Solutions: Core Mathematics 1
January 2008

1 $\frac{4}{3-\sqrt{7}} = \frac{4}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$ 1

$= \frac{12+4\sqrt{7}}{9-3\sqrt{7}+3\sqrt{7}-7}$

3 $= \frac{12+4\sqrt{7}}{2}$ 1

$= 6+2\sqrt{7}$ 1

2 (i) The equation of a circle centre (0, 0) radius r is $x^2 + y^2 = r^2$ (Learn this!) 1

1 So if the radius is 7, the equation is $x^2 + y^2 = 49$

(ii) The equation of the circle is $x^2 + y^2 - 6x - 10y - 30 = 0$

2 Centre (a,b) a = ½ coefft x, change sign = 3 b = ½ coefft y, change sign = 5 1

$r^2 = a^2 + b^2 -$ no. $= 3^2 + 5^2 - -30 = 64$

r = 8 1

3 The right hand side can be expanded out: note a = 3 in this case

$$3(x+3)^2 + c = 3(x+3)(x+3) + c = 3(x^2 + 6x + 9) + c$$

(Remember that a bracket is squared by multiplying it by itself.)

So

$$3(x+3)^2 + c = 3x^2 + 18x + 27 + c\,.$$ 2

Compare this with the left hand side:

$$3x^2 + bx + 10 = 3x^2 + 18x + 27 + c$$

Comparing coefficients of x: b = 18 1

4 Comparing the constant terms: 10 = 27 + c

i.e. c = 10 – 27 = -17. 1

So a = 3, b = 18 and c = -17.

4 (i) $10^p = 0.1 \quad \Rightarrow \quad p = -1$ 1

(Remember that a power of -1 means the reciprocal so $10^{-1} = \frac{1}{10}$)

1

(ii) A power of ½ means the square root.

3 So $\left(25k^2\right)^{1/2} = \sqrt{25k^2} = 5k$ 1

So, we need to solve ±5k = 15 i.e. k = ±3. 2

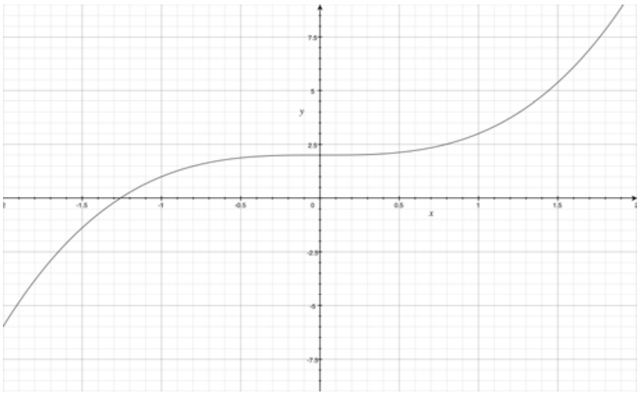
(iii) $t^{-1/3} = \frac{1}{t^{1/3}} = \frac{1}{\sqrt[3]{t}}$

So we need to solve

$$\frac{1}{\sqrt[3]{t}} = \frac{1}{2}$$ 1

2 i.e. $\sqrt[3]{t} = 2 \quad \Rightarrow \quad t = 8$ 1

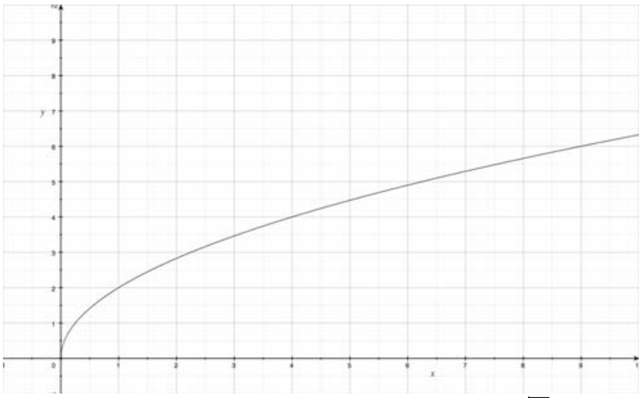
5 (i)



The graph of $y = x^3 + 2$ is formed by translating the graph of $y = x^3$ by two units upwards. 2

2

(ii)



The graph of $y = 2\sqrt{x}$ is formed by stretching the graph of $y = \sqrt{x}$ in the direction of the y-axis by a scale factor of 2. 2

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(iii) The transformation that transforms $y = 2\sqrt{x}$ onto the curve $y = 3\sqrt{x}$ is a stretch parallel to the y-axis scale factor 1.5. 3

3

6 (i) The solutions of the quadratic equation $ax^2 + bx + c = 0$ can be found using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here a = 1, b = 8 and c = 10.

