4751 (C1) Introduction to Advanced Mathematics

Section A

1	$x > 6/4$ o.e. isw	2	M1 for $4x > 6$ or for 6/4 o.e. found or for their final ans ft their $4x > k$ or $kx > 6$	2
2	(i) (0, 4) and (6, 0) (ii) −4/6 o.e. or ft their (i) isw	2 2	1 each; allow $x = 0$, $y = 4$ etc; condone $x = 6$, $y = 4$ isw but 0 for (6, 4) with no working 1 for $-\frac{4}{6}x$ or 4/−6 or 4/6 o.e. or ft (accept 0.67 or better) 0 for just rearranging to $y = -\frac{2}{3}x + 4$	4
3	(i) 0 or −3/2 o.e. (ii) $k < -9/8$ o.e. www	2 3	1 each M2 for 3^2 (−)(−8k) < 0 o.e. or −9/8 found or M1 for attempted use of $b^2 - 4ac$ (may be in quadratic formula); SC: allow M1 for $9 - 8k < 0$ and M1 ft for $k > 9/8$	5
4	(i) T (ii) E (iii) T (iv) F	3	3 for all correct, 2 for 3 correct. 1 for 2 correct	3
5	$y(x - 2) = (x + 3)$ $xy - 2y = x + 3$ or ft [ft from earlier errors if of comparable difficulty – no ft if there are no xy terms] $xy - x = 2y + 3$ or ft $[x =]\dfrac{2y+3}{y-1}$ o.e. or ft <u>alt method:</u> $y = 1 + \dfrac{5}{x-2}$ $y - 1 = \dfrac{5}{x-2}$ $x - 2 = \dfrac{5}{y-1}$ $x = 2 + \dfrac{5}{y-1}$	M1 M1 M1 M1 M1 M1 M1 M1	for multiplying by $x - 2$; condone missing brackets for expanding bracket and being at stage ready to collect x terms for collecting x and 'other' terms on opposite sides of eqn for factorising and division for either method: award 4 marks only if fully correct	4

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6	(i) 5 www (ii) $8x^{10}y^{13}z^4$ or $2^3x^{10}y^{13}z^4$	2 3	allow 2 for ±5; M1 for $25^{1/2}$ seen or for 1/5 seen or for using $25^{1/2} = 5$ with another error (ie M1 for coping correctly with fraction and negative index or with square root) mark final answer; B2 for 3 elements correct, B1 for 2 elements correct; condone multn signs included, but −1 from total earned if addn signs	5
7	(i) $\frac{5-\sqrt{3}}{22}$ or $\frac{5+(-1)\sqrt{3}}{22}$ or $\frac{5-1\sqrt{3}}{22}$ (ii) 37 − 12√ 7 isw www	2 3	or $a = 5$, $b = -1$, $c = 22$; M1 for attempt to multiply numerator and denominator by $5-\sqrt{3}$ 2 for 37 and 1 for − 12√ 7 or M1 for 3 correct terms from 9 − 6√ 7 − 6√ 7 + 28 or 9 − 3√ 28 − 3√ 28 + 28 or 9 − √ 252 − √ 252 + 28 o.e. eg using 2√ 63 or M2 for 9 − 12√ 7 + 28 or 9 − 6√ 28 + 28 or 9 − 2√ 252 + 28 or 9 − √ 1008 + 28 o.e.; 3 for 37 − √ 1008 but not other equivs	5
8	−2000 www	4	M3 for $10 \times 5^2 \times (-2[x])^3$ o.e. or M2 for two of these elements or M1 for 10 or (5×4×3)/(3×2×1) o.e. used [$^5C_3$ is not sufficient] or for 1 5 10 10 5 1 seen; or B3 for 2000; condone x^3 in ans; equivs: M3 for e.g $5^5 \times 10 \times \left(-\frac{2}{5}[x]\right)^3$ o.e. [5^5 may be outside a bracket for whole expansion of all terms], M2 for two of these elements etc similarly for factor of 2 taken out at start	4
9	(y − 3)(y − 4) [= 0] y = 3 or 4 cao $x = \pm\sqrt{3}$ or ± 2 cao	M1 A1 B2	for factors giving two terms correct or attempt at quadratic formula or completing square or B2 (both roots needed) B1 for 2 roots correct or ft their y (condone √ 3 and √ 4 for B1)	4

