Core 1 January 2009

1 $\sqrt{45} + \frac{20}{\sqrt{5}} = 3\sqrt{5} + 20\frac{\sqrt{5}}{5} = 7\sqrt{5}$ k = 7

2 a) $(\sqrt[3]{x})^6 = (x^{\frac{1}{3}})^6 = x^2$ b) $\frac{3y^4 \times (10y)^3}{2y^5} = \frac{3y^4 \times 1000y^3}{2y^5} = \frac{3000}{2}y^{7-5} = 1500y^2$

3 $3x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$, let $y = x^{\frac{1}{3}}$ $3y^2 + y - 2 = 0$

$(3y - 2)(y + 1) = 0$

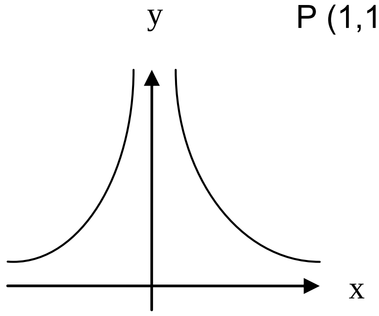
either $y = \frac{2}{3}$, or $y = -1$

$x^{\frac{1}{3}} = \frac{2}{3}$ $x = \frac{8}{27}$ $x^{\frac{1}{3}} = -1$ $x = -1$

4

$y = \frac{1}{x^2}$ $f(x) = \frac{1}{x^2}$ $f(x + 3) = \frac{1}{(x+3)^2}$ $4f(x) = \frac{4}{x^2}$

P (1,1) Q (1,4)



5 i) $\frac{dy}{dx} = -50x^{-6}$

ii) $\frac{dy}{dx} = \frac{1}{4}x^{\frac{-3}{4}}$

iii) $y = x(x - 5x^2 + 3 - 15x) = 3x - 14x^2 - 5x^3$

$\frac{dy}{dx} = 3 - 28x - 15x^2$

6 i) $5x^2 + 20x - 8 = 5(x + q)^2 + r = 5x^2 + 10qx + 5q^2 + r$

matching up q = 2 and r = –28

Or $5\left[(x + 2)^2 - 4 - \frac{8}{5})\right] = 5\left[(x + 2)^2 - \frac{28}{5}\right] = 5(x + 2)^2 - 28$

ii) line of symmetry x = –2

iii) $b^2 - 4ac = 400 - (4 \times 5 \times -8) = 560$

iv) $b^2 - 4ac > 0$ $5x^2 + 20x - 8 = 0$ has 2 real roots.

7 i) $3x + 4y - 10 = 0$ when $x = 10$ $4y = -20$, $y = -5$ B(10, k) $k = -5$

ii) A (2,1) B (10, -5) $AB^2 = (2-10)^2 + (1- -5)^2 = 64 + 36 = 100$ AB = 10

iii) $(x-6)^2 + (y+2)^2 = 25$ circle centre (6, -2) radius 5 diameter 10

iv) mid point AB $\left(\frac{2+10}{2}, \frac{1-5}{2}\right) = (6,-2)$

which is centre of circle making AB the diameter.

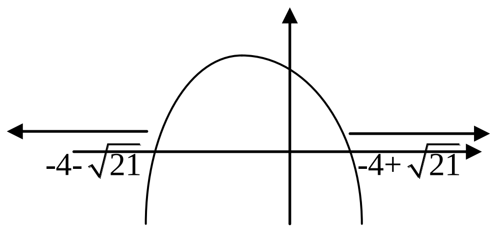
8 i) $5 - 8x - x^2 = 0$ $x^2 + 8x - 5 = 0$ $x = \frac{-8 \pm \sqrt{64+20}}{2} = \frac{-8 \pm 2\sqrt{21}}{2}$

$x = -4 \pm \sqrt{21}$

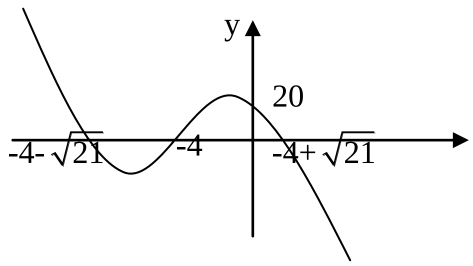
$5 - 8x - x^2 \le 0$

$x \le -4 - \sqrt{21}$

$x \ge -4 + \sqrt{21}$



iii) $-(x + 4 + \sqrt{21})(x + 4 - \sqrt{21})(x + 4)$



9 $y = x^3 + px^2 + 2$ $\frac{dy}{dx} = 3x^2 + 2px$ at the stationary point $\frac{dy}{dx} = 0$, given x = 4

$48 + 8p = 0$ p = -6. $\frac{d^2y}{dx^2} = 6x + 2(-6) = 6x - 12$

When x = 4 $\frac{d^2y}{dx^2} = 12$ $\frac{d^2y}{dx^2} > 0$ there is a minimum at x = 4

10 i) $y = x^2 + x$ $\frac{dy}{dx} = 2x + 1$ x = 2 gradient = 5

ii) x = 2, y = 6 gradient of normal $\frac{-1}{5}$

equation of normal $y - 6 = \frac{-1}{5}(x - 2)$ $x + 5y - 32 = 0$

iii) $y = kx - 4$ $kx - 4 = x^2 + x$

$x^2 + x(1-k) + 4 = 0$ but $b^2 - 4ac = 0$

$(1-k)^2 - 16 = 0$

$(1-k)^2 = 16$

$1 - k = \pm 4$

$1 - 4 = k$ or $1 + 4 = k$ k = -3, k = 5