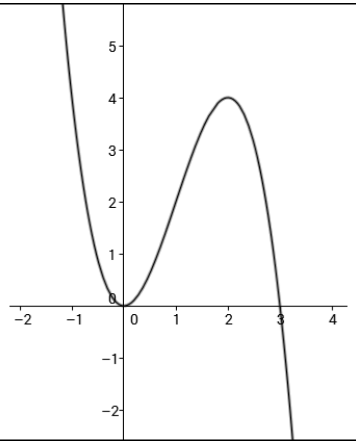
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

4. Here is the mark scheme for this question paper.
=

Question			Answer	Marks	Guidance	
1	(i)		$4x^2 - 12x + 9 - 2(9 - 6x + x^2)$ $2x^2 - 9$	**M1** **A1** **[2]**	Square to get at least one 3/4 term quadratic Fully correct **www**	**ISW** after correct answer
1	(ii)		$-6x^3 - 4x^3$ -10	**B1** **B1** **[2]**	$-6x^3$ **or** $-4x^3$ soi **www** in these terms Condone $-10x^3$	Ignore other terms If only embedded in full expansion then award **B1B0**
2			$\frac{3+\sqrt{20}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ $\frac{-1+3\sqrt{5}}{9-5}$ $-\frac{1}{4}+\frac{3}{4}\sqrt{5}$	**M1** **B1** **A1** **A1** **[4]**	Attempt to rationalise the denominator – must attempt to multiply $\sqrt{20} = 2\sqrt{5}$ soi Either numerator or denominator correct and simplified to no more than two terms Fully correct and fully simplified. Allow $\frac{-1+3\sqrt{5}}{4}$, order reversed etc. **Do not ISW** if then multiplied by 4 etc.	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{5}+b$ **B1** $\sqrt{20} = 2\sqrt{5}$ soi **A1** Either a or b correct **A1** Both correct
3			$x^2 + (3x + 4)^2 = 34$ $10x^2 + 24x - 18 = 0$ $5x^2 + 12x - 9 = 0$ $(5x - 3)(x + 3) = 0$ $x = \frac{3}{5}, x = -3$ $y = \frac{29}{5}, y = -5$	**M1*** **A1** **M1dep*** **A1** **A1** **[5]**	Substitute for x/y or valid attempt to eliminate one of the variables Correct three term quadratic in solvable form Attempt to solve resulting three term quadratic Correct x values Correct y values	If x eliminated: $10y^2 - 8y + 290 = 0$ $5y^2 - 4y + 145 = 0$ $(5y - 29)(y + 5) = 0$ Award **A1 A0** for one pair correctly found from correct quadratic **Spotted solutions:** If **M0 DM0** **SC B1** $x = \frac{3}{5}, y = \frac{29}{5}$ **www** **SC B1** $x = -3, y = -5$ **www** Must show on both line and curve (Can then get 5/5 if both found **www** and exactly two solutions justified)

Question			Answer	Marks	Guidance	
4			Let $y^{\frac{1}{4}} = x$ $2x^2 - 7x + 3 = 0$ $(2x - 1)(x - 3) = 0$ $x = \frac{1}{2}, x = 3$ $y = \left(\frac{1}{2}\right)^4, y = 3^4$ $y = \frac{1}{16}, y = 81$	**M1*** **M1dep*** **A1** **M1dep*** **A1** **[5]**	Use a substitution to obtain a quadratic or factorise into two brackets each containing $y^{\frac{1}{4}}$ Correct method to solve resulting quadratic Both values correct Attempt to raise to the fourth power Correct final answers	**No marks** if whole equation raised to fourth power etc. **No marks** if straight to formula with no evidence of substitution at start and no raising to fourth power/fourth rooting at end. **No marks** if $y^{\frac{1}{4}} = x$ and then $2x - 7x^2 + 3 = 0$. **Spotted solutions:** If **M0 DM0** or **M1 DM0** **SC B1** $y = 81$ **www** **SC B1** $y = \frac{1}{16}$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)
			<u>Alternative by rearrangement and squaring:</u> $2y^{\frac{1}{2}} - 7y^{\frac{1}{4}} + 3 = 0,\ 7y^{\frac{1}{4}} = 2y^{\frac{1}{2}} + 3$ $49y^{\frac{1}{2}} = 4y + 12y^{\frac{1}{2}} + 9,\ 37y^{\frac{1}{2}} = 4y + 9$ $16y^2 - 1297y + 81 = 0$ $(16y - 1)(y - 81) = 0$ $y = \frac{1}{16}, y = 81$	**M2*** **A1** **M1dep*** **A1**	Rearrange and square both sides twice Correct quadratic obtained Correct method to solve resulting quadratic Correct final answers	
			OR methods may be combined: e.g. after $37y^{\frac{1}{2}} = 4y + 9$ $4y - 37y^{\frac{1}{2}} + 9 = 0$ $4x^2 - 37x + 9 = 0$ $(4x - 1)(x - 9) = 0$ $x = \frac{1}{4}, x = 9$ $y = \left(\frac{1}{4}\right)^2, y = 9^2$	**M1*** **M1dep*** **A1** **M1dep*** **A1** **[5]**	Rearrange, square both sides and substitute Correct method to solve resulting quadratic Attempt to square Correct final answers	

