Core 1 May 2009

1) $y = x^5 + x^{-2}$

$\frac{dy}{dx} = 5x^4 - 2x^{-3} = 5x^4 - \frac{2}{x^3}$ **(3)**

$\frac{d^2y}{dx^2} = 20x^3 + 6x^{-4} = 20x^3 + \frac{6}{x^4}$ **(2)**

2) $\frac{(8+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})} = \frac{16-6\sqrt{7}-7}{4-7} = \frac{9-6\sqrt{7}}{-3} = 2\sqrt{7}-3$ **(4)**

3) i) $\frac{1}{9} = \frac{1}{3^2} = 3^{-2}$ **(1)**

ii) $\sqrt[3]{3} = 3^{\frac{1}{3}}$ **(1)**

iii) $3^{10} \times (3^2)^{15} = 3^{40}$ **(2)**

4) $y = 2x - 4$ $\quad 4x^2 + (2x - 4)^2 = 10$ $\quad 4x^2 + 4x^2 - 16x + 16 = 10$

$4x^2 - 8x + 3 = 0$ $\quad (2x - 3)(2x - 1) = 0$

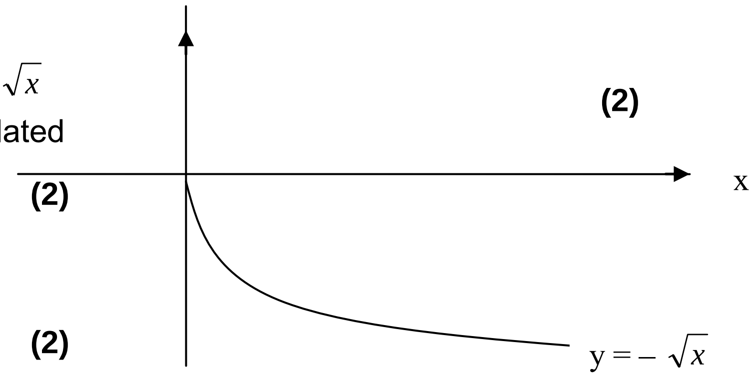
$x = \frac{3}{2}$ $\quad y = -1,$ $\quad x = \frac{1}{2}$ $\quad y = -3$ **(6)**

5)i) $(2x + 1)(x^2 + x - 12) = 2x^3 + 2x^2 - 24x + x^2 + x - 12$ **(3)**
$= 2x^3 + 3x^2 - 23x - 12$

ii) $x(x^2 + 2x + 3)(x^2 + 7x - 2)$
considering only the components making up the coefficient of x^4 **(2)**
$x(x^2 \times x + 2x \times x^2)$ hence coefficient of x^4 is $7 + 2 = 9$

6) $y = -\sqrt{x}$

ii) $f(x) = -\sqrt{x}$ $\quad 5 + f(x) = 5 - \sqrt{x}$ **(2)**
The graph of $y = -\sqrt{x}$ will be translated
5 units in a positive direction
parallel to the y axis. **(2)**



iii) $f(\frac{x}{2})$ $\quad y = -\sqrt{\frac{x}{2}}$ **(2)**

7) i) $x^2 - 5x + \frac{1}{4} = (x - \frac{5}{2})^2 - \frac{25}{4} + \frac{1}{4} = (x - \frac{5}{2})^2 - \frac{24}{4} = (x - \frac{5}{2})^2 - 6$ **(3)**

Or use matching method $(x - a)^2 - b = x^2 - 2ax + a^2 - b$

$-5x = -2ax$ so $a = 2\frac{1}{2}$ $\quad \frac{1}{4} = a^2 - b$ $\quad b = (2\frac{1}{2})^2 - \frac{1}{4} = 6.25 - 0.25 = 6$
as $2.5^2 = 6.25$

ii) $x^2 + y^2 - 5x + \frac{1}{4} = (x - \frac{5}{2})^2 - 6 + y^2 = 0$ $(x - \frac{5}{2})^2 + y^2 = 6$

