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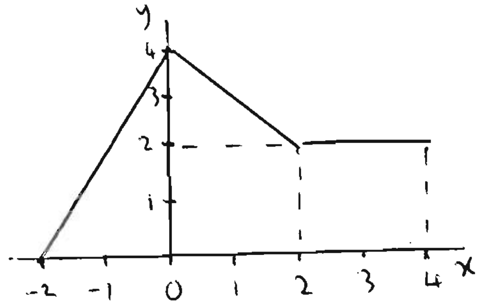
1) $x^2 - 12x + 1 \equiv (x-6)^2 - 36 + 1 \equiv (x-6)^2 - 35$ (3)

2) (i)



(2)

(ii) A translation through 1 unit to the right parallel to the x-axis. (2)

3) $y = x^3 - 4x^2 + 7$

$\frac{dy}{dx} = 3x^2 - 8x = -4$ when $x=2$

The product of the gradients of perpendicular lines is -1

$\therefore$ Gradient of normal $= \frac{-1}{-4} = \frac{1}{4}$

$y - y_1 = m(x - x_1)$ $\therefore$ equation of normal is at $(2, -1)$ $y - -1 = \frac{1}{4}(x-2)$

$4y + 4 = x - 2$

$-x + 4y + 6 = 0$ (7)

4) (i) $3^m = 81 = 3^4$ $\therefore m = 4$ (1)

(ii) $(36p^4)^{\frac{1}{2}} = 24$ $\therefore 36^{\frac{1}{2}} p^2 = 24$ $\therefore 6p^2 = 24$ $\therefore p^2 = 4$ $\therefore p = \pm 2$ (3)

(iii) $5^n \times 5^{n+4} = 25$ $\therefore 5^{2n+4} = 5^2$ $\therefore 2n+4 = 2$ $\therefore 2n = -2$ $\therefore n = -1$ (3)

5) $x - 8\sqrt{x} + 13 = 0$ Let $u = \sqrt{x}$ $\therefore u^2 - 8u + 13 = 0$

$u = \frac{--8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 13}}{2 \times 1} = \frac{8 \pm \sqrt{12}}{2} = \frac{8 \pm 2\sqrt{3}}{2} = 4 \pm \sqrt{3}$

$\therefore \sqrt{x} = 4 + \sqrt{3}$ or $\sqrt{x} = 4 - \sqrt{3}$ (7)

$x = 16 + 8\sqrt{3} + 3$ or $x = 16 - 8\sqrt{3} + 3$

$x = 19 \pm 8\sqrt{3}$

6 (i) $y = x^2 + 5$ $\therefore \frac{dy}{dx} = 2x$ $\therefore$ at $A(1,6)$ gradient $= 2$. (2)

(ii) Gradient of AB $= \frac{a^2 + 5 - 6}{a - 1} = \frac{a^2 - 1}{a - 1} = \frac{(a+1)(a-1)}{a-1} = a + 1 = 2.3$ $\therefore a = 1.3$ (4)

(iii) Gradient of AC is a number between 2 and 2.3 e.g. 2.1 (1)

7) (i) (a) $y = (3-x)^2$ corresponds to Fig. 3 (1)

(b) $y = x^2 + 9$ corresponds to Fig. 1 (1)

(c) $y = (3-x)(x+3)$ corresponds to Fig. 4 (1)

(ii) The equation of the curve is $y = -(x-3)^2$ (2)

8) (i) $x^2 + y^2 + 6x - 4y - 4 = 0$

$(x+3)^2 - 9 + (y-2)^2 - 4 - 4 = 0$ $\therefore$ centre is $(-3, 2)$ radius $= \sqrt{17}$. (3)

$(x+3)^2 + (y-2)^2 = 17$

(ii) To find the co-ordinates of the points of intersection it is necessary to solve $(x+3)^2 + (y-2)^2 = 17$ and $y = 3x + 4$ as simultaneous equations.