Question			Answer	Marks	Guidance	
5	(i)		$(2^{-2})^3$ or $2^{15} \div 2^{21}$ 2^{-6}	**B1** **B1** **[2]**	Valid attempt to simplify Correct answer. Accept $p = -6$.	Correct use of either index law $\left(\frac{1}{2}\right)^6$ **oe** is **B1**
5	(ii)		$5\times\left(2^2\right)^{\frac{2}{3}}+3\times\left(2^4\right)^{\frac{1}{3}}$ $=5\times2^{\frac{4}{3}}+3\times2^{\frac{4}{3}} or 10\times2^{\frac{1}{3}}+6\times2^{\frac{1}{3}}$ $=8\times2^{\frac{4}{3}}$ $=2^{\frac{13}{3}}$	**M1** **B1** **A1** **[3]**	Attempts to express both terms or a combined term as a power of 2 Correctly obtains $2^{\frac{4}{3}}$ or $2^{\frac{1}{3}}$ for **either** term Correct final answer	e.g. **Both** $4 = 2^2$ **and** $16 = 2^4$ **soi** If **M0** **SC B1** for $8\times16^{\frac{1}{3}}$ or $8\times4^{\frac{2}{3}}$
6	(i)		$-2(x^2-6x-2)$ $=-2[(x-3)^2-2-9)]$ $=-2(x-3)^2+22$	**B1** **B1** **M1** **A1** **[4]**	or $a=-2$ $b=-3$ $4+2b^2$ $c=22$ If a, b and c found correctly, then **ISW** slips in format. **If signs of all terms changed at start, can only score SC B1** for fully correct working to obtain $2(x-3)^2-22$ If done correctly and then signs changed at end, do not **ISW**, award **B1B1M1A0**	$-2(x-3)^2-22$ **B1 B1 M0 A0** $-2(x-3)+22$ **4/4 (BOD)** $-2(x-3x)^2+22$ **B1 B0 M1 A0** $-2(x^2-3)^2+22$ **B1 B0 M1 A0** $-2(x+3)^2+22$ **B1 B0 M1 A0** $-2x(x-3)^2+22$ **B0 B1 M1 A0** $-2(x^2-3)+22$ **B1 B0 M1 A0**
6	(ii)		(3, 22)	**B1ft** **B1ft** **[2]**	Allow follow through "– their b" Allow follow through "their c"	May restart. Follow through marks are for **their final answer** to (i)

