Core1 2006 June

1) i gradient AB $\quad \frac{21-3}{4-1} = \frac{18}{3} = 6.$ [2]

ii $y = x^2+x+1 \quad \frac{dy}{dx} = 2x+1 \quad x = 3. \quad \frac{dy}{dx} = 7.$ [2]

2 (i) $27^{-2/3} = \frac{1}{27^{2/3}} = \frac{1}{9}$ [2]

(ii) $5 \times 5^{1/2} = 5^{3/2}$ [1]

(iii) $\frac{(1-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \frac{3-4\sqrt{5}+5}{9-5} = \frac{8-4\sqrt{5}}{4} = 2-\sqrt{5}$ [3]

3 i $2x^2+12x+13 \quad = \quad 2(x+3)^2-5.$ [4]

$ax^2 + 2abx + ab^2+c$

$a=2 \quad 2ab=12 \quad 4b=12 \quad b=3 \quad ab^2+c=13 \quad 18+c=13 \quad c=-5$

$2x^2+12x+13=0$

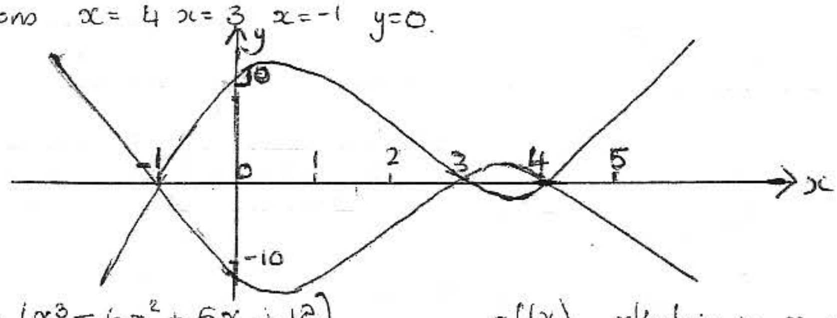
ii $x = \frac{-12 \pm \sqrt{144-104}}{4} = \frac{-12 \pm \sqrt{40}}{4} = \frac{-12 \pm 2\sqrt{10}}{4} = -3 \pm \frac{1}{2}\sqrt{10}$ [3]

4 $(x-4)(x-3) = x^2-4x-3x+12 = x^2-7x+12$

$(x^2-7x+12)(x+1) = x^3-7x^2+12x+x^2-7x+12 = x^3-6x^2+5x+12$ [3]

ii. solutions $x=4 \quad x=3 \quad x=-1 \quad y=0.$

[3]



[2]

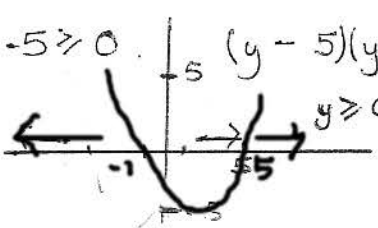
$y = -(x^3-6x^2+5x+12) \qquad -f(x)$ reflection in x axis.

5. i $1 < 4x-9 \quad 10 < 4x \quad \frac{5}{2} < x \qquad 4x-9 < 5 \quad 4x < 14 \quad x < \frac{7}{2}$ [3]

$\frac{5}{2} < x < \frac{7}{2}$

ii $y^2 \geqslant 4y+5 \quad y^2-4y-5 \geqslant 0. \quad (y-5)(y+1) \geqslant 0 \quad$ critical values $y=5 \; y=-1$.

$y \geqslant 5 \quad y \leqslant -1$ [5]



(1) (1)

6(i) $x^4 - 10x^2 + 25 = 0$ let $y = x^2$ $y^2 - 10y + 25 = 0$ $(y-5)(y-5) = 0$ [4]

$y = 5$ $x^2 = 5$

$(x^2 - 5)(x^2 - 5) = 0$ $x^2 = 5$ $x = \pm\sqrt{5}$ (2)

(ii) $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3.$

$\frac{dy}{dx} = 2x^4 - 20x^2 + 50.$ $2x^4 - 20x^2 + 50 = 2(x^4 - 10x^2 + 25)$ [2]

$\frac{dy}{dx} = 0$ $x = \pm\sqrt{5}$. 2 stationary points. [2)]

7.i $x^2 - 5x + 4 = x - 1$ $x^2 - 6x + 5 = 0.$ (1)

$(x-5)(x-1) = 0$ $x = 5$ $x = 1$ (2)

(1)

$x = 5$ $y = 4$ $x = 1$ $y = 0.$ [4]

ii 2points (1]

iii $x^2 - 5x + 4 = x + c$ (1) (2)

$x^2 - 6x + 4 - c = 0.$ 1 solution for tangent. $b^2 - 4ac = 0$ $36 - 4 \times 1 \times (4 - c)$

$36 - 16 + 4c = 0$

$20 + 4c = 0$ $4c = -20$ $c = -5.$

[4]

$y = x - 5$ is tgt

(3)

8.i $8 = x^2h$ $A = 2x^2 + 4xh.$ $A = 2x^2 + 4x\frac{8}{x^2} = 2x^2 + \frac{32}{x}$

ii $A = 2x^2 + 32x^{-1}$ $\frac{dA}{dx} = 4x - 32x^{-2}$ (3) $\frac{dA}{dx} = 0$

iii $4x - \frac{32}{x^2} = 0$ $4x^3 - 32 = 0$ $x^3 = 8$ $x = 2.$ (2)

$\frac{d^2A}{dx^2} = 4 + 64x^{-3} = 4 + \frac{64}{x^3}$ $x = 2$ $\frac{d^2A}{dx^2} = 12$ +ve $\therefore$ min. (2)

Smallest surface area $A = 2 \times 2^2 + \frac{32}{2} = 8 + 16 = 24\ m^2.$

(2) $\overset{A}{(4, -2)}$ $\overset{C}{(7, 2)}$

9 i midpt $\left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$ $(7, 2)$. length $AC = \sqrt{(7-4)^2 + (2+2)^2}$

ii $AC = \sqrt{9 + 16} = \sqrt{25} = 5.$ (2)

iii AB diameter C is centre. $(x-7)^2 + (y-2)^2 = 5^2.$ (3)

iv gradient AC $\frac{2+2}{7-4} = \frac{4}{3}.$ grad tgt at A $= -\frac{3}{4}$ (2)

$y + 2 = -\frac{3}{4}(x - 4)$ $4y + 8 = -3x + 12.$ (3)

$3x + 4y = 4.$