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Section B

10	**i**	$(x-3)^2-7$	3	mark final answer; 1 for $a = 3$, 2 for $b = 7$ or M1 for $-3^2 + 2$; bod 3 for $(x - 3) - 7$	3	
	ii	(3, −7) or ft from (i)	1+1		2	
	iii	sketch of quadratic correct way up and through (0, 2)	G1	accept (0, 2) o.e. seen in this part [eg in table] if 2 not marked as intercept on graph		
		t.p. correct or ft from (ii)	G1	accept 3 and −7 marked on axes level with turning pt., or better; no ft for (0, 2) as min	2	
	iv	$x^2 - 6x + 2 = 2x - 14$ o.e.	M1	or their (i) = $2x - 14$		
		$x^2 - 8x + 16$ [= 0]	M1	dep on first M1; condone one error		
		$(x - 4)^2$ [= 0]	M1	or correct use of formula, giving equal roots; allow $(x + 4)^2$ o.e. ft $x^2 + 8x + 16$		
		$x = 4$, $y = -6$	A1	if M0M0M0, allow SC2 for showing (4, −6) is on both graphs (need to go on to show line is tgt to earn more)		
		equal/repeated roots [implies tgt] - must be explicitly stated; condone ‘only one root [so tgt]’ or ‘line meets curve only once, so tgt’ or ‘line touches curve only once’ etc]	A1	or for use of calculus to show grad of line and curve are same when $x = 4$	5	12

11	**i**	f(−4) used	M1			
		−128 + 112 + 28 − 12 [= 0]	A1	or B2 for $(x + 4)(2x^2 - x - 3)$ here; or correct division with no remainder	2	
	ii	division of f(x) by (x + 4)	M1	as far as $2x^3 + 8x^2$ in working, or two terms of $2x^2 - x - 3$ obtained by inspection etc (may be earned in (i)), or f(−1) = 0 found		
		$2x^2 - x - 3$	A1	$2x^2 - x - 3$ seen implies M1A1		
		$(x + 1)(2x - 3)$	A1			
		[f(x) =] $(x + 4)(x + 1)(2x - 3)$	A1	or B4; allow final A1 ft their factors if M1A1A0 earned	4	
	iii	sketch of cubic correct way up	G1	ignore any graph of $y = f(x - 4)$		
		through −12 shown on y axis	G1	or coords stated near graph		
		roots −4, −1, 1.5 or ft shown on x axis	G1	or coords stated near graph if no curve drawn, but intercepts marked on axes, can earn max of G0G1G1	3	
	iv	$x(x - 3)(2[x - 4] - 3)$ o.e. or $x(x - 3)(x - 5.5)$ or ft their factors	M1	or $2(x - 4)^3 + 7(x - 4)^2 - 7(x - 4) - 12$ or stating roots are 0, 3 and 5.5 or ft; condone one error eg $2x - 7$ not $2x - 11$		
		correct expansion of one pair of brackets ft from their factors	M1	or for correct expn of $(x - 4)^3$ [allow unsimplified]; or for showing g(0) = g(3) = g(5.5) = 0 in given ans g(x)		
		correct completion to given answer	M1	allow M2 for working backwards from given answer to $x(x - 3)(2x - 11)$ and M1 for full completion with factors or roots	3	12

12	**i**	grad AB = $\frac{9-1}{3--1}$ or 2	M1		
		$y - 9 = 2(x - 3)$ or $y - 1 = 2(x + 1)$	M1	ft their *m*, or subst coords of A or B in y = their $m\,x + c$	
		$y = 2x + 3$ o.e.	A1	or B3	3
	ii	mid pt of AB = (1, 5)	M1	condone not stated explicitly, but used in eqn	
		grad perp = −1/grad AB	M1	soi by use eg in eqn	
		$y - 5 = -\frac{1}{2}(x - 1)$ o.e. or ft [no ft for just grad AB used]	M1	ft their grad and/or midpt, but M0 if their midpt not used; allow M1 for $y = -\frac{1}{2}x + c$ and then their midpt subst	
		at least one correct interim step towards given answer $2y + x = 11$, and correct completion NB ans $2y + x = 11$ given	M1	no ft; correct eqn only	
		<u>alt method working back from ans:</u>		mark one method or the other, to benefit of cand, not a mixture	
		$y = \frac{11-x}{2}$ o.e.	M1		
		grad perp = −1/grad AB and showing/stating same as given line	M1	eg stating $-\frac{1}{2} \times 2 = -1$	
		finding intn of their $y = 2x + 3$ and $2y + x = 11$ [= (1, 5)]	M1	or showing that (1, 5) is on $2y + x = 11$, having found (1, 5) first	4
		showing midpt of AB is (1, 5)	M1	[for both methods: for M4 must be fully correct]	
	iii	showing $(-1 - 5)^2 + (1 - 3)^2 = 40$	M1	at least one interim step needed for each mark; M0 for just $6^2 + 2^2 = 40$ with no other evidence such as a first line of working or a diagram; condone marks earned in reverse order	
		showing B to centre = $\sqrt{40}$ or verifying that (3, 9) fits given circle	M1		2
	iv	$(x - 5)^2 + 3^2 = 40$	M1	for subst $y = 0$ in circle eqn	
		$(x - 5)^2 = 31$	M1	condone slip on rhs; or for rearrangement to zero (condone one error) <u>and</u> attempt at quad. formula [allow M1 M0 for $(x - 5)^2 = 40$ or for $(x - 5)^2 + 3^2 = 0$]	
		$x = 5 \pm \sqrt{31}$ or $\frac{10 \pm \sqrt{124}}{2}$ isw	A1	or $5 \pm \frac{\sqrt{124}}{2}$	3

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