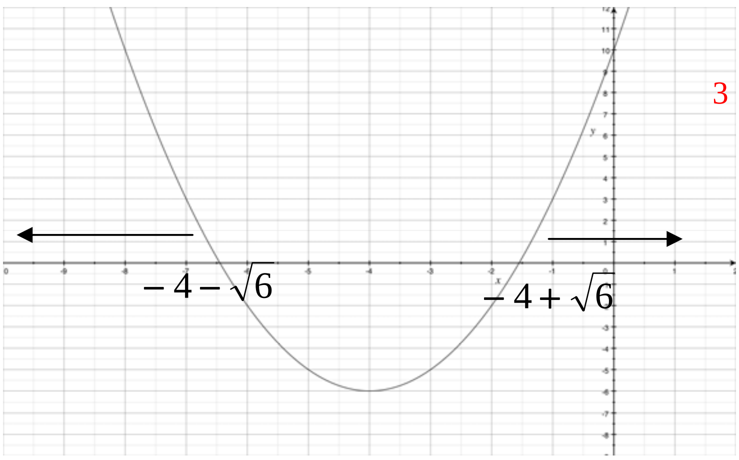
So $x = \dfrac{-8 \pm \sqrt{64 - 4 \times 1 \times 10}}{2} = \dfrac{-8 \pm \sqrt{24}}{2}$ 2

But $\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$

3

Therefore the solutions are $x = \dfrac{-8 \pm 2\sqrt{6}}{2} = -4 \pm \sqrt{6}$ 1

(ii) The curve cuts the y-axis at the point (0, 10).
The curve is a U graph and cuts the x-axis at $-4 + \sqrt{6},\ -4 - \sqrt{6}$ (both of which are negative values). 3



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(iii) From the sketch, the solutions of the inequality are: 2

2 $x \geq -4 + \sqrt{6}$ or $x \leq -4 - \sqrt{6}$

7 (i) To find the equation of the line, rearrange to the form y = mx + c:

x+ 2y = 4 i.e 2y = 4 – x i.e. y = 2 – 0.5x.

1

So the gradient is -0.5. 1

(ii) A parallel line will have the same gradient, so the parallel line will have gradient -0.5.
The equation of a line with gradient m which passes through the point (a, b) is:

$$y - y_1 = m(x - x_1)$$

So

$$y - 5 = -0.5(x - 6)$$

Multiply by 2:

$$2y - 10 = -x + 6$$

Rearrange to get:

3 $x + 2y - 16 = 0$ 3

(iii) Rearrange the equation x + 2y = 4 to make y the subject: y = 2 – 0.5x. 1

Substitute this into the equation of the curve:

$$2 - 0.5x = x^2 + x + 1$$

Multiply by 2 to remove the fraction:

$$4 - x = 2x^2 + 2x + 2$$

Rearrange to make one side equal to 0:

$2x^2 + 3x - 2 = 0$. 1

This equation can be solved by factorising:

$$2x^2 + 4x - 1x - 2 = 0$$
$$2x(x + 2) - 1(x + 2) = 0$$
$$(x + 2)(2x - 1) = 0$$

Therefore x = -2 or x = 0.5. 1

If x = -2, y = 2 – 0.5(-2) = 3 (–2, 3)
4 If x = 0.5, y = 2 – 0.5(0.5) = 1.75 (0.5, 1.75) 1

8 (i) To find the coordinates of the stationary points, the steps are:

1) Differentiate the equation of the curve to get $\frac{dy}{dx}$
2) Find the x-coordinates of the stationary points by solving $\frac{dy}{dx}$=0;
3) Find the y-coordinates of each point using the equation of the curve.

Here $y = x^3 + x^2 - x + 3$

Therefore:

$\frac{dy}{dx} = 3x^2 + 2x - 1$ 1

To find the coordinates of the stationary points we solve $3x^2 + 2x - 1$=0.
This can be solved by factorising:

$3x^2 + 2x - 1 = 3x^2 + 3x - 1x - 1$
= 3x(x + 1) – 1(x + 1)
= (3x – 1)(x + 1) 1

So the solutions are x = $^1/_3$ or x = -1. 2

When x = $\frac{1}{3}$, $y = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \frac{1}{3} + 3 = \frac{1}{27} + \frac{1}{9} - \frac{1}{3} + 3 = 2\frac{22}{27}$

When x = -1, $y = (-1)^3 + (-1)^2 - (-1) + 3 = 4$

6 The coordinates of the stationary points are (-1, 4) and $\left(\frac{1}{3}, 2\frac{22}{27}\right)$. 2

(ii) To decide whether the stationary points are maximums or minimums the steps are:

1) Find the second derivative $\frac{d^2y}{dx^2}$
2) Substitute each x value into the second derivative
3) If $\frac{d^2y}{dx^2} > 0$, then it is a minimum; If $\frac{d^2y}{dx^2} < 0$, then it is a maximum.