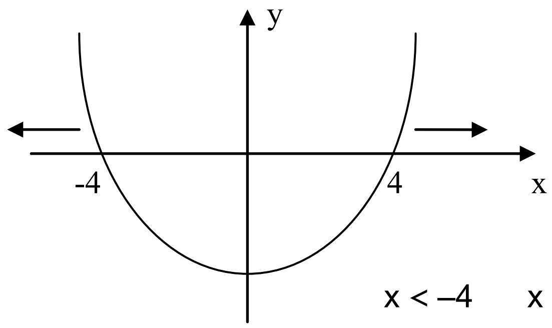
Centre $(\frac{5}{2}, 0)$ radius $\sqrt{6}$ or use a = $\frac{1}{2}$ coefft x change sign b = $\frac{1}{2}$ coefft y change sign

or $a = 2\frac{1}{2}$ $b = 0$ $r^2 = a^2 + b^2 - \text{no.} = (2\frac{1}{2})^2 + 0^2 - \frac{1}{4}$ $r = \sqrt{6}$ **(3)**

8) i) $-35 < 6x + 7 < 1$ either $-42 < 6x$ $-7 < x$
0r $6x < -6$ $x < -1$

So $-7 < x < -1$ **(3)**

ii) $3x^2 > 48$ $x^2 > 16$ $x^2 - 16 > 0$ $(x - 4)(x + 4) > 0$



$x < -4$ $x > 4$ **(3)**

9) i) A (4, -3) B (-1,9) AB = $\sqrt{((4--1)^2 + (-3-9)^2)}$ AB = $\sqrt{169}$ = 13 **(2)**

ii) midpoint AB $(\frac{4-1}{2}, \frac{-3+9}{2}) = (\frac{3}{2}, 3)$ **(2)**

iii) gradient AB = $\frac{-3-9}{4--1} = \frac{-12}{5}$

Equation of line gradient $\frac{-12}{5}$, through (1, 3)

$y - 3 = \frac{-12}{5}(x - 1)$ $5y - 15 = -12x + 12$

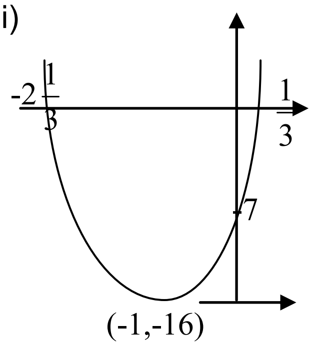
$12x + 5y - 27 = 0$ **(4)**

10) i) $(3x + 7)(3x - 1) = 0$ $x = -2\frac{1}{3}$, $x = \frac{1}{3}$ **(3)**

ii) At stationary point $\frac{dy}{dx} = 0$

$y = 9x^2 + 18x - 7$ $\frac{dy}{dx} = 18x + 18 = 0$ $x = -1$ $y = -16$ (-1,-16) **(4)**

iii)



(3)

(iv) y increases as x increases for x > -1 **(2)**

11 i) $y = k\sqrt{x}$ $2x + 3y = 0$ gradient normal at P is $\frac{-2}{3}$, $y = -{}^2/_3x$

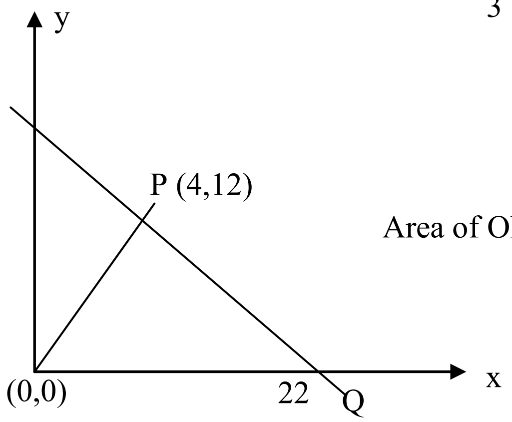
gradient tangent is $\frac{3}{2}$ (flip and change sign)

Gradient of tangent to curve at P is given by $\frac{dy}{dx} = \frac{1}{2}k\,x^{-\frac{1}{2}}$

Gradient of tangent is $\frac{3}{2}$ $\frac{1}{2}k\,x^{-\frac{1}{2}} = \frac{3}{2}$ at P x =4, hence k = 6 **(6)**

ii) P (4,12) gradient normal = $\frac{-2}{3}$

Equation of normal $(y - 12) = \frac{-2}{3}(x - 4)$ $3y = -2x + 44$



Area of OPQ = $\frac{1}{2} \times 22 \times 12 = 132\ u^2$ **(5)**