Substituting for y $(x+3)^2 + (3x+4-2)^2 = 17$

$(x+3)^2 + (3x+2)^2 = 17$

$x^2 + 6x + 9 + 9x^2 + 12x + 4 = 17$

8 cont.)

$$10x^2 + 18x + 13 = 17$$
$$10x^2 + 18x - 4 = 0$$
$$5x^2 + 9x - 2 = 0$$
$$(5x - 1)(x + 2) = 0$$
$$x = \tfrac{1}{5} \quad \text{or} \quad x = -2$$

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(6)

when $x = \frac{1}{5}$, $y = 3 \times \frac{1}{5} + 4 = 4\frac{3}{5}$; when $x = -2$, $y = 3 \times -2 + 4 = -2$

$\therefore$ the circle meets the line $y = 3x + 4$ at $(\frac{1}{5}, 4\frac{3}{5})$ and $(-2, -2)$.

9) $f(x) = \frac{1}{x} - \sqrt{x} + 3 = x^{-1} - x^{\frac{1}{2}} + 3$

(i) $\therefore f'(x) = -x^{-2} - \frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{x^2} - \frac{1}{2\sqrt{x}}$ (3)

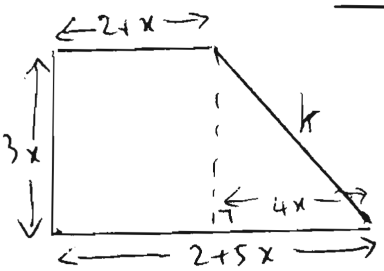
(ii) $f''(x) = 2x^{-3} + \frac{1}{4}x^{-3/2} = \frac{2}{x^3} + \frac{1}{4(\sqrt{x})^3}$ (5)

when $x = 4$, $f''(4) = \frac{2}{4^3} + \frac{1}{4(\sqrt{4})^3} = \frac{2}{64} + \frac{1}{32} = \frac{2}{32} = \frac{1}{16}$

10) If the quadratic $kx^2 - 30x + 25k = 0$ has equal roots then the discriminant $b^2 - 4ac = 0$ i.e. $(-30)^2 - 4 \times k \times 25k = 0$ (4)

$$900 - 100k^2 = 0$$
$$900 = 100k^2 \quad \therefore k^2 = 9 \quad \therefore k = \pm 3$$

11)



Using Pythagoras' Theorem

$k^2 = (3x)^2 + (4x)^2 = 9x^2 + 16x^2 = 25x^2$ (2)

$\therefore k = 5x$

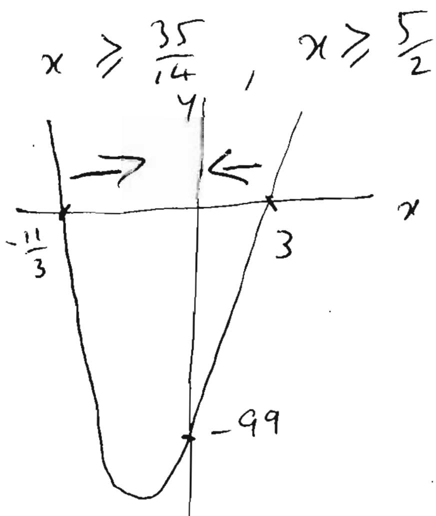
$\therefore$ perimeter $P = (2+5x) + 5x + (2+x) + 3x = 4 + 14x$

(ii) Area of a trapezium $= \frac{1}{2}(a+b)h = \frac{1}{2}[2+x + 2+5x] \times 3x = \frac{1}{2}[4+6x]3x = 6x + 9x^2$ (2)

$P \geqslant 39\,m \qquad A < 99\,m^2$

$\therefore 4 + 14x \geqslant 39\,m \qquad \therefore 14x \geqslant 35\,m \quad x \geqslant \frac{35}{14}, \quad x \geqslant \frac{5}{2}\,m$

$$9x^2 + 6x < 99$$
$$9x^2 + 6x - 99 < 0$$
$$3(3x^2 + 2x - 33) < 0$$
$$3(3x + 11)(x - 3) < 0$$



It can be seen from the graph that

$$-\tfrac{11}{3} < x < 3$$

(7)

To satisfy both conditions

$$\tfrac{5}{2} \leq x < 3.$$