3 Here, $\frac{d^2y}{dx^2} = 6x + 2$ When x = -1, $\frac{d^2y}{dx^2} = 6(-1) + 2 = -4 < 0$, i.e. a maximum point

When x = 1/3, $\frac{d^2y}{dx^2} = 6(\frac{1}{3}) + 2 = 4 > 0$ i.e. a minimum point.

(iii) We can sketch the graph of $y = x^3 + x^2 - x + 3$:

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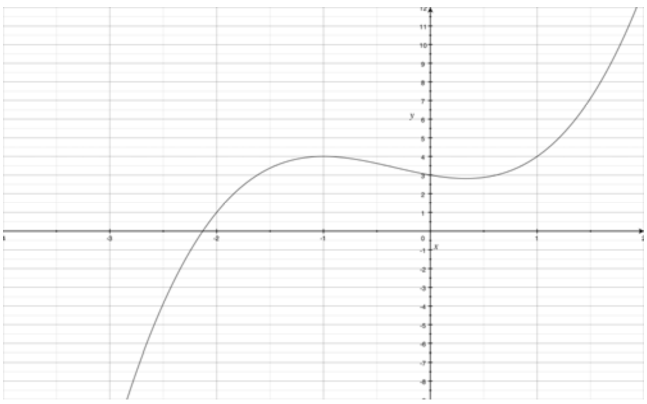

This shows that the curve is decreasing if -1 < x < 1/3.

9 (i) The gradient of the line AB is $m = \frac{y_2 - y_1}{x_2 - x_1}$ (LEARN THIS!)

So the gradient is $m = \frac{1-(-2)}{3-(-5)} = \frac{3}{8}$ 1

Using the formula $y - y_1 = m(x - x_1)$ for the equation of a straight line, we get:

$y - 1 = \frac{3}{8}(x - 3)$ 1

Multiply by 8 to remove the fraction:

$8y - 8 = 3x - 9$

3 Therefore: -3x + 8y + 1 = 0. or 3x – 8y –1 = 0 1

(ii) The coordinates of the midpoint of AB are: $\left(\frac{-5+3}{2}, \frac{-2+1}{2}\right) = (-1, -\frac{1}{2})$ 2

2

(iii) We can calculate the distance between two points using Pythagoras's theorem (if we draw a sketch of the diagram) OR we can use the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3

So: $d = \sqrt{(-3 - -5)^2 + (4 - -2)^2} = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$ 3

(iv) Gradient of AC is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{-3-(-5)} = \frac{6}{2} = 3$

1

Gradient of BC is $\frac{4-(1)}{-3-3} = \frac{3}{-6} = -\frac{1}{2}$ 1

4 AC is not perpendicular to BC as the product of their gradients is not -1. 2

(Note: Two lines are perpendicular if the product of their gradients is -1, $m_1 \times m_2 = -1$ i.e. if the gradient of one line is the negative reciprocal of the gradient of the other).

10(i) $f(x) = 8x^3 + \frac{1}{x^3} = 8x^3 + x^{-3}$ (Write as a negative power)

Therefore

$f'(x) = 24x^2 + -3x^{-4} = 24x^2 - 3x^{-4}$ 3

(drop the power down infront and make the power 1 less).

5 So $f''(x) = 48x + 12x^{-5}$

(ii) We have to solve:

$$8x^3 + \frac{1}{x^3} = -9$$

Multiply by x^3:

$$8x^6 + 1 = -9x^3$$

So: $8x^6 + 9x^3 + 1 = 0$ 1

This can be turned into a quadratic is we substitute $y = x^3$:

$8y^2 + 9y + 1 = 0$ 1

This can be solved by factorising;

$8y^2 + 8y + 1y + 1 = 8y(y+1) + 1(y+1) = (8y+1)(y+1)$ 1

So the solutions are y = -1 or $y = -\frac{1}{8}$

5 So:

$x = \sqrt[3]{y} = \sqrt[3]{-1} = -1$ *or* $x = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$ 2