

C1 June 2007

$$1) (2x+5)^2 - 4x^2 + 20x + 25$$

$$(x-3)^2 = x^2 - 6x + 9$$

$$4x^2 + 20x + 25 - (x^2 - 6x + 9)$$

$$= 3x^2 + 26x + 16$$

6 cont

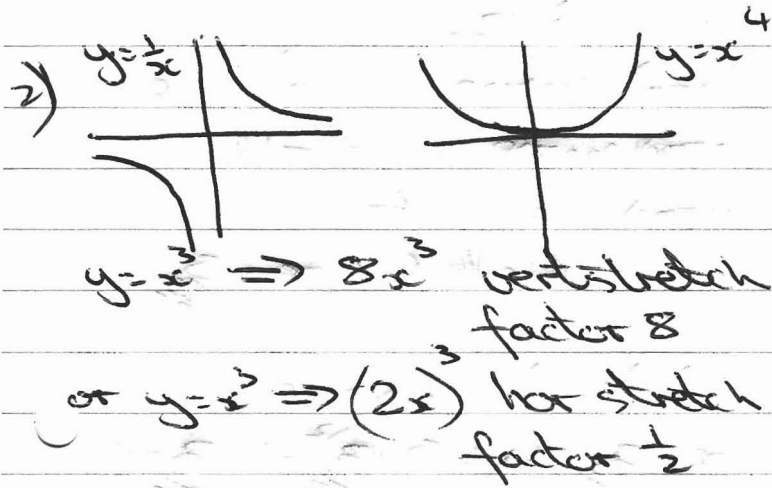
$$y = -6 \text{ or } +1$$

$$\text{so } (x+2)^2 = -6 \text{ or } +1$$

$$x+2 = \sqrt{-6} \text{ (imposs)}$$

$$\text{or } x+2 = \sqrt{1} = \pm 1$$

$$x = \pm 1 - 2 = -1 \text{ or } -3$$



7)

$$f(x) = x + 3x^{-1}$$

$$f'(x) = 1 - 3x^{-2}$$

8)

$$y = x^{\frac{5}{2}} \quad \frac{dy}{dx} = \frac{5x^{\frac{3}{2}}}{2}$$

subst $x=4$

$$\frac{dy}{dx} = \frac{5 \times 4^{\frac{3}{2}}}{2}$$

$$= \frac{5}{2} \times 8 = 20$$

3)

$$3\sqrt{10} \times \sqrt{2} = 3\sqrt{20} = 3\sqrt{4 \times 5}$$

$$= 6\sqrt{5}$$

$$\sqrt{500} + \sqrt{125} = \sqrt{5 \times 100} + \sqrt{25 \times 5}$$

$$= 10\sqrt{5} + 5\sqrt{5} = 15\sqrt{5}$$

8)

$$x^2 + 8x + 15 = (x+a)^2 - b$$

$$= x^2 + 2ax + a^2 - b$$

$$8x = 2ax \Rightarrow a = 4$$

$$15 = a^2 - b \Rightarrow 15 = 16 - b$$

$$b = 1$$

4) discriminant = $b^2 - 4ac$

$$= 16 - 4k^2$$

i) for equal roots $b^2 - 4ac = 0$

$$16 - 4k^2 = 0 \quad 4k^2 = 16$$

$$k^2 = 4 \quad k = \pm 2$$

$$x^2 + 8x + 15 = (x+4)^2 - 1$$

ii) vertex $(-4, -1)$

iii) $x^2 + 8x + 15 > 0$

$$(x+5)(x+3) > 0$$

$$x < -5 \text{ or } x > -3$$

5) i) $b = 20 - 2x$

$$A = x(20 - 2x) = 20x - 2x^2$$

ii) $\frac{dA}{dx} = 20 - 4x = 0$ for max

$$20 = 4x \quad x = 5$$

if $x = 5$ $A = 20 \times 5 - 2 \times 5^2$

$$= 50 \text{ m}^2$$

6)

$$y = (x+2)^2$$

so $y^2 + 5y - 6 = 0$

$$(y+6)(y-1) = 0$$

9) $a = +3$ $b = 0$
 centre $(3, 0)$
 $r^2 = a^2 + b^2 - 10$
 $16 = 9 - k$ $k = +7$

10 contd

$$324 + 60 + 12c = 0$$

$$384 = -12c$$

$$c = -32$$

method 2

grad curve = grad line

grad curve = $6x - 14$

grad line = 4

$$6x - 14 = 4 \quad x = 3$$

subst into curve

$$x = 3 \quad y = 3 \times (3)^2 - 14 \times 3 - 5$$

$$= -20$$

subst into line

$$-20 = 4 \times 3 + c$$

$$c = -32$$

ii) if $x = 3$ $y = a$ subst in \odot
 $3^2 + a^2 - 18 - 7 = 0$
 $a^2 = 16$ $a = 4$ as $a > 0$

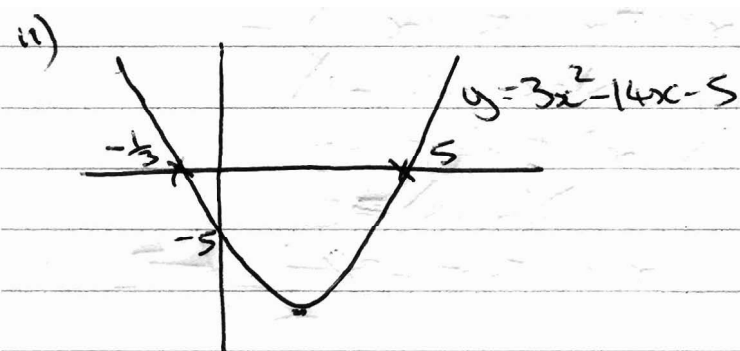
$$AB = \sqrt{(-1-3)^2 + (0-4)^2}$$

$$= \sqrt{32} = 4\sqrt{2}$$

iii) grad AB = $\frac{0-4}{-1-3} = 1$
 pt $(-1, 0)$
 $y - 0 = 1(x - (-1))$
 $y = x + 1$

10) i) $3x^2 - 14x - 5 = 0$
 $\begin{matrix} 1 & | & 1 \\ 3 & | & 5 \end{matrix} (3x+1)(x-5) = 0$

$$x = -\frac{1}{3} \text{ or } +5$$



iii) method 1
 If line is a tangent / root

$$3x^2 - 14x - 5 = 4x + c$$

$$3x^2 - 18x - 5 - c = 0$$

$$b^2 - 4ac = 0 \text{ for 1 root}$$

$$18^2 - 4 \times 3 \times (-5 - c) = 0$$