Question			Answer	Marks	Guidance	
7	(i)			B1 B1 B1 [3]	Negative cubic with a max and a min Cubic that meets y-axis at (0, 0) only Double root at (0,0) and single root at (3, 0) and no other roots	For **first mark** must clearly be a cubic – must not stop at or before x axis, do not allow straight line sections drawn with a ruler/tending to extra turning points etc. Must not be a finite plot.
7	(ii)		$y=(x-2)^2(5-x)$ or $y=3(x-2)^2-(x-2)^3$	M1 A1 [2]	Translates curve by +2 or – 2 parallel to the x-axis; must be consistent Fully correct, must have "$y =$". **ISW** expansions	e.g. for M1 $(x-2)^2(3-(x-2))$ but not $(x-2)^2(3-x-2)$
7	(iii)		Stretch Scale factor one-half parallel to the y-axis	B1 B1 [2]	**Must** use the word "stretch" **Must** have "factor" or "scale factor". For "parallel to the y axis" allow "vertically", "in the y direction".	Do **not** accept "in/on/across/up the y axis". Allow second B1 after "squash" etc. but not after "translate" etc.
8	(i)		$y_1 = 50$, $y_2 = 2(5+h)^2$ $\dfrac{(50+20h+2h^2)-50}{(5+h)-5}$ $20 + 2h$	B1 M1 A1 [3]	Finds y coordinates at 5 and $5 + h$ Correct method to find gradient of a line segment; at least 3/4 values correct Fully correct working to give answer **AG**	Need not be simplified
8	(ii)		e.g. "As h tends to zero, the gradient will be 20"	B1 [1]	Indicates understanding of limit **See Appendix 2 for examples**	e.g. refer to h tending to zero or substitute $h = 0$ into $20 + 2h$ to obtain gradient at A
8	(iii)		Gradient of normal $=-\dfrac{1}{20}$ $y-50=-\dfrac{1}{20}(x-5), x=0$ 50¼	B1 M1 A1 [3]	Gradient of line must be numerical negative reciprocal of their gradient at A through their A Correct coordinate in any form e.g. $\dfrac{201}{4}, \dfrac{1005}{20}$	Any correct method e.g. labelled diagram.

Question		Answer	Marks	Guidance	
9		$x^2 + (2 - 2k)x + 11 + k = 0$ $(2 - 2k)^2 - 4(11 + k)$ $4k^2 - 12k - 40 > 0$ $k^2 - 3k - 10 > 0$ $(k - 5)(k + 2)$ $k < -2, k > 5$	**M1*** **M1dep*** **A1** **M1dep*** **A1** **M1dep*** **A1** **[7]**	Attempt to rearrange to a three-term quadratic Uses $b^2 - 4ac$, involving k and not involving x Correct simplified inequality obtained **www** Correct method to find roots of 3-term quadratic 5 and – 2 seen as roots $b^2 - 4ac > 0$ and chooses "outside region" Fully correct, strict inequalities.	Each Ms depend on the previous M $-2 > k > 5$ scores **M1A0** Allow "$k < -2$ or $k > 5$" for A1 **Do not allow** "$k < -2$ **and** $k > 5$"
10	(i)	Centre of circle (4, 3) $(x-4)^2 - 16 + (y-3)^2 - 9 - 20 = 0$ $r^2 = 45$ $r = \sqrt{45}$	**B1** **M1** **A1** **[3]**	Correct centre $(x \pm 4)^2 - 4^2$ and $(y \pm 3)^2 - 3^2$ seen (or implied by correct answer) $\sqrt{45}$ or better **www**	Or $r^2 = 4^2 + 3^2 + 20$ **soi** **ISW** after $\sqrt{45}$
10	(ii)	At A, $y = 0$ so $x^2 - 8x - 20 = 0$ $(x - 10)(x + 2) = 0$ A = (10, 0) Gradient of radius = $\frac{3-0}{4-10} = -\frac{1}{2}$ Gradient of tangent = 2 $y - 0 = 2(x - 10)$ $y = 2x - 20$	**M1** **A1** **M1** **B1** **M1** **A1** **[6]**	Valid method to find A e.g. put $y = 0$ and attempt to solve quadratic (allow slips) or Pythagoras' theorem Correct answer found Attempts to find gradient of radius (3 out of 4 terms correct for their centre, their A) Equation of line **through their A**, any non-zero gradient Correct answer in any three-term form	Alterative for finding gradient: **M1** Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x + 2y\frac{dy}{dx} - 8 - 6\frac{dy}{dx} = 0$ **and** substitution of (10, 0) to obtain 2.
10	(iii)	A' = (–2, 6) $y - 6 = 2(x + 2)$ $y = 2x + 10$	**B1** **M1** **A1** **[3]**	Finds the opposite end of the diameter Line through their A' parallel to their line in (ii) Correct answer in any three-term form	 Not through centre of circle
10	(iv)	OC = $\sqrt{3^2 + 4^2} = 5$ $(0 <)\ r < \sqrt{45} - 5$	**M1** **A1** **[2]**	Attempts to find the distance from O to their centre **and subtract from their radius** Correct inequality, condone $\le$	**ISW** incorrect simplification

Question			Answer	Marks	Guidance	
11			$y = 4x^2 + ax^{-1} + 5$	B1	ax^{-1} **soi**	
			$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct	
				A1	Fully correct	
			At stationary point, $8x - ax^{-2} = 0$	M1	Sets their derivative to 0	
			$a = 8x^3$ **oe**	A1	Obtains expression for a in terms of x, or x in terms of a **www**	$x = \frac{\sqrt[3]{a}}{2}$ **oe**, $a = 18x$ **oe** also fine
			When $a = 8x^3$, $y = 32$ $32 = 4x^2 + 8x^2 + 5$	M1	Substitutes their expression and 32 into equation of the curve to form single variable equation	
			$x = \frac{3}{2}$ **oe**	A1	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	or expression for a e.g. $a^{\frac{2}{3}} = 9$
			$a = 27$	A1 **[8]**	Obtains correct value for a. Ignore –27 given as well.	
			OR			
			$y = 4x^2 + ax^{-1} + 5$	B1	ax^{-1} **soi**	
			$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct	
				A1	Fully correct	
			$32 = 4x^2 + ax^{-1} + 5$	M1	Substitutes 32 into equation of the curve to find expression for a	
			$a = 27x - 4x^3$	A1	Obtains expression for a in terms of x **www**	
			At stationary point, $8x - ax^{-2} = 0$ $8x - (27x - 4x^3)x^{-2} = 0$	M1	Sets derivate to zero **and** forms single variable equation	
			$x = \frac{3}{2}$ **oe**	A1	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	
			$a = 27$	A1	Obtains correct value for a. Ignore –27 given as well.	

APPENDIX 1

Allocation of method mark for solving a quadratic

e.g. $2x^2 - x - 6 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x - 3)(x + 2)$	**M1**	$2x^2$ and -6 obtained from expansion
$(2x - 3)(x + 1)$	**M1**	$2x^2$ and $-x$ obtained from expansion
$(2x + 3)(x + 2)$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$$\frac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times 2}$$ earns **M1** (minus sign incorrect at start of formula)

$$\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$$ earns **M1** (6 for c instead of -6)

$$\frac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$$ **M0** (2 sign errors: initial sign and c incorrect)

$$\frac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times -6}$$ **M0** ($2c$ on the denominator)

Notes – for equations such as $2x^2 - x - 6 = 0$, then $b^2 = 1^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - x - 6 = 0$$

$$2\left(x^2 - \frac{1}{2}x\right) - 6 = 0$$

$$2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6 = 0$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{49}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{1}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last (complete) attempt.

APPENDIX 2 – this section contains additional subject specific information

Example responses to 8ii	
h is zero so the gradient is 20 **B1**	The gradient at A is 20 **B0**
At A x = 5, h = 0 so gradient equals 20 **B1**	The gradient at A is 20 so h = 0 **B0**
As h approaches 0, the gradient of AB approaches 20 which is the gradient of A **B1**	At A, gradient is 20 so it's 2h more **B0**
As h were infinitely small, 20 + 2h is the same as the gradient at A, otherwise it's greater than the gradient at A **B1**	$\frac{dy}{dx} = 20$, so it is the gradient of A plus a bit more **B0**
The smaller h is the closer the gradient of AB is to the gradient of the curve at A **B1**	2h + 20 = 20 so h = 0 **B0**
As h tends to zero the gradient gets closer and closer to the actual value **B1**	They're getting closer to each other **B0**
The gradient of AB tends to the gradient of the tangent of the curve as h tends to zero **B1**	
The answer of (i) is converging towards the gradient at A **B1**	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored