Mark Scheme 4751
June 2005

Section A

1	40	2	M1 subst of 3 for x or attempt at long divn with $x^3 - 3x^2$ seen in working; 0 for attempt at factors by inspection	2
2	$[x =]\dfrac{6y}{3+m}$ as final answer	3	M1 for $3x + mx = y + 5y$ o.e. and M1 for $x(3 + m)$ or ft sign error	3
3	$n + 1$ and $n + 2$ both seen $3n + 3$ $=3(n + 1)$ o.e.	1 M1 A1	condone e.g. a instead of n for last 2 marks or starting again with full method for middle number = y etc or 3 a factor of both terms so divisible by 3	3
4	–0.6 o.e. (4, 0) (0, 12/5) o.e.	2 1 1	M1 for 0.6 or $-0.6x$ o.e. or rearrangement to 'y =' form [need not be correct] condone values of x and y given	4
5	$8 - 12x + 6x^2 - x^3$ isw	4	B3 for 3 terms correct or all correct except for signs; B2 for two terms correct including at least one of $-12x$ and $6x^2$; B1 for 1 3 3 1 soi or for 8 and $-x^3$	4
6	(i) 1 (ii) a^8 cao (iii) $\dfrac{1}{3a^3b}$ or $\frac{1}{3}a^{-3}b^{-1}$ isw	1 1 3	 M2 for two 'terms' correct or M1 for $3a^3b$ or $\dfrac{1}{(9a^6b^2)^{\frac{1}{2}}}$ or $\dfrac{1}{\sqrt{9a^6b^2}}$; ignore ±	5
7	(i) $3\sqrt{6}$ or $\sqrt{54}$ isw (ii) $10 + 2\sqrt{7}$	2 3	M1 for $\sqrt{(4\times6)}$ or $2\sqrt{6}$ or $3\sqrt{2}\sqrt{3}$seen M1 for attempt to multiply num. and denom. by $5 + \sqrt{7}$ and M1 for 18 or 25 – 7 seen	5
8	$x(30 - 2x) = 112$ $x(15 - x) = 56$ or $30x - 2x^2 = 112$ $(x - 7)(x - 8)$ $x = 7$ or 8 7 by 16 or 8 by 14	M1 A1 1 1 1	allow M1 for length = $30 - 2x$ soi NB answer given 0 for formula or completing sq etc must be explicit; both values required allow for 16 and 14 found following 7 and 8; both required	5
9	$[y=]\ 3x + 2 = 3x^2 - 7x + 1$ $[0 =]\ 3x^2 - 10x - 1$ or $-3x^2 + 10x + 1$ $x = \dfrac{10 \pm \sqrt{100+12}}{6}$ $= \dfrac{10 \pm \sqrt{112}}{6}$ or $\dfrac{5 \pm \sqrt{28}}{3}$ o.e. isw	M1 M1 M1 A2	or rearrangement of linear and subst for x in quadratic attempted condone one error; dep on first M1 attempt at formula [dep. on first M1 and quadratic = 0]; M2 for whole method for completing square or M1 to stage before taking roots A1 for two of three 'terms' correct [with correct fraction line] or for one root	5

Section B

10	**i**	$(x-4)^2+9$	3	B1 for 4, B2 for 9 or M1 for 25 – 16	3
	ii	(4, 9) or ft	1+1		
		parabola right way up 25 at intersection on *y*-axis (mark intent)	G1 G1	condone stopping at *y* axis ignore posn of min: can ft theirs	4
	iii	$x > 7$ or $x < 1$	3	M1 for x^2-8x+7 [> 0] and M1 for $(x-7)(x-1)$ [>0] <u>or</u> M1 for $(x-4)^2$ [>] 9 and M1 for $x-4>3$ and $x-4<-3$ or B2 for 1 and 7	3
	iv	[y =] x^2-8x+5	1	or [y =] $(x-4)^2-11$	1
11	**i**	$(6-0)^2+(10-2)^2$ AC = 10 AB = √98 and BC = √2 clear correct use of Pythagoras's theorem	M1 A1 1 1	 or 1 for grad AB = 1 and grad BC = –1 and 1 for comment/ showing $m_1m_2=-1$ o.e.	4
	ii	[angle in a semicircle so]AC diameter [so radius = 5] midpt of AC = (6/2, [10+2]/2) $(x-3)^2+(y-6)^2=5^2$ o.e. isw	1 1 2	*d* or diameter needed; NB ans given method must be shown; NB ans givn B1 for one side correct	4
	iii	[grad AC =] 8/6 or 4/3 grad tgt = –3/4 $y-10=[-3/4](x-6)$ o.e. [e.g. $3x+4y=58$] or ft (58/3, 0) and (0, 58/4) o.e. isw	1 M1 M1 A2	 for grad tgt = –1/their grad AC or M1 for y = their $m\,x+c$ then subst (6, 10) to find c 1 each cao; condone not as coords	5
12	**i**	$(x+1)(x-2)(x-5)$ $(x+1)(x^2-7x+10)$ correct step shown towards completion [answer given]	M1 A1 A1	 o.e. with two other factors; condone missing brackets if expanded correctly; A2 for $x^3-5x^2-2x^2+x^2+10x-5x-2x+10$	3
	ii	cubic the right way up –1, 2 and 5 indicated on *x* axis 10 indicated at intn on *y* axis	G1 G1 G1	must extend beyond $x=-1$ and 5 at intersections of curve and axis	3
	iii	f(4) attempted = 64 – 96+ 12 + 10 attempt at long division of $x^3-6x^2+3x+20$ by $x-4$ as far as x^3-4x^2 in working $x^2-2x-5=0$	M1 A1 M2 A2	or f(4) + 10; or '4 a root implies $(x-4)$ a factor' or vv or 5 × 2 × –1 etc or correct long division if first M1 earned or M2 for $(x-4)(x^2+..-5)$ or $(x-4)(x^2-2x+k)$ seen; M1 for realising long divn by $x-4$ needed but not doing it A1 for x^2-2x-5 SC2 for finding f(x) ÷ $(x-4)=x^2-2x-5$ rem − 10 without further explanation	6

Mark Scheme 4751
January 2006

Section A

1	$n(n+1)$ seen = odd × even and/or even × odd = even	M1 A1	<u>or</u> B1 for n odd $\Rightarrow n^2$ odd, and comment eg odd + odd = even B1 for n even$\Rightarrow n^2$ even, and comment eg even + even = even allow A1 for 'any number multiplied by the consecutive number is even'	2
2	(i) translation of $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (ii) $y = \mathrm{f}(x - 2)$	1 1 2	 or '2 to the right' or '$x \to x + 2$' or 'all x values are increased by 2' 1 for $y = \mathrm{f}(x + 2)$	4
3	$16 + 32x + 24x^2 + 8x^3 + x^4$ isw	4	3 for 4 terms correct, 2 for 3 terms correct, or M1 for 1 4 6 4 1 s.o.i. and M1 for expansion with correct powers of 2	4
4	$x > -4.5$ o.e. isw www [M1 for × 4 M1 expand brackets or divide by 3 M1 subtract constant from LHS M1 divide to find x]	4	accept −27/6 or better; 3 for $x = -4.5$ etc or Ms for each of the four steps carried out correctly with inequality [−1 if working with equation] (ft from earlier errors if of comparable difficulty)	4
5	$[C =]\dfrac{4P}{1-P}$ or $\dfrac{-4P}{P-1}$ o.e.	4	M1 for $PC + 4P = C$ M1 for $4P = C - PC$ or ft M1 for $4P = C(1 - P)$ or ft B3 for $[C =]\dfrac{4}{\frac{1}{P}-1}$ o.e. unsimplified	4
6	f(1) used $1^3 + 3 \times 1 + k = 6$ $k = 2$	M1 A1 A1	or division by $x - 1$ as far as $x^2 + x$ or remainder = $4 + k$ B3 for $k = 2$ www	3
7	grad BC = – ¼ soi $y - 3 = -¼(x - 2)$ o.e. cao 14 or ft from their BC	2 1 2	M1 for $m_1m_2 = -1$ soi or for grad AB = 4 or grad BC = 1/4 e.g. $y = -0.25x + 3.5$ M1 for subst $y = 0$ in their BC	5
8	(i) $30\sqrt{2}$ (ii) $\dfrac{1}{11}+\dfrac{2}{11}\sqrt{3}$ or $\dfrac{3}{33}+\dfrac{6}{33}\sqrt{3}$ or mixture of these	2 3	M1 for $\sqrt{8}=2\sqrt{2}$ or $\sqrt{50} = 5\sqrt{2}$ soi B1 for $6\sqrt{50}$ or other correct $a\sqrt{b}$ M1 for mult num and denom by $6+\sqrt{3}$ and M1 for denom = 11 or 33	5

			B2 for $\frac{3+6\sqrt{3}}{33}$ or $\frac{1+2\sqrt{3}}{11}$	
9	(i) $k \leq 25/4$ (ii) −2.5	3 2	M2 for $5^2 - 4k \geq 0$ or B2 for 25/4 obtained isw or M1 for $b^2 - 4ac$ soi or completing square accept −20/8 or better, isw; M1 for attempt to express quadratic as $(2x + a)^2$ or for attempt at quadratic formula	5

Section B

10	**i**	(0, 0), √45 isw or 3√5	1+1		2
	ii	$x = 3 - y$ or $y = 3 - x$ seen or used subst in eqn of circle to eliminate variable $9 - 6y + y^2 + y^2 = 45$ $2y^2 - 6y - 36 = 0$ or $y^2 - 3y - 18 = 0$ $(y - 6)(y + 3) = 0$ $y = 6$ or -3 $x = -3$ or 6 $\sqrt{(6 - -3)^2 + (3 - -6)^2}$	M1 M1 M1 M1 M1 A1 A1 M1	for correct expn of $(3 - y)^2$ seen oe condone one error if quadratic or quad. formula attempted [complete sq attempt earns last 2 Ms] or A1 for (6, –3) and A1 for (–3, 6) no ft from wrong points (A.G.)	8
11	**i**	$(x - 3.5)^2 - 6.25$	3	B1 for $a = 7/2$ o.e, B2 for $b = -25/4$ o.e. or M1 for $6 - (7/2)^2$ or $6 - (\text{their } a)^2$	3
	ii	(3.5, –6.25) o.e. or ft from their (i)	1+1	allow $x = 3.5$ and $y = -6.25$ or ft; allow shown on graph	2
	iii	(0, 6) (1, 0) (6, 0) curve of correct shape fully correct intns and min in 4th quadrant	3 G1 G1	1 each [stated or numbers shown on graph]	5
	iv	$x^2 - 7x + 6 = x^2 - 3x + 4$ $2 = 4x$ $x = ½$ or 0.5 or 2/4 cao	M1 M1 A1	or $4x - 2 = 0$ (simple linear form; condone one error) condone no comment re only one intn	3
12	**i**	sketch of cubic the correct way up curve passing through (0, 0) curve touching x axis at (3, 0)	G1 G1 G1		3
	ii	$x(x^2 - 6x + 9) = 2$ $x^3 - 6x^2 + 9x = 2$	M1 M1	or $(x^2 - 3x)(x - 3) = 2$ [for one step in expanding brackets] for 2nd step, dep on first M1	2
	iii	subst $x = 2$ in LHS of their eqn or in $x(x - 3)^2 = 2$ o.e. working to show consistent division of their eqn by $(x - 2)$ attempted $x^2 - 4x + 1$	1 1 M1 A1	or 2 for division of their eqn by $(x - 2)$ and showing no remainder or inspection attempted with $(x^2 + kx + c)$ seen	

		soln of their quadratic by formula or completing square attempted $x = 2 \pm \sqrt{3}$ or $(4 \pm \sqrt{12})/2$ isw locating the roots on intersection of their curve and $y = 2$	M1 A2 G1	condone ignoring remainder if they have gone wrong A1 for one correct must be 3 intns; condone $x = 2$ not marked; mark this when marking sketch graph in (i)	7 G1

Mark Scheme 4751
June 2006

Section A

1	$[r] = [\pm]\sqrt{\dfrac{3V}{\pi h}}$ o.e. 'double-decker'	3	2 for $r^2 = \dfrac{3V}{\pi h}$ or $r = \sqrt{\dfrac{V}{\frac{1}{3}\pi h}}$ o.e. or M1 for correct constructive first step or for $r = \sqrt{k}$ ft their $r^2 = k$	3
2	a = ¼	2	M1 for subst of –2 or for −8 + 4a + 7 = 0 o.e. obtained eg by division by (x + 2)	2
3	3x + 2y = 26 or y = –1.5x + 13 isw	3	M1 for 3x + 2y = c or y = –1.5x + c M1 for subst (2, 10) to find c or for or for y − 10 = their gradient × (x − 2)	3
4	(i) P ⇐ Q (ii) P ⇔ Q	1 1	condone omission of P and Q	2
5	x + 3(3x + 1) = 6 o.e. 10x = 3 or 10y = 19 o.e. (0.3, 1.9) or x = 0.3 <u>and</u> y = 1.9 o.e.	M1 A1 A1	for subst <u>or</u> for rearrangement and multn to make one pair of coefficients the same <u>or</u> for both eqns in form 'y =' (condone one error) graphical soln: (must be on graph paper) M1 for each line, A1 for (0.3, 1.9) o.e cao; allow B3 for (0.3, 1.9) o.e.	3
6	–3 < x < 1 [condone x < 1, x > −3]	4	B3 for –3 and 1 or M1 for x^2 + 2x – 3 [< 0]or (x + 1)^2 < / = 4 and M1 for (x + 3)(x – 1) or x= (−2 ±4)/2 or for (x + 1) and ±2 on opp. sides of eqn or inequality; if 0, then SC1 for one of x < 1, x > −3	4
7	(i) 28√6 (ii) 49 – 12√5 isw	2 3	1 for 30√6 or 2√6 or 2√2√3 or 28√2√3 2 for 49 and 1 for – 12√5 or M1 for 3 correct terms from 4 − 6√5 − 6√5 + 45	5
8	20 –160 or ft for –8 × their 20	2 2	0 for just 20 seen in second part; M1 for 6!/(3!3!) or better condone −160x^3; M1 for [–]2^3 × [their] 20 seen or for [their] 20 × (−2x)^3; allow B1 for 160	4
9	(i) 4/27 (ii) 3$a^{10}b^8c^{-2}$ or $\dfrac{3a^{10}b^8}{c^2}$	2 3	1 for 4 or 27 2 for 3 'elements' correct, 1 for 2 elements correct, −1 for any adding of elements; mark final answer; condone correct but unnecessary brackets	5
10	x^2 + 9x^2 = 25 10x^2 = 25 x= ±(√10)/2 or.±√(5/2) or ±5/√10 oe y = [±] 3√(5/2) o.e. eg y = [±] √22.5	M1 M1 A2 B1	for subst for x or y attempted or x^2 = 2.5 o.e.; condone one error from start [allow 10x^2 − 25 = 0 + correct substn in correct formula] allow ±√2.5; A1 for one value ft 3 × their x value(s) if irrational; condone not written as coords.	5

Section B

11	**i**	grad AB = 8/4 or 2 or $y = 2x - 10$ grad BC = 1/–2 or – ½ or $y = -\frac{1}{2}x + 2.5$ product of grads = –1 [so perp] (allow seen or used)	1 1 1	or M1 for $AB^2 = 4^2 + 8^2$ or 80 and $BC^2 = 2^2 + 1^2$ or 5 and $AC^2 = 6^2 + 7^2$ or 85; M1 for $AC^2 = AB^2 + BC^2$ and 1 for [Pythag.] true so AB perp to BC; if 0, allow G1 for graph of A, B, C	3
	ii	midpt E of AC = (6, 4.5) $AC^2 = (9-3)^2 + (8-1)^2$ or 85 rad = ½ √85 o.e. $(x-6)^2 + (y-4.5)^2 = 85/4$ o.e. $(5-6)^2 + (0-4.5)^2 = 1 + 81/4$ [= 85/4]	1 M1 A1 B2 1	allow seen in (i) only if used in (ii); or $AE^2 = (9 - \text{their } 6)^2 + (8 - \text{their } 4.5)^2$ or rad.^{2} = 85/4 o.e. e.g. in circle eqn M1 for $(x-a)^2 + (y-b)^2 = r^2$ soi or for lhs correct some working shown; or 'angle in semicircle [=90°]'	6
	iii	$\overrightarrow{BE} = \overrightarrow{ED} = \begin{pmatrix} 1 \\ 4.5 \end{pmatrix}$ D has coords (6 + 1, 4.5 + 4.5) ft or (5 + 2, 0 + 9) = (7, 9)	M1 M1 A1	o.e. ft their centre; or for $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ or (9 − 2, 8 + 1); condone mixtures of vectors and coords. throughout part iii allow B3 for (7,9)	3
12	**i**	f(–2) used –8 + 36 – 40 + 12 = 0	M1 A1	or M1 for division by $(x + 2)$ attempted as far as $x^3 + 2x^2$ then A1 for $x^2 + 7x + 6$ with no remainder	2
	ii	divn attempted as far as $x^2 + 3x$ $x^2 + 3x + 2$ or $(x + 2)(x + 1)$	M1 A1	or inspection with $b = 3$ or $c = 2$ found; B2 for correct answer	2
	iii	$(x + 2)(x + 6)(x + 1)$	2	allow seen earlier; M1 for $(x + 2)(x + 1)$	2
	iv	sketch of cubic the right way up <u>through</u> 12 marked on y axis intercepts –6, –2, –1 on x axis	G1 G1 G1	with 2 turning pts; no 3rd tp curve must extend to $x > 0$ condone no graph for $x < -6$	3
	v	$[x](x^2 + 9x + 20)$ $[x](x + 4)(x + 5)$ $x = 0, -4, -5$	M1 M1 A1	or other partial factorisation or B1 for each root found e.g. using factor theorem	3
13	**i**	$y = 2x + 3$ drawn on graph $x = 0.2$ to 0.4 and – 1.7 to –1.9	M1 A2	 1 each; condone coords; must have line drawn	3
	ii	$1 = 2x^2 + 3x$ $2x^2 + 3x - 1$ [= 0] attempt at formula or completing square $x = \dfrac{-3 \pm \sqrt{17}}{4}$	M1 M1 M1 A2	for multiplying by x correctly for correctly rearranging to zero (may be earned first) or suitable step re completing square if they go on ft, but no ft for factorising A1 for one soln	5
	iii	branch through (1,3), branch through (−1,1),approaching $y = 2$ from below	1 1	and approaching $y = 2$ from above and extending below x axis	2
	iv	–1 and ½ or ft intersection of their curve and line [tolerance 1 mm]	2	1 each; may be found algebraically; ignore y coords.	2

Mark Scheme 4751
January 2007

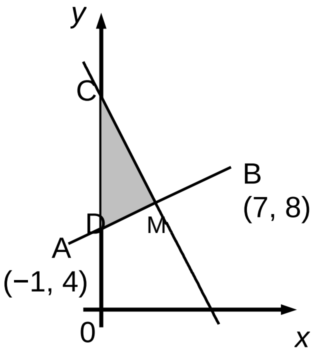
Section A

1	$y = 2x + 4$	3	M1 for $m = 2$ stated [M0 if go on to use $m = -½$] or M1 for $y = 2x + k$, $k \neq 7$ and M1indep for $y - 10 = m(x - 3)$ or (3, 10) subst in $y = mx + c$; allow 3 for $y = 2x + k$ and $k = 4$	3
2	neg quadratic curve intercept (0, 9) <u>through</u> (3, 0) and (−3, 0)	1 1 1	condone (0, 9) seen eg in table	3
3	$[a =]\dfrac{2c}{2-f}$ or $\dfrac{-2c}{f-2}$ as final answer	3	M1 for attempt to collect *a*s and *c*s on different sides and M1 ft for $a(2 - f)$ or dividing by $2 - f$; allow M2 for $\dfrac{7c-5c}{2-f}$ etc	3
4	f(2) = 3 seen or used $2^3 + 2k + 5 = 3$ o.e. $k = -5$	M1 M1 B1	allow M1 for divn by $(x - 2)$ with $x^2 + 2x + (k + 4)$ or $x^2 + 2x - 1$ obtained <u>alt:</u> M1 for $(x - 2)(x^2 + 2x - 1) + 3$ (may be seen in division) then M1dep (and B1) for $x^3 - 5x + 5$ <u>alt</u> divn of $x^3 + kx + 2$ by $x - 2$ with no rem.	3
5	375	3	allow $375x^4$; M1 for 5^2 or 25 used or seen with x^4 and M1 for 15 or $\dfrac{6\times5}{2}$ oe eg $\dfrac{6!}{4!2!}$ or 1 6 15 … seen [$^6C_4$ not sufft]	3
6	(i) 125 (ii) $\dfrac{9}{49}$ as final answer	2 2	M1 for $25^{\frac{1}{2}} = \sqrt{25}$ soi or for $\sqrt{25^3}$ M1 for $a^{-1} = \dfrac{1}{a}$ soi eg by 3/7 or 3/49	4
7	showing $a + b + c = 6$ o.e $bc = \dfrac{9^2 - 17}{16}$ =64/16 o.e. correctly obtained completion showing $abc = 6$ o.e.	1 M1 A1 A1	simple equiv fraction eg 192/32 or 24/4 correct expansion of numerator; may be unsimplified 4 term expansion; M0 if get no further than $\left(\sqrt{17}\right)^2$; M0 if no evidence before 64/16 o.e. may be implicit in use of factors in completion	4

8	$b^2 - 4ac$ soi use of $b^2 - 4ac$ **<** 0 $k^2 < 16$ [may be implied by $k < 4$] $-4 < k < 4$ or $k > -4$ and $k < 4$ isw	M1 M1 A1 A1	may be implied by $k^2 < 16$ deduct one mark in qn for ≤ instead of <; allow equalities earlier if final inequalities correct; condone *b* instead of *k*; if M2 not earned, give SC2 for qn [or M1 SC1] for *k* [=] 4 and − 4 as answer]	4
9	(i) $12a^5b^3$ as final answer (ii) $\frac{(x+2)(x-2)}{(x-2)(x-3)}$ $\frac{x+2}{x-3}$ as final answer	2 M2 A1	1 for 2 'terms' correct in final answer M1 for each of numerator or denom. correct or M1, M1 for correct factors seen separately	5
10	correct expansion of both brackets seen (may be unsimplified), or difference of squares used $4m^2$ correctly obtained [p =] [±]2m cao	M2 A1 A1	M1 for one bracket expanded correctly; for M2, condone done together and lack of brackets round second expression if correct when we insert the pair of brackets	4

Section B

11	**iA**	0.2 to 0.3 and 3.7 to 3.8	1+1	[tol. 1mm or 0.05 throughout qn]; if 0, allow M1 for drawing down lines at both values	2
	iB	$x + \frac{1}{x} = 4 - x$	M1	condone one error	
		their $y = 4 - x$ drawn	M1	allow M2 for plotting positive branch of $y = 2x + 1/x$ [plots at (1,3) and (2,4.5) and above other graph] or for plot of $y = 2x^2 - 4x + 1$	
		0.2 to 0.35 and 1.65 to 1.8	B2	1 each	4
	ii	(0, ±√3)	2	condone y = ±√3 isw; 1 each or M1 for $1 + y^2 = 4$ or $y^2 = 3$ o.e.	2
	iii	centre (1, 0) radius 2 touches at (1, 2) [which is distance 2 from centre] all points on other branch > 2 from centre	1+1 1 1	allow seen in (ii) allow ft for both these marks for centre at (−1, 0), rad 2; allow 2 for good sketch or compass-drawn circle of rad 2 centre (±1, 0)	4

12	**i**	(3, 6) grad AB = (8 − 4)/(7 − −1) or 4/8 grad normal = −2 or ft perp bisector is $y - 6 = -2(x - 3)$ or ft their grad. of normal (not AB) and/or midpoint correct step towards completion	2 M1 M1 M1 A1	1 each coord indep obtained for use of $m_1m_2 = -1$; condone stated/used as −2 with no working only if 4/8 seen or M1 for showing grad given line = −2 and M1 for showing (3, 6) fits given line	 6
	ii	Bisector crosses *y* axis at C (0, 12) seen or used AB crosses *y* axis at D (0, 4.5) seen or used ½ × (12 − their 4.5) × 3 (may be two triangles M1 each) 45/4 o.e. without surds, isw  alt allow integration used: $\int_0^3 (-2x+12)\mathrm{d}x\ [=27]$ obtaining AB is $y - 8$ = their ½ $(x - 7)$ oe [$y = \frac{1}{2}x + 4.5$] $\int_0^3 (\frac{1}{2}x + 4.5)\mathrm{d}x$ = 63/4 o.e. cao their area under CB − their area under AB = 45/4 o.e. cao	M1 B2 M2 A1 M1 M1 M1 A1 M1 A1	may be implicit in their area calcn M1 for 4 + their grad AB or for eqn AB is $y - 8$ = their ½ $(x - 7)$ oe with coords of A or their M used or M1 for $[MC]^2 = 3^2 + 6^2$ or 45 or $[MD]^2 = 3^2 + 1.5^2$ or 11.25 oe and M1 for ½ × their MC × MD; all ft their M <u>MR</u>: AMC used not DMC: lose B2 for D but then allow ft M1 for MC^2 or MA^2 [$=4^2 + 2^2$] and M1 for ½ × MA × MC and A1 for 15 <u>MR</u>: intn used as D(0, 4) can score a max of M1, B0, M2 (eg M1 for their DM = √13), A0 condone poor notation allow if seen, with correct line and limits seen/used as above ft from their AB allow only if at least some valid integration/area calculations for these trapezia seen if combined integration, so 63/4 not found separately, mark equivalently for Ms and allow A2 for final answer	 6
13	**i**	$x - 2$ is factor soi attempt at divn by $x - 2$ as far as $x^3 - 2x^2$ seen in working $x^2 + 2x - 1$ obtained attempt at quad formula or comp square $-1 \pm \sqrt{2}$ as final answer	M1 M1 A1 M1 A2	eg may be implied by divn or other factor (x^2 ….−1) or ($x^2 + 2x$...) or B3 www ft their quadratic A1 for $\frac{-2 \pm \sqrt{8}}{2}$ seen; or B3 www	 6

	ii	$f(x-3) = (x-3)^3 - 5(x-3) + 2$	B1	or $(x-5)(x-2+\sqrt{2})(x-2-\sqrt{2})$ soi or ft from their (i)	
		$(x-3)(x^2-6x+9)$ or other constructive attempt at expanding $(x-3)^3$ eg 1 3 3 1 soi	M1	for attempt at multiplying out 2 brackets or valid attempt at multiplying all 3	
		$x^3 - 9x^2 + 27x - 27$ $-5x + 15$ [+2]	A1 B1	alt: A2 for correct full unsimplified expansion or A1 for correct 2 bracket expansion eg $(x-5)(x^2-4x+2)$	4
	iii	5 $2 \pm \sqrt{2}$ or ft	B1 B1	condone factors here, not roots if B0 in this part, allow SC1 for their roots in (i) − 3	2

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT
MATHEMATICS (MEI)

4751/01

Introduction to Advanced Mathematics (C1)

INSERT

TUESDAY 16 JANUARY 2007

Morning
Time: 1 hour 30 minutes

Candidate Name

Centre Number

Candidate Number

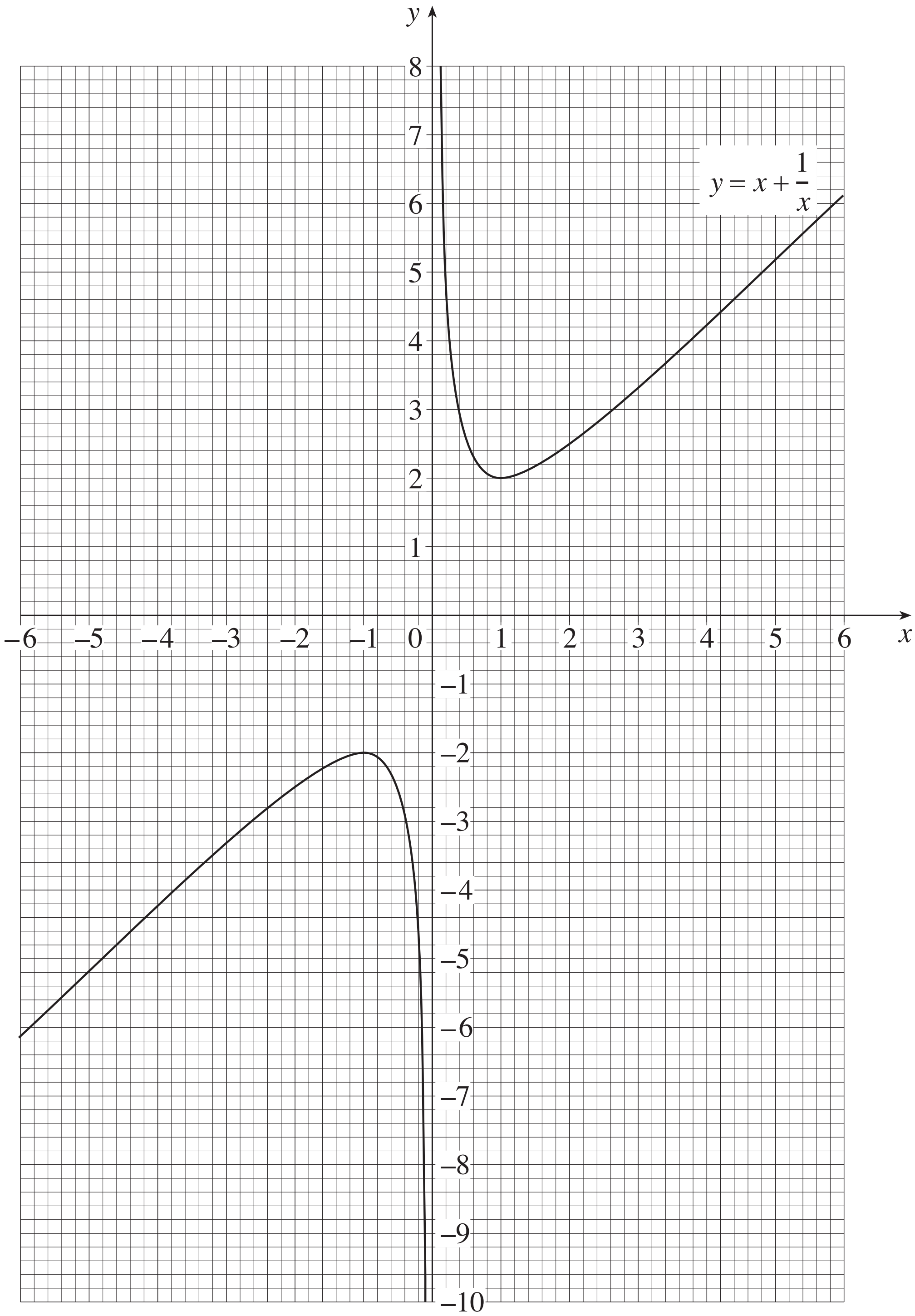
INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **11**.
- Write your name, centre number and candidate number in the spaces provided above and **attach the page to your answer booklet**.

This insert consists of 2 printed pages.

[Turn over

11 (i)



Mark Scheme 4751
January 2007

Section A

1	$y = 2x + 4$	3	M1 for $m = 2$ stated [M0 if go on to use $m = -½$] or M1 for $y = 2x + k$, $k \neq 7$ and M1indep for $y - 10 = m(x - 3)$ or (3, 10) subst in $y = mx + c$; allow 3 for $y = 2x + k$ and $k = 4$	3
2	neg quadratic curve intercept (0, 9) through (3, 0) and (−3, 0)	1 1 1	condone (0, 9) seen eg in table	3
3	$[a =]\dfrac{2c}{2-f}$ or $\dfrac{-2c}{f-2}$ as final answer	3	M1 for attempt to collect *a*s and *c*s on different sides and M1 ft for $a(2 - f)$ or dividing by $2 - f$; allow M2 for $\dfrac{7c-5c}{2-f}$ etc	3
4	f(2) = 3 seen or used $2^3 + 2k + 5 = 3$ o.e. $k = -5$	M1 M1 B1	allow M1 for divn by $(x - 2)$ with $x^2 + 2x + (k + 4)$ or $x^2 + 2x - 1$ obtained alt: M1 for $(x - 2)(x^2 + 2x - 1) + 3$ (may be seen in division) then M1dep (and B1) for $x^3 - 5x + 5$ alt divn of $x^3 + kx + 2$ by $x - 2$ with no rem.	3
5	375	3	allow $375x^4$; M1 for 5^2 or 25 used or seen with x^4 and M1 for 15 or $\dfrac{6\times5}{2}$ oe eg $\dfrac{6!}{4!2!}$ or 1 6 15 … seen [$^6C_4$ not sufft]	3
6	(i) 125 (ii) $\dfrac{9}{49}$ as final answer	2 2	M1 for $25^{\frac{1}{2}} = \sqrt{25}$ soi or for $\sqrt{25^3}$ M1 for $a^{-1} = \dfrac{1}{a}$ soi eg by 3/7 or 3/49	4
7	showing $a + b + c = 6$ o.e $bc = \dfrac{9^2 - 17}{16}$ =64/16 o.e. correctly obtained completion showing $abc = 6$ o.e.	1 M1 A1 A1	simple equiv fraction eg 192/32 or 24/4 correct expansion of numerator; may be unsimplified 4 term expansion; M0 if get no further than $\left(\sqrt{17}\right)^2$; M0 if no evidence before 64/16 o.e. may be implicit in use of factors in completion	4

4751 (C1) Introduction to Advanced Mathematics

Section A

1	$[v =][\pm]\sqrt{\frac{2E}{m}}$ www	3	M2 for $v^2 = \frac{2E}{m}$ or for $[v =][\pm]\sqrt{\frac{E}{\frac{1}{2}m}}$ or M1 for a correct constructive first step and M1 for $v = [\pm]\sqrt{k}$ ft their $v^2 = k$; if M0 then SC1 for $\sqrt{E/ \frac{1}{2} m}$ or $\sqrt{2E/m}$ etc	3
2	$\frac{3x-4}{x+1}$ or $3 - \frac{7}{x+1}$ www as final answer	3	M1 for $(3x - 4)(x - 1)$ and M1 for $(x + 1)(x - 1)$	3
3	(i) 1 (ii) 1/64 www	1 3	 M1 for dealing correctly with each of reciprocal, square root and cubing (allow 3 only for 1/64) eg M2 for 64 or −64 or 1/√4096 or $\frac{1}{4}^3$ or M1 for $1/16^{3/2}$ or 4^3 or -4^3 or 4^{-3} etc	4
4	$6x + 2(2x - 5) = 7$ $10x = 17$ $x = 1.7$ o.e. isw $y = -1.6$ o.e .isw	M1 M1 A1 A1	for subst or multn of eqns so one pair of coeffts equal (condone one error) simplification (condone one error) or appropriate addn/subtn to eliminate variable allow as separate or coordinates as requested graphical soln: M0	4
5	(i) −4/5 or −0.8 o.e. (ii) (15, 0) or 15 found www	2 3	M1 for 4/5 or 4/−5 or 0.8 or −4.8/6 or correct method using two points on the line (at least one correct) (may be graphical) or for $-0.8x$ o.e. M1 for y = their (i) $x + 12$ o.e. or $4x + 5y = k$ and (0, 12) subst and M1 for using $y = 0$ eg $-12 = -0.8x$ or ft their eqn <u>or</u> M1 for given line goes through (0, 4.8) and (6, 0) and M1 for 6 × 12/4.8 graphical soln: allow M1 for correct required line drawn and M1 for answer within 2mm of (15, 0)	5

19

6	f(2) used $2^3 + 2k + 7 = 3$ $k = -6$	M1 M1 A1	or division by $x - 2$ as far as $x^2 + 2x$ obtained correctly or remainder $3 = 2(4 + k) + 7$ o.e. 2nd M1 dep on first	3
7	(i) 56 (ii) −7 or ft from −their (i)/8	2 2	M1 for $\frac{8\times7\times6}{3\times2\times1}$ or more simplified M1 for 7 or ft their (i)/8 or for $56 \times (-1/2)^3$ o.e. or ft; condone x^3 in answer or in M1 expression; 0 in qn for just Pascal's triangle seen	4
8	(i) $5\sqrt{3}$ (ii) common denominator = $(5 - \sqrt{2})(5 + \sqrt{2})$ =23 numerator = 10	2 M1 A1 B1	M1 for $\sqrt{48} = 4\sqrt{3}$ allow M1A1 for $\frac{5-\sqrt{2}}{23}+\frac{5+\sqrt{2}}{23}$ allow 3 only for 10/23	5
9	(i) $n = 2m$ $3n^2 + 6n = 12m^2 + 12m$ or $= 12m(m + 1)$ (ii) showing false when n is odd e.g. $3n^2 + 6n$ = odd + even = odd	M1 M2 B2	or any attempt at generalising; M0 for just trying numbers <u>or</u> M1 for $3n^2 + 6n = 3n(n + 2) = 3 \times$ even × even <u>and</u> M1 for explaining that 4 is a factor of even × even <u>or</u> M1 for 12 is a factor of $6n$ when n is even <u>and</u> M1 for 4 is a factor of n^2 so 12 is a factor of $3n^2$ or $3n(n + 2)$ =3 × odd × odd = odd or counterexample showing not always true; M1 for false with partial explanation or incorrect calculation	5

17

Section B

10	**i**	correct graph with clear asymptote $x = 2$ (though need not be marked) $(0, -½)$ shown	G2 G1	G1 for one branch correct; condone $(0, -½)$ not shown SC1 for both sections of graph shifted two to left allow seen calculated	3	
	ii	11/5 or 2.2 o.e. isw	2	M1 for correct first step	2	
	iii	$x = \frac{1}{x-2}$ $x(x-2) = 1$ o.e. $x^2 - 2x - 1$ [= 0]; ft their equiv eqn attempt at quadratic formula $1 \pm \sqrt{2}$ cao position of points shown	M1 M1 M1 M1 A1 B1	or equivs with *y*s or $(x-1)^2 - 1 = 1$ o.e. or $(x-1) = \pm\sqrt{2}$ (condone one error) on their curve with $y = x$ (line drawn or $y = x$ indicated by both coords); condone intent of diagonal line with gradient approx 1through origin as $y = x$ if unlabelled	6	11
11	**i**	$(x-2.5)^2$ o.e. $-2.5^2 + 8$ $(x-2.5)^2 + 7/4$ o.e. min $y = 7/4$ o.e. [so above *x* axis] or commenting $(x-2.5)^2 \geq 0$	M1 M1 A1 B1	for clear attempt at -2.5^2 allow M2A0 for $(x-2.5) + 7/4$ o.e. with no $(x-2.5)^2$ seen ft, dep on $(x-a)^2 + b$ with *b* positive; condone starting again, showing $b^2 - 4ac < 0$ or using calculus	4	
	ii	correct symmetrical quadratic shape 8 marked as intercept on *y* axis tp (5/2, 7/4) o.e. or ft from (i)	G1 G1 G1	 or (0, 8) seen in table	3	
	iii	$x^2 - 5x - 6$ seen or used -1 and 6 obtained $x < -1$ and $x > 6$ isw or ft their solns	M1 M1 M1	or $(x-2.5)^2$ [> or =] 12.25 or ft $14 - b$ also implies first M1 if M0, allow B1 for one of $x < -1$ and $x > 6$	3	
	iv	min = (2.5, − 8.25) or ft from (i) so yes, crosses	M1 A1	or M1 for other clear comment re translated 10 down and A1 for referring to min in (i) or graph in (ii); or M1 for correct method for solving $x^2 - 5x - 2 = 0$ or using $b^2 - 4ac$ with this and A1 for showing real solns eg $b^2 - 4ac = 33$; allow M1A0 for valid comment but error in −8.25 ft; allow M1 for showing *y* can be neg eg (0, −2) found and A1 for correct conclusion	2	12

12	**i**	$(x-4)^2 - 16 + (y-2)^2 - 4 = 9$ o.e. rad = $\sqrt{29}$	M2 B1	M1 for one completing square or for $(x-4)^2$ or $(y-2)^2$ expanded correctly <u>or</u> starting with $(x-4)^2 + (y-2)^2 = r^2$: M1 for correct expn of at least one bracket and M1 for $9 + 20 = r^2$ o.e. <u>or</u> using $x^2 - 2gx + y^2 - 2fy + c = 0$ M1 for using centre is (g, f) [must be quoted] and M1 for $r^2 = g^2 + f^2 - c$	3
	ii	$4^2 + 2^2$ o.e = 20 which is less than 29	M1 A1	allow 2 for showing circle crosses *x* axis at −1 and 9 or equiv for *y* (or showing one positive; one negative); 0 for graphical solutions (often using A and B from (iii) to draw circle)	2
	iii	showing midpt of AB = (4, 2) and showing AB = $2\sqrt{29}$ or showing AC or BC = $\sqrt{29}$ or that A or B lie on circle <u>or</u> showing both A and B lie on circle (or AC = BC = $\sqrt{29}$), and showing AB = $2\sqrt{29}$ or that C is midpt of AB or that C is on AB or that gradients of AB and AC are the same or equiv. <u>or</u> showing C is on AB and showing both A and B are on circle or AC = BC = $\sqrt{29}$	2 2 2 2 2 2	in each method, two things need to be established. Allow M1 for the concept of what should be shown and A1 for correct completion with method shown allow M1A0 for AB just shown as $\sqrt{116}$ not $2\sqrt{29}$ allow M1A0 for stating mid point of AB = (4,2) without working/method shown NB showing AB = $2\sqrt{29}$ and C lies on AB is not sufficient – earns 2 marks only if M0, allow SC2 for accurate graph of circle drawn with compasses and AB joined with ruled line through C.	4
	iv	grad AC or AB or BC = −5/2 o.e. grad tgt = −1/their grad AC tgt is $y - 7 =$ their $m(x-2)$ o.e. $y = 2/5\,x + 31/5$ o.e.	M1 M1 M1 A1	may be seen in (iii) but only allow this M1 if they go on to use in this part allow for $m_1m_2 = -1$ used eg $y =$ their $mx + c$ then (2, 7) subst; M0 if grad AC used condone $y = 2/5x + c$ and $c = 31/5$ o.e.	4

13

4751 (C1) Introduction to Advanced Mathematics

Section A

1	$x > 6/4$ o.e. isw	2	M1 for $4x > 6$ or for 6/4 o.e. found or for their final ans ft their $4x > k$ or $kx > 6$	2
2	(i) (0, 4) and (6, 0) (ii) −4/6 o.e. or ft their (i) isw	2 2	1 each; allow $x = 0$, $y = 4$ etc; condone $x = 6$, $y = 4$ isw but 0 for (6, 4) with no working 1 for $-\frac{4}{6}x$ or 4/−6 or 4/6 o.e. or ft (accept 0.67 or better) 0 for just rearranging to $y = -\frac{2}{3}x + 4$	4
3	(i) 0 or −3/2 o.e. (ii) $k < -9/8$ o.e. www	2 3	1 each M2 for 3^2 (−)(−8k) < 0 o.e. or −9/8 found or M1 for attempted use of $b^2 - 4ac$ (may be in quadratic formula); SC: allow M1 for $9 - 8k < 0$ and M1 ft for $k > 9/8$	5
4	(i) T (ii) E (iii) T (iv) F	3	3 for all correct, 2 for 3 correct. 1 for 2 correct	3
5	$y(x - 2) = (x + 3)$ $xy - 2y = x + 3$ or ft [ft from earlier errors if of comparable difficulty – no ft if there are no xy terms] $xy - x = 2y + 3$ or ft $[x =]\dfrac{2y+3}{y-1}$ o.e. or ft <u>alt method:</u> $y = 1 + \dfrac{5}{x-2}$ $y - 1 = \dfrac{5}{x-2}$ $x - 2 = \dfrac{5}{y-1}$ $x = 2 + \dfrac{5}{y-1}$	M1 M1 M1 M1 M1 M1 M1 M1	for multiplying by $x - 2$; condone missing brackets for expanding bracket and being at stage ready to collect x terms for collecting x and ‘other’ terms on opposite sides of eqn for factorising and division for either method: award 4 marks only if fully correct	4

18

6	(i) 5 www (ii) $8x^{10}y^{13}z^4$ or $2^3x^{10}y^{13}z^4$	2 3	allow 2 for ±5; M1 for $25^{1/2}$ seen or for 1/5 seen or for using $25^{1/2} = 5$ with another error (ie M1 for coping correctly with fraction and negative index or with square root) mark final answer; B2 for 3 elements correct, B1 for 2 elements correct; condone multn signs included, but −1 from total earned if addn signs	5
7	(i) $\frac{5-\sqrt{3}}{22}$ or $\frac{5+(-1)\sqrt{3}}{22}$ or $\frac{5-1\sqrt{3}}{22}$ (ii) 37 − 12√ 7 isw www	2 3	or $a = 5$, $b = -1$, $c = 22$; M1 for attempt to multiply numerator and denominator by $5-\sqrt{3}$ 2 for 37 and 1 for − 12√ 7 or M1 for 3 correct terms from 9 − 6√ 7 − 6√ 7 + 28 or 9 − 3√ 28 − 3√ 28 + 28 or 9 − √ 252 − √ 252 + 28 o.e. eg using 2√ 63 or M2 for 9 − 12√ 7 + 28 or 9 − 6√ 28 + 28 or 9 − 2√ 252 + 28 or 9 − √ 1008 + 28 o.e.; 3 for 37 − √ 1008 but not other equivs	5
8	−2000 www	4	M3 for $10 \times 5^2 \times (-2[x])^3$ o.e. or M2 for two of these elements or M1 for 10 or (5×4×3)/(3×2×1) o.e. used [$^5C_3$ is not sufficient] or for 1 5 10 10 5 1 seen; or B3 for 2000; condone x^3 in ans; equivs: M3 for e.g $5^5 \times 10 \times \left(-\frac{2}{5}[x]\right)^3$ o.e. [5^5 may be outside a bracket for whole expansion of all terms], M2 for two of these elements etc similarly for factor of 2 taken out at start	4
9	$(y - 3)(y - 4)$ [= 0] $y = 3$ or 4 cao $x = \pm\sqrt{3}$ or ± 2 cao	M1 A1 B2	for factors giving two terms correct or attempt at quadratic formula or completing square or B2 (both roots needed) B1 for 2 roots correct or ft their y (condone √ 3 and √ 4 for B1)	4

18

Section B

10	**i**	$(x-3)^2-7$	3	mark final answer; 1 for a = 3, 2 for b = 7 or M1 for $-3^2 + 2$; bod 3 for $(x - 3) - 7$	3
	ii	(3, −7) or ft from (i)	1+1		2
	iii	sketch of quadratic correct way up and through (0, 2)	G1	accept (0, 2) o.e. seen in this part [eg in table] if 2 not marked as intercept on graph	
		t.p. correct or ft from (ii)	G1	accept 3 and −7 marked on axes level with turning pt., or better; no ft for (0, 2) as min	2
	iv	$x^2 - 6x + 2 = 2x - 14$ o.e.	M1	or their (i) = $2x - 14$	
		$x^2 - 8x + 16$ [= 0]	M1	dep on first M1; condone one error	
		$(x - 4)^2$ [= 0]	M1	or correct use of formula, giving equal roots; allow $(x + 4)^2$ o.e. ft $x^2 + 8x + 16$	
		$x = 4$, $y = -6$	A1	if M0M0M0, allow SC2 for showing (4, −6) is on both graphs (need to go on to show line is tgt to earn more)	
		equal/repeated roots [implies tgt] - must be explicitly stated; condone ‘only one root [so tgt]’ or ‘line meets curve only once, so tgt’ or ‘line touches curve only once’ etc]	A1	or for use of calculus to show grad of line and curve are same when $x = 4$	5

12

11	**i**	f(−4) used	M1		
		$-128 + 112 + 28 - 12$ [= 0]	A1	or B2 for $(x + 4)(2x^2 - x - 3)$ here; or correct division with no remainder	2
	ii	division of f(x) by (x + 4)	M1	as far as $2x^3 + 8x^2$ in working, or two terms of $2x^2 - x - 3$ obtained by inspection etc (may be earned in (i)), or f(−1) = 0 found	
		$2x^2 - x - 3$	A1	$2x^2 - x - 3$ seen implies M1A1	
		$(x + 1)(2x - 3)$	A1		
		[f(x) =] $(x + 4)(x + 1)(2x - 3)$	A1	or B4; allow final A1 ft their factors if M1A1A0 earned	4
	iii	sketch of cubic correct way up	G1	ignore any graph of $y = f(x - 4)$	
		through −12 shown on y axis	G1	or coords stated near graph	
		roots −4, −1, 1.5 or ft shown on x axis	G1	or coords stated near graph	
				if no curve drawn, but intercepts marked on axes, can earn max of G0G1G1	3
	iv	$x(x - 3)(2[x - 4] - 3)$ o.e. or $x(x - 3)(x - 5.5)$ or ft their factors	M1	or $2(x - 4)^3 + 7(x - 4)^2 - 7(x - 4) - 12$ or stating roots are 0, 3 and 5.5 or ft; condone one error eg $2x - 7$ not $2x - 11$	
		correct expansion of one pair of brackets ft from their factors	M1	or for correct expn of $(x - 4)^3$ [allow unsimplified]; or for showing g(0) = g(3) = g(5.5) = 0 in given ans g(x)	
		correct completion to given answer	M1	allow M2 for working backwards from given answer to $x(x - 3)(2x - 11)$ and M1 for full completion with factors or roots	3

12

12	**i**	grad AB = $\frac{9-1}{3--1}$ or 2 $y - 9 = 2(x - 3)$ or $y - 1 = 2(x + 1)$ $y = 2x + 3$ o.e.	M1 M1 A1	ft their *m*, or subst coords of A or B in y = their $m\,x + c$ or B3	3	
	ii	mid pt of AB = (1, 5) grad perp = −1/grad AB $y - 5 = -\frac{1}{2}(x - 1)$ o.e. or ft [no ft for just grad AB used] at least one correct interim step towards given answer $2y + x = 11$, and correct completion NB ans $2y + x = 11$ given <u>alt method working back from ans:</u> $y = \frac{11-x}{2}$ o.e. grad perp = −1/grad AB and showing/stating same as given line finding intn of their $y = 2x + 3$ and $2y + x = 11$ [= (1, 5)] showing midpt of AB is (1, 5)	M1 M1 M1 M1 M1 M1 M1 M1	condone not stated explicitly, but used in eqn soi by use eg in eqn ft their grad and/or midpt, but M0 if their midpt not used; allow M1 for $y = -\frac{1}{2}x + c$ and then their midpt subst no ft; correct eqn only mark one method or the other, to benefit of cand, not a mixture eg stating $-\frac{1}{2} \times 2 = -1$ or showing that (1, 5) is on $2y + x = 11$, having found (1, 5) first [for both methods: for M4 must be fully correct]	4	
	iii	showing $(-1 - 5)^2 + (1 - 3)^2 = 40$ showing B to centre = √40 or verifying that (3, 9) fits given circle	M1 M1	at least one interim step needed for each mark; M0 for just $6^2 + 2^2 = 40$ with no other evidence such as a first line of working or a diagram; condone marks earned in reverse order	2	
	iv	$(x - 5)^2 + 3^2 = 40$ $(x - 5)^2 = 31$ $x = 5 \pm \sqrt{31}$ or $\frac{10 \pm \sqrt{124}}{2}$ isw	M1 M1 A1	for subst $y = 0$ in circle eqn condone slip on rhs; or for rearrangement to zero (condone one error) <u>and</u> attempt at quad. formula [allow M1 M0 for $(x - 5)^2 = 40$ or for $(x - 5)^2 + 3^2 = 0$] or $5 \pm \frac{\sqrt{124}}{2}$	3	12

4751 (C1) Introduction to Advanced Mathematics

Section A

1	(i) 0.125 or 1/8 (ii) 1	1 1	as final answer	2
2	$y = 5x - 4$ www	3	M2 for $\frac{y-11}{-9-11} = \frac{x-3}{-1-3}$ o.e. or M1 for grad = $\frac{11-(-9)}{3-(-1)}$ or 5 eg in $y = 5x + k$ and M1 for $y - 11$ = their $m(x - 3)$ o.e. or subst (3, 11) or (−1, −9) in y = their $mx + c$ or M1 for $y = kx - 4$ (eg may be found by drawing)	3
3	$x > 9/6$ o.e. or $9/6 < x$ o.e. www isw	3	M2 for $9 < 6x$ or M1 for $-6x < -9$ or $k < 6x$ or $9 < kx$ or $7 + 2 < 5x + x$ [condone ≤ for Ms]; if 0, allow SC1 for 9/6 o.e found	3
4	$a = -5$ www	3	M1 for f(2) = 0 used and M1 for $10 + 2a = 0$ or better long division used: M1 for reaching $(8 + a)x - 6$ in working and M1 for $8 + a = 3$ equating coeffts method: M2 for obtaining $x^3 + 2x^2 + 4x + 3$ as other factor	3
5	(i) $4[x^3]$ (ii) $84[x^2]$ www	2 3	ignore any other terms in expansion M1 for $-3[x^3]$ and $7[x^3]$ soi; M1 for $\frac{7 \times 6}{2}$ or 21 or for Pascal's triangle seen with 1 7 21 … row and M1 for 2^2 or 4 or $\{2x\}^2$	5

16

6	1/5 or 0.2 o.e. www	3	M1 for $3x + 1 = 2x \times 4$ and M1 for $5x = 1$ o.e. <u>or</u> M1 for $1.5 + \frac{1}{2x} = 4$ and M1 for $\frac{1}{2x} = 2.5$ o.e.	3
7	(i) $5^{3.5}$ or $k = 3.5$ or 7/2 o.e. (ii) $16a^6b^{10}$	2 2	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ SC1 for $5^{\frac{3}{2}}$ o.e. as answer without working M1 for two 'terms' correct and multiplied; mark final answer only	4
8	$b^2 - 4ac$ soi $k^2 - 4 \times 2 \times 18 < 0$ o.e. $-12 < k < 12$	M1 M1 A2	allow in quadratic formula or clearly looking for perfect square condone ≤; or M1 for 12 identified as boundary may be two separate inequalities; A1 for ≤ used or for one 'end' correct if two separate correct inequalities seen, isw for then wrongly combining them into one statement; condone *b* instead of *k*; if no working, SC2 for $k < 12$ and SC2 for $k > -12$ (ie SC2 for each 'end' correct)	4
9	$y + 5 = xy + 2x$ $y - xy = 2x - 5$ oe or ft $y(1 - x) = 2x - 5$ oe or ft $[y =]\frac{2x-5}{1-x}$ oe or ft as final answer	M1 M1 M1 M1	for expansion for collecting terms for taking out *y* factor; dep on *xy* term for division and no wrong work after ft earlier errors for equivalent steps if error does not simplify problem	4
10	(i) $9\sqrt{3}$ (ii) $6 + 2\sqrt{2}$ www	2 3	M1 for $5\sqrt{3}$ or $4\sqrt{3}$ seen M1 for attempt to multiply num. and denom. by $3 + \sqrt{2}$ and M1 for denom. 7 or 9 – 2 soi from denom. mult by $3 + \sqrt{2}$	5

20

Section B

11	**i**	C, mid pt of AB = $\left(\frac{11+(-1)}{2}, \frac{4}{2}\right)$ = (5, 2)	B1	evidence of method required – may be on diagram, showing equal steps, or start at A or B and go half the difference towards the other		
		[AB^2 =] $12^2 + 4^2$ [= 160] oe or [CB^2 =] $6^2 + 2^2$ [=40] oe with AC	B1	or square root of these; accept unsimplified		
		quote of $(x-a)^2 + (y-b)^2 = r^2$ o.e with different letters	B1	or (5, 2) clearly identified as centre and $\sqrt{40}$ as r (or 40 as r^2) www or quote of *gfc* formula and finding c = −11		
		completion (ans given)	B1	dependent on centre (or midpt) and radius (or radius^2) found independently and correctly	4	
	ii	correct subst of $x = 0$ in circle eqn soi	M1			
		$(y-2)^2 = 15$ or $y^2 - 4y - 11$ [= 0]	M1	condone one error		
		$y - 2 = \pm\sqrt{15}$ or ft	M1	or use of quad formula (condone one error in formula); ft only for 3 term quadratic in y		
		$[y =] 2 \pm \sqrt{15}$ cao	A1	if $y = 0$ subst, allow SC1 for (11, 0) found alt method: M1 for y values are $2 \pm a$ M1 for $a^2 + 5^2 = 40$ soi M1 for $a^2 = 40 - 5^2$ soi A1 for $[y =] 2 \pm \sqrt{15}$ cao	4	
	iii	grad AB = $\frac{4}{11-(-1)}$ or 1/3 o.e.	M1	or grad AC (or BC)		
		so grad tgt = – 3	M1	or ft −1/their gradient of AB		
		eqn of tgt is $y - 4 = -3(x - 11)$	M1	or subst (11, 4) in $y = -3x + c$ or ft (no ft for their grad AB used)		
		$y = -3x + 37$ or $3x + y = 37$	A1	accept other simplified versions		
		(0, 37) and (37/3, 0) o.e. ft isw	B2	B1 each, ft their tgt for grad ≠ 1 or 1/3; accept $x = 0$, $y = 37$ etc NB alt method: intercepts may be found first by proportion then used to find eqn	6	14

12	**i**	$3x^2 + 6x + 10 = 2 - 4x$	M1	for subst for *x* or *y* or subtraction attempted	
		$3x^2 + 10x + 8$ [= 0]	M1	or $3y^2 - 52y + 220$ [=0]; for rearranging to zero (condone one error)	
		$(3x + 4)(x + 2)$ [=0]	M1	or $(3y - 22)(y - 10)$; for sensible attempt at factorising or formula or completing square	
		$x = -2$ or $-4/3$ o.e. $y = 10$ or $22/3$ o.e.	A1 A1	or A1 for each of (−2, 10) and (−4/3, 22/3) o.e.	5
	ii	$3(x + 1)^2 + 7$	4	1 for $a = 3$, 1 for $b = 1$, 2 for $c = 7$ or M1 for $10 - 3 \times$ their b^2 soi or for 7/3 or for $10/3 -$ their b^2 soi	4
	iii	min at $y = 7$ or ft from (ii) for positive *c* (ft for (ii) only if in correct form)	B2	may be obtained from (ii) or from good symmetrical graph or identified from table of values showing symmetry condone error in *x* value in stated min ft from (iii) [getting confused with 3 factor] B1 if say turning pt at $y = 7$ or ft without identifying min <u>or</u> M1 for min at $x = -1$ [e.g. may start again and use calculus to obtain $x = -1$] or min when $(x + 1)^{[2]} = 0$; and A1 for showing *y* positive at min <u>or</u> M1 for showing discriminant neg. so no real roots and A1 for showing above axis not below eg positive x^2 term or goes though (0, 10) <u>or</u> M1 for stating bracket squared must be positive [or zero] and A1 for saying other term is positive	2

11

13	**i**	any correct y value calculated from quadratic seen or implied by plots	B1	for $x \neq 0$ or 1; may be for neg x or eg min.at (2.5, −1.25)	
		(0, 5)(1, 1)(2, −1)(3, −1)(4,1) and (5,5) plotted	P2	tol 1 mm; P1 for 4 correct [including (2.5, −1.25) if plotted]; plots may be implied by curve within 1 mm of correct position	
		good quality smooth parabola within 1mm of their points	C1	allow for correct points only [accept graph on graph paper, not insert]	4
	ii	$x^2 - 5x + 5 = \frac{1}{x}$	M1		
		$x^3 - 5x^2 + 5x = 1$ and completion to given answer	M1		2
	iii	divn of $x^3 - 5x^2 + 5x - 1$ by $x - 1$ as far as $x^3 - x^2$ used in working	M1	or inspection eg $(x - 1)(x^2 \ldots\ldots + 1)$ or equating coeffts with two correct coeffts found	
		$x^2 - 4x + 1$ obtained	A1		
		use of $b^2 - 4ac$ or formula with quadratic factor	M1	or $(x - 2)^2 = 3$; may be implied by correct roots or $\sqrt{12}$ obtained	
		$\sqrt{12}$ obtained and comment re shows other roots (real and) irrational <u>or</u> for $2 \pm \sqrt{3}$ or $\frac{4 \pm \sqrt{12}}{2}$ obtained isw	A2	[A1 for $\sqrt{12}$ and A1 for comment] NB A2 is available only for correct quadratic factor used; if wrong factor used, allow A1 ft for obtaining two irrational roots or for their discriminant and comment re irrational [no ft if their discriminant is negative]	5

11

4751 (C1) Introduction to Advanced Mathematics

Section A

1	(0, 14) and (14/4, 0) o.e. isw	4	M2 for evidence of correct use of gradient with (2, 6) eg sketch with 'stepping' or $y - 6 = -4(x - 2)$ seen or $y = -4x + 14$ o.e. or M1 for $y = -4x + c$ [accept any letter or number] and M1 for $6 = -4 \times 2 + c$; A1 for (0, 14) [$c = 14$ is not sufficient for A1] and A1 for (14/4, 0) o.e.; allow when $x = 0$, $y = 14$ etc isw	4
2	$[a =]\dfrac{2(s-ut)}{t^2}$ o.e. as final answer [condone $[a =]\dfrac{(s-ut)}{0.5t^2}$]	3	M1 for each of 3 complete correct steps, ft from previous error if equivalent difficulty [eg dividing by t does not count as step – needs to be by t^2] $[a =]\dfrac{(s-ut)}{\frac{1}{2}t^2}$ gets M2 only (similarly other triple-deckers)	3
3	10 www	3	M1 for f(3) = 1 soi and A1 for $31 - 3k = 1$ or $27 - 3k = -3$ o.e. [a correct 3-term or 2-term equation] long division used: M1 for reaching $(9 - k)x + 4$ in working and A1 for $4 + 3(9 - k) = 1$ o.e. equating coeffts method: M2 for $(x - 3)(x^2 + 3x - 1)$ [+ 1] o.e. (from inspection or division)	3
4	$x < 0$ or $x > 6$ (both required)	2	B1 each; if B0 then M1 for 0 and 6 identified;	2
5	(i) 10 www (ii) 80 www or ft 8 × their (i)	2 2	M1 for $\dfrac{5\times4\times3}{3\times2(\times1)}$ or $\dfrac{5\times4}{2(\times1)}$ or for 1 5 10 10 5 1 seen B2 for $80x^3$; M1 for 2^3 or $(2x)^3$ seen	4
				16

6	any general attempt at n being odd <u>and</u> n being even even n odd implies n^3 odd and odd − odd = even n even implies n^3 even and even − even = even	M1 A1 A1	M0 for just trying numbers, even if some odd, some even or $n(n^2 - 1)$ used with n odd implies $n^2 - 1$ even and odd × even = even etc [allow even × odd = even] or A2 for $n(n-1)(n+1)$ = product of 3 consecutive integers; at least one even so product even; odd^3 − odd = odd etc is not sufft for A1 SC1 for complete general method for only one of odd or even eg $n = 2m$ leading to $2(4m^3 - m)$	3
7	(i) 1 (ii) 1000	2 1	B1 for 5^0 or for 25 × 1/25 o.e.	3
8	(i) 2/3 www (ii) $43 - 30\sqrt{2}$ www as final answer	2 3	M1 for 4/6 or for $\sqrt{48} = 2\sqrt{12}$ or $4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$ or $\sqrt{108} = 3\sqrt{12}$ or for $\sqrt{\frac{4}{9}}$ M2 for 3 terms correct of $25 - 15\sqrt{2} - 15\sqrt{2} + 18$ soi, M1 for 2 terms correct	5
9	(i) $(x + 3)^2 - 4$ (ii) ft their $(-a, b)$; if error in (i), accept (−3, −4) if evidence of being independently obtained	3 2	B1 for $a = 3$, B2 for $b = -4$ or M1 for $5 - 3^2$ soi B1 each coord.; allow $x = -3$, $y = -4$; or M1 for $\begin{bmatrix}-3\\-4\end{bmatrix}$ o.e. oe for sketch with −3 and −4 marked on axes but no coords given	5
10	$(x^2 - 9)(x^2 + 4)$ $x^2 = 9$ [or −4] or ft for integers /fractions if first M1 earned $x = \pm 3$ cao	M2 M1 A1	or correct use of quad formula or comp sq reaching 9 and −4; allow M1 for attempt with correct eqn at factorising with factors giving two terms correct, or sign error, or attempt at formula or comp sq [no more than two errors in formula/substn]; for this first M2 or M1 allow use of y etc or of x instead of x^2 must have x^2; or M1 for $(x + 3)(x - 3)$; this M1 may be implied by $x = \pm 3$ A0 if extra roots if M0 then allow SC1 for use of factor theorem to obtain both 3 and −3 as roots or $(x + 3)$ and $(x - 3)$ found as factors and SC2 for $x^2 + 4$ found as other factor using factor theorem [ie max SC3]	4
				20

Section B

11	**i**	$y = 3x$	2	M1 for grad AB = $\frac{1-3}{6}$ or − 1/3 o.e.	2
	ii	eqn AB is $y = -1/3\,x + 3$ o.e. or ft	M1	need not be simplified; no ft from midpt used in (i); may be seen in (i) but do not give mark unless used in (ii)	
		$3x = -1/3x + 3$ or ft $x = 9/10$ or 0.9 o.e. cao	M1 A1	eliminating *x* or *y* , ft their eqns if find *y* first, cao for *y* then ft for *x*	
		$y = 27/10$ oe ft their 3 × their *x*	A1	ft dep on both Ms earned	4
	iii	$\left(\frac{9}{10}\right)^2(1+3^2)$ o.e and completion to given answer	2	or square root of this; M1 for $\left(\frac{9}{10}\right)^2+\left(\frac{27}{10}\right)^2$ or 0.81 + 7.29 soi or ft their coords (inc midpt) <u>or</u> M1 for distance = 3 cos *θ* and tan *θ* = 3 and M1 for showing $\sin\theta = \frac{3}{\sqrt{10}}$ and completion	2
	iv	$2\sqrt{10}$	2	M1 for $6^2 + 2^2$ or 40 or square roots of these	2
	v	9 www or ft their $a\sqrt{10}$	2	M1 for ½ × 3 × 6 or $\frac{1}{2}\times \text{their}\,2\sqrt{10}\times\frac{9}{10}\sqrt{10}$	2
					12

12	iA	expansion of one pair of brackets correct 6 term expansion	M1 M1	eg [$(x + 1)$]$(x^2 - 6x + 8)$; need not be simplified eg $x^3 - 6x^2 + 8x + x^2 - 6x + 8$; or M2 for correct 8 term expansion: $x^3 - 4x^2 + x^2 - 2x^2 + 8x - 4x - 2x + 8$, M1 if one error allow equivalent marks working backwards to factorisation, by long division or factor theorem etc or M1 for all three roots checked by factor theorem and M1 for comparing coeffts of x^3	2
	iB	cubic the correct way up x-axis: −1, 2, 4 shown y-axis 8 shown	G1 G1 G1	with two tps and extending beyond the axes at 'ends' ignore a second graph which is a translation of the correct graph	3
	iC	[y=]$(x - 2)(x - 5)(x - 7)$ isw or $(x - 3)^3 - 5(x - 3)^2 + 2(x - 3) + 8$ isw or $x^3 - 14x^2 + 59x - 70$ (0, −70) or $y = -70$	2 1	M1 if one slip or for [y =] $f(x - 3)$ or for roots identified at 2, 5, 7 or for translation 3 to the left allow M1 for complete attempt: $(x + 4)(x + 1)(x - 1)$ isw or $(x + 3)^3 - 5(x + 3)^2 + 2(x + 3) + 8$ isw allow 1 for (0, −4) or $y = -4$ after $f(x + 3)$ used	3
	ii	$27 - 45 + 6 + 8 = -4$ or $27 - 45 + 6 + 12 = 0$ long division of f(x) or their f(x) + 4 by $(x - 3)$ attempted as far as $x^3 - 3x^2$ in working $x^2 - 2x - 4$ obtained $[x =]\dfrac{2 \pm \sqrt{(-2)^2 - 4\times(-4)}}{2}$ or $(x - 1)^2 = 5$ $\dfrac{2 \pm \sqrt{20}}{2}$ o.e. isw or $1 \pm \sqrt{5}$	B1 M1 A1 M1 A1	or correct long division of $x^3 - 5x^2 + 2x + 12$ by $(x - 3)$ with no remainder or of $x^3 - 5x^2 + 2x + 8$ with rem −4 or inspection with two terms correct eg $(x - 3)(x^2$ ……….$- 4)$ dep on previous M1 earned; for attempt at formula or comp square on their other 'factor'	5 13

13	**i**	(5, 2) $\sqrt{20}$ or $2\sqrt{5}$	1 1	0 for $\pm\sqrt{20}$ etc	2
	ii	no, since $\sqrt{20}$ < 5 or showing roots of $y^2 - 4y + 9 = 0$ o.e. are not real	2	or ft from their centre and radius M1 for attempt (no and mentioning $\sqrt{20}$ or 5) or sketch or solving by formula or comp sq $(-5)^2 + (y-2)^2 = 20$ [condone one error] or SC1 for fully comparing distance from *x* axis with radius and saying yes	2
	iii	$y = 2x - 8$ or simplified alternative	2	M1 for $y - 2 = 2(x - 5)$ or ft from **(i)** or M1 for $y = 2x + c$ and subst their **(i)** or M1 for ans $y = 2x + k$, $k \neq 0$ or -8	2
	iv	$(x - 5)^2 + (2x)^2 = 20$ o.e.	M1	subst $2x + 2$ for *y* [oe for *x*]	
		$5x^2 - 10x + 5 [= 0]$ or better equiv.	M1	expanding brackets and rearranging to 0; condone one error; dep on first M1	
		obtaining $x = 1$ (with no other roots) or showing roots equal	M1		
		one intersection [so tangent]	A1	o.e.; must be explicit; or showing line joining (1,4) to centre is perp to $y = 2x + 2$	
		(1, 4) cao	A1	allow $y = 4$	
		<u>alt method</u> $y - 2 = -½ (x - 5)$ o.e. $2x + 2 - 2 = -½ (x - 5)$ o.e. $x = 1$ $y = 4$ cao showing (1, 4) is on circle	 M1 M1 A1 A1 B1	line through centre perp to $y = 2x + 2$ dep; subst to find intn with $y = 2x + 2$ by subst in circle eqn or finding dist from centre = $\sqrt{20}$ [a similar method earns first M1 for eqn of diameter, 2nd M1 for intn of diameter and circle A1 each for *x* and *y* coords and last B1 for showing (1, 4) on line – award onlyA1 if (1, 4) and (9, 0) found without (1, 4) being identified as the soln]	
		<u>alt method</u> perp dist between $y = 2x - 8$ and $y = 2x + 2 = 10 \cos \theta$ where $\tan \theta = 2$ showing this is $\sqrt{20}$ so tgt $x = 5 - \sqrt{20} \sin \theta$ $x = 1$ (1, 4) cao	 M1 M1 M1 A1 A1	 or other valid method for obtaining *x* allow $y = 4$	 5 11

4751 (C1) Introduction to Advanced Mathematics

1		$[a=]2c^2 - b$ www o.e.	3	**M1** for each of 3 complete correct steps, ft from previous error if equivalent difficulty
2		$5x-3<2x+10$ $3x<13$ $x<\frac{13}{3}$ o.e.	**M1** **M1** **M1**	condone '=' used for first two Ms **M0** for just $5x - 3 < 2(x + 5)$ or $-13 < -3x$ or ft or ft; isw further simplification of 13/3; M0 for just $x < 4.3$
3	**(i)**	(4, 0)	1	allow $y = 0$, $x = 4$ bod **B1** for $x = 4$ but do not isw: **0** for (0, 4) seen **0** for (4, 0) and (0, 10) **both** given (choice) unless (4, 0) clearly identified as the x-axis intercept
3	**(ii)**	$5x + 2(5 - x) = 20$ o.e. (10/3, 5/3) www isw	**M1** **A2**	for subst or for multn to make coeffts same and appropriate addn/subtn; condone one error or **A1** for $x = 10/3$ and **A1** for $y = 5/3$ o.e. isw; condone 3.33 or better and 1.67 or better **A1** for (3.3, 1.7)
4	**(i)**	translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ or 4 [units] to left	**B1** **B1**	**0** for shift/move or 4 units in negative x direction o.e.
4	**(ii)**	sketch of parabola right way up and with minimum on negative y-axis min at (0, −4) and graph through −2 and 2 on x-axis	**B1** **B1**	mark intent for both marks must be labelled or shown nearby
5	**(i)**	$\frac{1}{12}$ or $\pm\frac{1}{12}$	2	**M1** for $\frac{1}{144^{\frac{1}{2}}}$ o.e. or for $\sqrt{144}=12$ soi
5	**(ii)**	denominator = 18 numerator = $5-\sqrt{7}+4\left(5+\sqrt{7}\right)$ = $25+3\sqrt{7}$ as final answer	**B1** **M1** **A1**	**B0** if 36 after addition for **M1**, allow in separate fractions allow **B3** for $\frac{25+3\sqrt{7}}{18}$ as final answer www

6 (i)	cubic correct way up and with two turning pts touching x-axis at −1, and through it at 2.5 and no other intersections y- axis intersection at −5	**B1** **B1** **B1**	 intns must be shown labelled or worked out nearby
6 (ii)	$2x^3 - x^2 - 8x - 5$	**2**	**B1** for 3 terms correct or **M1** for correct expansion of product of two of the given factors
7	attempt at f(−3) $-27 + 18 - 15 + k = 6$ $k = 30$	**M1** **A1** **A1**	or **M1** for long division by $(x + 3)$ as far as obtaining $x^2 - x$ and **A1** for obtaining remainder as $k - 24$ (but see below) equating coefficients method: **M2** for $(x + 3)(x^2 - x + 8)$ [+6] o.e. (from inspection or division) eg M2 for obtaining $x^2 - x + 8$ as quotient in division
8	$x^3 + 15x + \frac{75}{x} + \frac{125}{x^3}$ www isw or $x^3 + 15x + 75x^{-1} + 125x^{-3}$ www isw	**4**	**B1** for **both** of x^3 **and** $\frac{125}{x^3}$ or $125x^{-3}$ isw and **M1** for 1 3 3 1 soi; **A1** for **each** of $15x$ **and** $\frac{75}{x}$ or $75x^{-1}$ isw **or** **SC2** for completely correct unsimplified answer

9	$x^2 - 5x + 7 = 3x - 10$	**M1**	or attempt to subst $(y + 10)/3$ for x
	$x^2 - 8x + 17$ [= 0] o.e or $y^2 - 4y + 13$ [= 0] o.e	**M1**	condone one error; allow **M1** for $x^2 - 8x = -17$ [oe for y] only if they go on to completing square method
	use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula)	**M1**	or $(x - 4)^2 = 16 - 17$ or $(x - 4)^2 + 1 = 0$ (condone one error)
	$b^2 - 4ac = 64 - 68$ or −4 cao [or 16 – 52 or −36 if y used]	**A1**	or $(x - 4)^2 = -1$ or $x = 4 \pm \sqrt{-1}$ [or $(y - 2)^2 = -9$ or $y = 2 \pm \sqrt{-9}$]
	[< 0] so no [real] roots [so line and curve do not intersect]	**A1**	or conclusion from comp. square; needs to be explicit correct conclusion and correct ft; allow '< 0 so no intersection' o.e.; allow '−4 so no roots' etc allow A2 for full argument from sum of two squares = 0; A1 for weaker correct conclusion some may use the condition $b^2 < 4ac$ for no real roots; allow equivalent marks, with first A1 for 64 < 68 o.e.
10 (i)	grad CD = $\frac{5-3}{3-(-1)}\left[=\frac{2}{4} \text{ o.e.}\right]$ isw	**M1**	NB needs to be obtained independently of grad AB
	grad AB = $\frac{3-(-1)}{6-(-2)}$ or $\frac{4}{8}$ isw	**M1**	
	same gradient so parallel www	**A1**	must be explicit conclusion mentioning 'same gradient' or 'parallel' if M0, allow **B1** for 'parallel lines have same gradient' o.e.
10 (ii)	[BC^2=] $3^2 + 2^2$ [BC^2 =] 13 showing $AD^2 = 1^2 + 4^2$ [=17] [≠BC^2] isw	**M1** **A1** **A1**	accept $(6 - 3)^2 + (3 - 5)^2$ o.e. or [BC =] $\sqrt{13}$ or [AD =] $\sqrt{17}$ or equivalent marks for finding AD or AD^2 first alt method: showing AC ≠ BD – mark equivalently

10 (iii)	[BD eqn is] $y = 3$ eqn of AC is $y - 5 = 6/5 \times (x - 3)$ o.e [$y = 1.2x + 1.4$ o.e.] M is (4/3, 3) o.e. isw	**M1** **M2** **A1**	eg allow for 'at M, $y = 3$' or for 3 subst in eqn of AC or **M1** for grad AC = 6/5 o.e. (accept unsimplified) and M1 for using their grad of AC with coords of A(−2, −1) or C (3, 5) in eqn of line or **M1** for 'stepping' method to reach M allow : at M, $x = 16/12$ o.e. [eg =4/3] isw A0 for 1.3 without a fraction answer seen
10 (iv)	midpt of BD = (5/2, 3) or equivalent simplified form cao midpt AC = (1/2, 2) or equivalent simplified form cao or 'M is 2/3 of way from A to C' conclusion 'neither diagonal bisects the other'	**M1** **M1** **A1**	or showing BM ≠ MD oe [BM = 14/3, MD = 7/3] or showing AM ≠ MC or $AM^2 \neq MC^2$ in these methods A1 is dependent on coords of M having been obtained in part (iii) or in this part; the coordinates of M need not be correct; it is also dependent on midpts of both AC and BD attempted, at least one correct alt method: show that mid point of BD does not lie on AC (M1) and vice-versa (M1), A1 for both and conclusion

11 (i)	centre C' = (3, −2) radius 5	**1** **1**	 0 for ±5 or −5
11 (ii)	showing $(6-3)^2+(-6+2)^2=25$ showing that $\overrightarrow{AC'}=\overrightarrow{C'B}=\begin{pmatrix}-3\\4\end{pmatrix}$ o.e.	**B1** **B2**	interim step needed **or B1** each for two of**:** showing midpoint of AB = (3, −2); showing B (0, 2) is on circle; showing AB = 10 **or B2** for showing midpoint of AB = (3, −2) and saying this is centre of circle **or B1** for finding eqn of AB as $y=-4/3\,x+2$ o.e. and **B1** for finding one of its intersections with the circle is (0, 2) **or B1** for showing C′B = 5 and **B1** for showing AB = 10 or that AC′ and BC′ have the same gradient **or B1** for showing that AC′ and BC′ have the same gradient and B1 for showing that B (0, 2) is on the circle
11 (iii)	grad AC' or AB = −4/3 o.e. grad tgt = −1/their AC' grad $y-(-6)=$ their $m(x-6)$ o.e. $y=0.75x-10.5$ o.e. isw	**M1** **M1** **M1** **A1**	or ft from their C', must be evaluated may be seen in eqn for tgt; allow M2 for grad tgt = ¾ oe soi as first step or **M1** for y = their $m\times x+c$ then subst (6, −6) eg **A1** for $4y=3x-42$ allow **B4** for correct equation www isw
11 (iv)	centre C is at (12, −14) cao circle is $(x-12)^2+(y+14)^2=100$	**B2** **B1**	B1 for each coord ft their C if at least one coord correct

12 (i)	10	**1**	
12 (ii)	[x =] 5 or ft their (i) ÷ 2 ht = 5[m] cao	**1** **1**	not necessarily ft from (i) eg they may start again with calculus to get $x = 5$
12 (iii)	$d = 7/2$ o.e. [y =] 1/5 × 3.5 × (10 − 3.5) o.e. or ft = 91/20 o.e. cao isw	**M1** **M1** **A1**	or ft their (ii) − 1.5 or their (i) ÷ 2 − 1.5 o.e. or 7 – 1/5 × 3.5^2 or ft or showing $y - 4 = 11/20$ o.e. cao
12 (iv)	4.5 = 1/5 × $x(10 - x)$ o.e. 22.5 = $x(10 - x)$ o.e. $2x^2 - 20x + 45$ [= 0] o.e. eg $x^2 - 10x + 22.5$ [=0] or $(x - 5)^2 = 2.5$ $[x =]\dfrac{20 \pm \sqrt{40}}{4}$ or $5 \pm \dfrac{1}{2}\sqrt{10}$ o.e. width = $\sqrt{10}$ o.e. eg $2\sqrt{2.5}$ cao	**M1** **M1** **A1** **M1** **A1**	 eg $4.5 = x(2 - 0.2x)$ etc cao; accept versions with fractional coefficients of x^2, isw or $x - 5 = [\pm]\sqrt{2.5}$ o.e.; ft their quadratic eqn provided at least M1 gained already; condone one error in formula or substitution; need not be simplified or be real accept simple equivalents only

GCE

Mathematics (MEI)

Advanced Subsidiary GCE **4751**

Introduction to Advanced Mathematics (C1)

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

SECTION A

1	$y = 3x + c$ or $y - y_1 = 3(x - x_1)$ $y - 5 =$ their $m(x - 4)$ o.e. $y = 3x - 7$ or simplified equiv.	**M1** **M1** **A1**	allow M1 for 3 clearly stated/ used as gradient of required line or (4, 5) subst in their $y = mx + c$; allow M1 for $y - 5 = m(x - 4)$ o.e. condone $y = 3x + c$ and $c = -7$ or **B3** www
2	(i) $250a^6b^7$ (ii) 16 cao (iii) 64	2 1 2	**B1** for two elements correct; condone multiplication signs left in SC1 for eg $250 + a^6 + b^7$ condone ±64 **M1** for [±]4^3 or for $\sqrt{4096}$ or for only –64
3	$ac = \sqrt{y} - 5$ o.e. $ac + 5 = \sqrt{y}$ o.e. $[y =](ac+5)^2$ o.e. isw	**M1** **M1** **M1**	**M1** for each of 3 correct or ft correct steps s.o.i. leading to y as subject or some/all steps may be combined; allow **B3** for $[y =](ac+5)^2$ o.e. isw or **B2** if one error
4 (i)	$2 - 2x > 6x + 5$ $-3 > 8x$ o.e. or ft $x < -3/8$ o.e. or ft isw	**M1** **M1** **M1**	or $1 - x > 3x + 2.5$ for collecting terms of their inequality correctly on opposite sides eg $-8x > 3$ allow **B3** for correct inequality found after working with equation allow **SC2** for –3/8 o.e. found with equation or wrong inequality
4 (ii)	$-4 < x < ½$ o.e.	2	accept as two inequalities **M1** for one 'end' correct or for −4 and ½
5 (i)	$7\sqrt{3}$	2	**M1** for $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$

5 (ii)	$\frac{10+15\sqrt{2}}{7}$ www isw	**3**	**B1** for 7 [B0 for 7 wrongly obtained] <u>and</u> **B2** for $10+15\sqrt{2}$ or **B1** for one term of numerator correct; if **B0**, then **M1** for attempt to multiply num and denom by $3+\sqrt{2}$
6	5 + 2k soi k = 12 attempt at f(3) 27 + 36 + m = 59 o.e. m = −4 cao	**M1** **A1** **M1** **A1** **A1**	allow M1 for expansion with $5x^3 + 2kx^3$ and no other x^3 terms or M1 for (29 – 5) / 2 soi must substitute 3 for x in cubic not product or long division as far as obtaining $x^2 + 3x$ in quotient or from division m – (–63) = 59 o.e. or for 27 + 3k + m = 59 or ft their k
7	$1+2x+\frac{3}{2}x^2+\frac{1}{2}x^3+\frac{1}{16}x^4$ oe (must be simplified) isw	**4**	**B3** for 4 terms correct, or **B2** for 3 terms correct or for all correct but unsimplified (may be at an earlier stage, but factorial or ^nC_r notation must be expanded/worked out) or **B1** for 1, 4, 6, 4, 1 soi or for $1+...+\frac{1}{16}x^4$ [must have at least one other term]
8	$5(x + 2)^2 - 14$	**4**	**B1** for a = 5, <u>and</u> **B1** for b = 2 <u>and</u> **B2** for c = −14 or **M1** for c = 6 − their ab^2 or **M1** for [their a](6/their a – their b^2) [no ft for a = 1]
9	mention of –5 as a square root of 25 or $(–5)^2 = 25$ –5 –5 ≠ 0 o.e. or x + 5 = 0	**M1** **M1**	condone $–5^2 = 25$ or, dep on first M1 being obtained, allow **M1** for showing that 5 is the only soln of x – 5 = 0 allow M2 for $x^2 – 25 = 0$ (x + 5)(x – 5) [= 0] so x – 5 = 0 or x + 5 = 0

Section A Total: 36

SECTION B

10 (i)	$(2x - 3)(x + 1)$ $x = 3/2$ and $- 1$ obtained	**M2** **B1**	**M1** for factors with one sign error or giving two terms correct allow M1 for $2(x - 1.5)(x + 1)$ with no better factors seen or ft their factors
10 (ii)	graph of quadratic the correct way up and crossing both axes crossing x-axis only at 3/2 and $- 1$ or ft from their roots in (i), or their factors if roots not given crossing y-axis at -3	**B1** **B1** **B1**	 for $x = 3/2$ condone 1 and 2 marked on axis and crossing roughly halfway between; intns must be shown labelled or worked out nearby
10 (iii)	use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula) $25 - 40 < 0$ or -15 obtained	**M1** **A1**	may be in formula or $(x - 2.5)^2 = 6.25 - 10$ or $(x - 2.5)^2 + 3.75 = 0$ oe (condone one error) or $\sqrt{-15}$ seen in formula or $(x - 2.5)^2 = -3.75$ oe or $x = 2.5 \pm \sqrt{-3.75}$ oe
10 (iv)	$2x^2 - x - 3 = x^2 - 5x + 10$ o.e. $x^2 + 4x - 13$ [= 0] use of quad. formula on resulting eqn (do not allow for original quadratics used) $-2 \pm \sqrt{17}$ cao	**M1** **M1** **M1** **A1**	attempt at eliminating y by subst or subtraction or $(x + 2)^2 = 17$; for rearranging to form $ax^2 + bx + c$ [= 0] or to completing square form condone one error for each of 2nd and 3rd **M1s** or $x + 2 = \pm\sqrt{17}$ o.e. 2nd and 3rd **M1s** may be earned for good attempt at completing square as far as roots obtained

11	(i)	grad AB = $\frac{1-3}{5-(-1)}$ [= −1/3] y − 3 = their grad (x – (–1)) or y − 1 = their grad (x − 5) y = −1/3x + 8/3 or 3y = −x + 8 o.e isw	**M1** **M1** **A1**	 or use of y = their gradient x + c with coords of A or B or **M2** for $\frac{y-3}{1-3} = \frac{x-(-1)}{5-(-1)}$ o.e. o.e. eg $x + 3y - 8 = 0$ or $6y = 16 - 2x$ allow **B3** for correct eqn www
11	(ii)	when y = 0, x = 8; when x = 0, y = 8/3 or ft their (i) [Area =] ½ × 8/3 × 8 o.e. cao isw	**M1** **M1**	allow y = 8/3 used without explanation if already seen in eqn in (i) NB answer 32/3 given; allow 4 × 8/3 if first M1 earned; or M1 for $\int_0^8 \left[\frac{1}{3}(8-x)\right]dx = \left[\frac{1}{3}\left(8x-\frac{1}{2}x^2\right)\right]_0^8$ and M1 dep for $\frac{1}{3}(64-32[-0])$
11	(iii)	grad perp = −1/grad AB stated, or used after their grad AB stated in this part midpoint [of AB] = (2, 2) y − 2 = their grad perp (x − 2) or ft their midpoint <u>alt method working back from ans</u>: grad perp = −1/grad AB and showing/stating same as given line finding intn of their y = −1/3x − 8/3and y = 3x − 4 is (2, 2) showing midpt of AB is (2, 2)	**M1** **M1** **M1** **or** **M1** **M1** **M1**	or showing 3 × −1/3 = −1 if (i) is wrong, allow the first M1 here ft, provided the answer is correct ft must state 'midpoint' or show working for **M3** this must be correct, starting from grad AB = –1/3, and also needs correct completion to given ans y = 3x − 4 mark one method or the other, to benefit of candidate, not a mixture eg stating − 1/3 × 3 = −1 or showing that (2, 2) is on y = 3x − 4, having found (2, 2) first [for both methods: for **M3** must be fully correct]

11	**(iv)**	subst $x = 3$ into $y = 3x - 4$ and obtaining centre = (3, 5) $r^2 = (5-3)^2 + (1-5)^2$ o.e. $r = \sqrt{20}$ o.e. cao eqn is $(x-3)^2 + (y-5)^2 = 20$ or ft their r and y-coord of centre	**M1** **M1** **A1** **B1**	or using $(-1-3)^2 + (3-b)^2 = (5-3)^2 + (1-b)^2$ and finding (3, 5) or $(-1-3)^2 + (3-5)^2$ or ft their centre using A or B condone $(x-3)^2 + (y-b)^2 = r^2$ o.e. or $(x-3)^2 + (y - \text{their } 5)^2 = r^2$ o.e. (may be seen earlier)
12	**(i)**	trials of at calculating f(x) for at least one factor of 30 details of calculation for f(2) or f(−3) or f(−5) attempt at division by $(x-2)$ as far as $x^3 - 2x^2$ in working correctly obtaining $x^2 + 8x + 15$ factorising a correct quadratic factor $(x-2)(x+3)(x+5)$	**M1** **A1** **M1** **A1** **M1** **A1**	**M0** for division or inspection used or equiv for $(x+3)$ or $(x+5)$; or inspection with at least two terms of quadratic factor correct or B2 for another factor found by factor theorem for factors giving two terms of quadratic correct; M0 for formula without factors found condone omission of first factor found; ignore '= 0' seen allow last four marks for $(x-2)(x+3)(x+5)$ obtained; for all 6 marks must see factor theorem use first
12	**(ii)**	sketch of cubic right way up, with two turning points values of intns on x axis shown, correct (−5, −3, and 2) or ft from their factors/ roots in (i) y-axis intersection at −30	**B1** **B1** **B1**	0 if stops at x-axis on graph or nearby in this part mark intent for intersections with both axes or $x = 0$, $y = -30$ seen in this part if consistent with graph drawn

12 (iii)	$(x - 1)$ substituted for x in either form of eqn for $y = f(x)$	**M1**	correct or ft their (i) or (ii) for factorised form; condone one error; allow for new roots stated as −4,−2 and 3 or ft
	$(x - 1)^3$ expanded correctly (need not be simplified) or two of their factors multiplied correctly	**M1 dep**	or **M1** for correct or correct ft multiplying out of all 3 brackets at once, condoning one error [$x^3 - 3x^2 + 4x^2 + 2x^2 + 8x - 6x - 12x - 24$]
	correct completion to given answer [condone omission of '$y =$']	**M1**	unless all 3 brackets already expanded, must show at least one further interim step allow **SC1** for $(x + 1)$ subst <u>and</u> correct exp of $(x + 1)^3$ or two of their factors ft <u>or</u>, for those using given answer: **M1** for roots stated or used as −4,−2 and 3 or ft **A1** for showing all 3 roots satisfy given eqn **B1** for comment re coefft of x^3 or product of roots to show that eqn of translated graph is not a multiple of RHS of given eqn

Section B Total: 36

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for January 2011

Marking instructions for GCE Mathematics (MEI): Pure strand

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.

2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.

3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)

4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

9. **Rules for crossed out and/or replaced work**

   If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

   If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

   If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

   If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

    Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

    The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

    For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

    Award NR (no response) if:
    - Nothing is written at all in the answer space
    - There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
    - There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

    The hash key [#] on your keyboard will enter NR.

    Award 0 if:
    - There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

M1	method mark (M2, etc, is also used)
A1	accuracy mark
B1	independent mark
E1	mark for explaining
U1	mark for correct units
G1	mark for a correct feature on a graph
M1 dep*	method mark dependent on a previous mark, indicated by *
cao	correct answer only
ft	follow through
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
sc	special case
soi	seen or implied
www	without wrong working

14. Annotating scripts. The following annotations are available:

✓and ✗
BOD Benefit of doubt
FT Follow through
ISW Ignore subsequent working (after correct answer obtained)
M0, M1 Method mark awarded 0, 1
A0, A1 Accuracy mark awarded 0, 1
B0, B1 Independent mark awarded 0,1
SC Special case
^ Omission sign
MR Misread
Highlighting is also available to highlight any particular points on a script.

15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

    Please do not type in the comments box yourself. Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.

17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see scoris assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.

18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

SECTION A

1	$y = 5x + 3$	**3**	**M2** for $y - 13 = 5(x - 2)$ oe or **M1** for $y = 5x$ [+ k] [k = letter or number other than −4] and **M1** for 13 = their $m \times 2 + k$	or **M1** for $y - b = 5(x - a)$ with wrong a, b or for $y - 13 =$ their $5(x - 2)$ oe **M0** for first M if –1/5 used as gradient even if 5 seen first; second M still available if earned
2	(i)(A) 1/16	**1**	isw attempted conversion of 1/16 to decimals	accept 0.0625
2	(i)(B) 1	**1**		set image 'fit to height' so that in marking this question you also check that there is no working on the back page attached to the image
2	(ii) 256/625	**2**	**M1** for num or denom correct or for 4/5 or 0.8	accept 0.4096
3	$\frac{9y^{10}}{2x^2}$ oe as final answer	**3**	**1** for each 'term'; 27/6 gets 0 for first term if **0**, allow **B1** for $(3xy^4)^3 = 27x^3y^{12}$	allow eg $4.5x^{-2}y^{10}$
4	$x > 5/2$ oe (–5/–2 oe not sufft)	**2**	**M1** for $5 < 2x$ or for 5/2 oe obtained with equation or wrong inequality	**M0** for just $-2x < -5$ (not sufft) ; **M1** for $x > -5/-2$

5	$\frac{3V}{\pi r^2}=\sqrt{l^2-r^2}$	**M1**	for correctly getting non- '$l^2 - r^2$' terms on other side[**M0** for 'triple decker' fraction]	may be done in several steps, if so, condone omission of brackets in eg $9V^2 = \pi^2 r^4\ l^2 - r^2$ if they recover – if not, do not give 1st **M1** [but can earn the 2nd **M1**]
	$\left(\frac{3V}{\pi r^2}\right)^2=l^2-r^2$	**M1**	oe or ft; for squaring correctly	for combined steps, allow credit for correct process where possible;
	$l^2=\left(\frac{3V}{\pi r^2}\right)^2+r^2$	**M1**	oe or ft; for getting l term as subject	eg $\pi^2 r^4 l^2$ as the term on one side
	$[l=]\sqrt{\left(\frac{3V}{\pi r^2}\right)^2+r^2}$	**M1**	oe. or ft; mark final answer; for finding square root (and dealing correctly with coefficient of l term if needed at this stage); condone ±√etc	For **M4**, the final expression must be totally correct, [condoning omission of l and insertion of ±] eg **M4** for $\frac{\sqrt{9V^2+\pi^2 r^6}}{\pi r^2}$
6	$32 - 240x + 720x^2$ isw	**4**	**B3** for all correct except for sign error(s) **B2** for 2 terms correct numerically, ignoring any sign error or for 32, –240 and 720 found or B2 for all correct, including signs, but unsimplified **B1** for binomial coeffts 1, 5, 10 used or 1 5 10 10 5 1 seen **SC3** for $-240x + 720x^2 - 1080x^3$ isw or for $-243x^5 + 810x^4 - 1080x^3$ or **SC2** for these terms with sign error(s)	accept terms listed separately; condone $-240x^1$ expressions left in ${}^nC_r$ form or with factorials not sufft

7	(i) $3^{7/2}$ oe or k = 7/2 oe	**2**	**M1** for $\frac{3^4}{\sqrt{3}}$ or $\frac{81}{3^{1/2}}$ or $81 \times 3^{-1/2}$ or $3^3\sqrt{3}$ or 27 × $3^{1/2}$ or better or for 81 = 3^4 or √3 = $3^{1/2}$ or $\frac{1}{\sqrt{3}} = 3^{-1/2}$ or (following correct rationalisation of denominator) for 27 = 3^3 isw conversion of 7/2 oe	**M0** for just 81 = 3 × 3 × 3 × 3 oe – indices needed allow an M mark for partially correct work still seen in fraction form eg $\frac{3^4}{3^{-1/2}}$ gets mark for 81 = 3^4
7	(ii) $\frac{14+5\sqrt{3}}{11}$ or $\frac{28+10\sqrt{3}}{22}$ www isw	**3**	**M1** for multiplying num and denom by 5 + √3 and **M1** for num or denom correct in final answer (M0 if wrongly obtained)	2nd **M1** is not dependent on 1st **M1**
8	(7/11, 24/11) oe www	**3**	**B2** for one coord correct; condone not expressed as coords, isw or **M1** for subst or elimination; eg x + 2(5x − 1) = 5 oe; condone one error **SC2** for mixed fractions and decimals eg (3.5/5.5, 12/5.5)	
9	(i) ½ × 2x × (x + 2 + 3x + 6) oe x(4x + 8) = 140 oe and given ans $x^2 + 2x - 35 = 0$ obtained correctly with at least one further interim step	**M1** **A1**	correct statement of area of trap; may be rectangle ± triangle, or two triangles	eg 2x(x + 2) + ½ × 2x × (2x + 4) condone missing brackets for **M1;** condone also for **A1** if expansion is treated as if they were there

	(ii) [AB =] 21 www	**3**	or **B2** for x = [−7 or] 5 cao www or for AB = 21 or −15 or **M1** for $(x + 7)(x - 5)$ [= 0]or formula or completing square used eg $(x + 1)^2 - 36$ [= 0]; condone one error eg factors with sign wrong or which give two terms correct when expanded or **M1** for showing f(5) = 0 without stating $x = 5$	may be done in (i) if not here – allow the marks if seen in either part of the image – some candidates are omitting the request in (i) and going straight to solving the equation (in which case give 0 [not NR] for (i), but annotate when the image appears again in (ii)) 5 on its own or AB = 5 with no working scores 0; we need to see $x = 5$
10	(i) $P \Leftarrow Q$ (ii) none [of the above] (iii) $P \Rightarrow Q$	**1** **1** **1**	or $\Leftarrow$ or ‘$Q \Rightarrow P$’ or $\Rightarrow$	Condone single arrows

Section A Total: 36

SECTION B

11	(i) grad AB = $\frac{0-6}{1-(-1)}$ oe [= −3] isw grad BC = $\frac{0-4}{1-13}$ oe [= 1/3] isw product of grads = −1 [so lines perp] stated or shown numerically	**M1** **M1** **M1**	for full marks, it should be clear that grads are independently obtained or 'one grad is neg. reciprocal of other' **or** **M1** for length of one side (or square of it) **M1** for length of other two sides (or their squares) found independently **M1** for showing or stating that Pythag holds [so triangle rt angled]	eg grads of −3 and 1/3 without earlier working earn **M1M0** for **M3**, must be fully correct, with gradients evaluated at least to −6/2 and −4/−12 stage $AB^2 = 6^2 + 2^2 = 40$, $BC^2 = 4^2 + 12^2 = 160$, $AC^2 = 14^2 + 2^2 = 200$
11	(ii) AB = √40 or BC = √160 ½ × √40 × √160 oe or ft their AB, BC 40	**M1** **M1** **A1**	**or M1** for one of area under AC (=70), under AB (=6) under BC (=24) (accept unsimplified) and **M1** for their trap. – two triangles	allow **M1** for $\sqrt{(1-(-1))^2+(6-0)^2}$ or for $\sqrt{(13-1)^2+(4-0)^2}$ **or** for rectangle – 3 triangles method, [$6\times14-\frac{1}{2}(2)(6)-\frac{1}{2}(4)(12)-\frac{1}{2}(2)(14)$ =84 – 6 – 24 – 14] **M1** for two of the 4 areas correct and **M1** for the subtraction

11	(iii) angle subtended by diameter = 90° soi	**B1**	or angle at centre = twice angle at circumf = 2 × 90 = 180 soi or showing BM = AM or CM, where M is midpt of AC; or showing that BM = ½ AC	condone 'AB and BC are perpendicular' or 'ABC is right angled triangle' provided no spurious extra reasoning
	mid point M of AC = (6, 5)	**B2**	allow if seen in circle equation ; **M1** for correct working seen for **both** coords	
	rad of circle = $\frac{1}{2}\sqrt{14^2+2^2}\,[=]\,\frac{1}{2}\sqrt{200}$ oe or equiv using r^2	**M1**	accept unsimplified; or eg $r^2 = 7^2 + 1^2$ or $5^2 + 5^2$; may be implied by correct equation for circle or by correct method for AM, BM or CM ft their M	allow **M1** bod intent for AC = $\sqrt{200}$ followed by $r = \sqrt{100}$
	$(x - a)^2 + (y - b)^2 = r^2$ seen or $(x$ – their $6)^2 + (y$ – their $5)^2 = k$ used, with $k > 0$	**M1**		
	$(x - 6)^2 + (y - 5)^2 = 50$ cao	**A1**	or $x^2 + y^2 - 12x - 10y + 11 = 0$	must be simplified (no surds)
11	(iv) (11, 10) cao	**1**		
12	(i)(A) sketch of cubic correct way up and with two tps, crossing x-axis in 3 distinct points	**B1**	**0** if stops at x-axis; condone not crossing y-axis	No section to be ruled; no curving back; condone slight 'flicking out' at ends; condone some doubling (eg erased curves may continue to show)
	crossing x axis at 1, 2.5 and 4	**B1**	intersections labelled on graph or shown nearby in this part; mark intent for intersections with both axes (eg condone graphs stopping at axes)	allow 2.5 indicated by graph crossing halfway between their marked 2 and 3 on scale; allow if no graph but 0 if graph inconsistent with values
	crossing y axis at −20	**B1**	or $x = 0$, $y = -20$ seen in this part if consistent with graph drawn	allow if no graph, but eg **B0** for graph with intn on +ve y-axis or nowhere near their indicated –20

12	(i)(B) correct expansion of two brackets correct interim step(s) multiplying out linear and quadratic factors before given answer **or** showing that 1, 2.5 and 4 all satisfy f(x) = 0 for cubic in $2x^3$ ... form comparing coeffts of eg x^3 in the two forms	**M1** **M1** **or** **M1** **M1**	or **M2** for all 3 brackets multiplied at once, showing all 8 terms (**M1** if error in one term): $2x^3 - 8x^2 - 2x^2 - 5x^2 + 8x + 5x + 20x - 20$ **or** **M1** for dividing $2x^3$ ... form by one of the linear factors and **M1** for factorising the resultant quadratic factor	eg **M1** for $(2x - 5)(x^2 - 5x + 4)$ condone missing brackets if intent clear /used correctly
12	(ii)(A) 250 − 375 + 165 − 40 isw	**B1**	**or** showing that $x - 5$ is a factor by eg division and then stating that $x = 5$ is root or that g(5) = 0	'2 × 125 + 15 × 25 + 33 × 5 – 40' is not sufft **or** [g(5) =] f(5) – 20 = 5 × 4 × 1 – 20 [= 0]
12	(ii) (B) $(x - 5)$ seen or used as linear factor division by $(x - 5)$ as far as $2x^3 - 10x^2$ seen in working $2x^2 - 5x + 8$ obtained isw	**M1** **M1** **A1**	may be in attempt at division or inspection/equating coefficients with two terms correct eg $(2x^2$ ….. + 8) eg may be seen in grid; condone g(x) not expressed as product	allow if seen in (ii)(A) for division: condone signs of $2x^3 - 10x^2$ changed for subtraction, or subtraction sign in front of first term

12	(ii)(C) $b^2 - 4ac$ used on their quadratic factor $(-5)^2 - 4 \times 2 \times 8$ oe and negative [or −39] so no [real] root [may say only one [real] root, thinking of $x = 5$]	**M1** **A1**	may be in formula [or allow **2** marks for complete correct attempt at completing square and conclusion, or using calculus to show min value is above x-axis and comment re curve all above x-axis]	no ft for A mark from wrong quadratic factor condone error in working out –39 if correct unsimplified expression seen and neg result obtained $-5^2 - 4 \times 2 \times 8$ evaluated correctly with comment is eligible for **A1**, otherwise bod for the **M1** only
12	(iii) translation $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$	**B1** **B1**	NB 'Moves' not sufficient for this first mark or 20 down;	**B0** for second mark if choice of one wrong, one right description
13	(i) (0, −2) or 'crosses y-axis at −2' oe isw $(\pm 2^{\frac{1}{4}}, 0)$ oe isw	**B1** **B2**	or [when $y = 0$], $[x =] \pm 2^{\frac{1}{4}}$ or $\pm\sqrt{\sqrt{2}}$ or $\pm\sqrt[4]{2}$ isw **B1** for one root correct	condone $y = -2$

13	(ii) [y =] $x^2 = x^4 - 2$ oe and rearrangement to $x^4 - x^2 - 2$ [= 0] or $y^2 - y - 2$ [=0]	**M1**		
	$(x^2 - 2)(x^2 + 1) = 0$ oe in y	**M1**	or formula or completing square; condone one error; condone replacement of x^2 by another letter or by x for 2nd M1 (but not the 3rd M1)	if completing square, and haven't arranged to zero, can earn first **M1** as well for an attempt such as $(x^2 - 0.5)^2 = 2.25$
	$x^2 = 2$ [or −1] or $y = 2$ or −1 or ft or $x = \sqrt{2}$ or $x = -\sqrt{2}$ or ft	**M1**	dep on 2nd **M1;** allow inclusion of correct complex roots; **M0** if any incorrect roots are included for x^2 or x	NB for second and third M: **M0** for $x^2 - 2 = 0$ or $x^2 = 2$ oe straight from quartic eqn – some candidates probably thinking $x^4 - x^2$ simplifies to x^2; last two marks for roots are available as B marks
	$(\sqrt{2}, 2)$ and $(-\sqrt{2}, 2)$; with no other intersections given	**B2**	or **B1** for one of these two intersections (even if extra intersections given) or for $x = \pm\sqrt{2}$ (and no other roots) or for y = 2 (and no other roots), marking to candidates' advantage	some candidates having several attempts at solving this equation – mark the best in this particular case

13	(iii) from $x^4 - kx^2 - 2$ [= 0]: $k^2 + 8 > 0$ oe $k + \sqrt{k^2+8} \geq 0$ for all k [so there is a positive root for x^2 and hence real root for x and so intersection]	 **B1** **B1**	Allow x^2 replaced by other letters or x or from $y^2 - k^2y - 2k^2$ [= 0] $k^4 + 8k^2 > 0$ oe $k^2 + \sqrt{k^4+8k^2} > 0$ oe for all k [so there is a positive root for y and hence real root for x and so intersection] if **B0B0**, allow **SC1** for $\frac{k \pm \sqrt{k^2+8}}{2}$ or $\frac{k^2 \pm \sqrt{k^4+8k^2}}{2}$ obtained [need not be simplified]	[alt methods: may use completing square to show similarly, or comment that at $x = 0$ the quadratic is above the quartic and that as $x \to \infty$, $x^4 - 2 > kx^2$ for all k] condone lack of brackets in $(-k)^2$

Section B Total: 36

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for June 2011

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

SECTION A

1	$x > -13/4$ o.e. isw www	**3**	condone $x > 13/-4$ or $13/-4 < x$; **M2** for $4x > -13$ or **M1** for one side of this correct with correct inequality, and **B1** for final step ft from their $ax > b$ or $c > dx$ for $a \neq 1$and $d \neq 1$; if no working shown, allow **SC1** for $-13/4$ oe with equals sign or wrong inequality	**M1** for $13 > -4x$ (may be followed by $13/-4 > x$, which earns no further credit); $6x + 3 > 2x + 5$ is an error not an MR; can get **M1** for $4x >$ ... following this, and then a possible **B1**
2	7	**2**	condone $y = 7$ or (5, 7); **M1** for $\frac{k-(-5)}{5-1} = 3$ or other correct use of gradient eg triangle with 4 across, 12 up	condone omission of brackets; or **M1** for correct method for eqn of line and $x = 5$ subst in their eqn and evaluated to find k; or **M1** for both of $y - k = 3(x - 5)$ oe and $y - (-5) = 3(x - 1)$ oe
3	(i) 4/3 isw	**2**	condone ±4/3; **M1** for numerator or denominator correct or for $\frac{3}{4}$ or $\frac{1}{\left(\frac{3}{4}\right)}$ oe or for $\left(\frac{16}{9}\right)^{\frac{1}{2}}$ soi	**M1** for just $-4/3$; allow **M1** for $\sqrt{16} = 4$ and $\sqrt{9} = 3$ soi; condone missing brackets

3	(ii) $\frac{2a}{c^5}$ or $2ac^{-5}$	**3**	**B1** for each 'term' correct; mark final answer; if B0, then **SC1** for $(2ac^2)^3 = 8a^3c^6$ or $72a^5c^7$ seen	condone a^1; condone multiplication signs but **0** for addition signs
4	(i) (10, 4)	**2**	**0** for (5, 4); otherwise **1** for each coordinate	ignore accompanying working / description of transformation; condone omission of brackets; (Image includes back page for examiners to check that there is no work there)
4	(ii) (5, 11)	**2**	**0** for (5, 4); otherwise **1** for each coordinate	ignore accompanying working / description of transformation; condone omission of brackets
5	6000	**4**	**M3** for $15 \times 5^2 \times 2^4$; or **M2** for two of these elements correct with multiplication or all three elements correct but without multiplication (e.g. in list or with addition signs); or **M1** for 15 soi or for 1 6 15 ... seen in Pascal's triangle; **SC2** for 20000[x^3]	condone inclusion of x^4 eg $(2x)^4$; condone omission of brackets in $2x^4$ if 16 used; allow **M3** for correct term seen (often all terms written down) but then wrong term evaluated or all evaluated and correct term not identified; $15 \times 5^2 \times (2x)^4$ earns **M3** even if followed by $15 \times 25 \times 2$ calculated; no MR for wrong power evaluated but **SC** for fourth term evaluated

6	$2x^3 + 9x^2 + 4x - 15$	**3**	as final answer; ignore '= 0'; **B2** for 3 correct terms of answer seen or for an 8-term or 6 term expansion with at most one error: or **M1** for correct quadratic expansion of one pair of brackets; or **SC1** for a quadratic expansion with one error then a good attempt to multiply by the remaining bracket	correct 8-term expansion: $2x^3 + 6x^2 - 2x^2 + 5x^2 - 6x + 15x - 5x - 15$ correct 6-term expansions: $2x^3 + 4x^2 + 5x^2 - 6x + 10x - 15$ $2x^3 + 6x^2 + 3x^2 + 9x - 5x - 15$ $2x^3 + 11x^2 - 2x^2 + 15x - 11x - 15$ for **M1**, need not be simplified; ie **SC1** for knowing what to do and making a reasonable attempt, even if an error at an early stage means more marks not available
7	$b^2 - 4ac$ soi	**M1**		allow seen in formula; need not have numbers substituted but discriminant part must be correct;
	1 www	**A1**	or **B2**	clearly found as discriminant, or stated as $b^2 - 4ac$, not just seen in formula eg **M1A0** for $\sqrt{b^2 - 4ac} = \sqrt{1} = 1$;
	2 [distinct real roots]	**B1**	**B0** for finding the roots but not saying how many there are	condone discriminant not used; ignore incorrect roots found

8	$yx + 3y = 1 - 2x$ oe or ft	**M1**	for multiplying to eliminate denominator <u>and</u> for expanding brackets, or for correct division by y <u>and</u> writing as separate fractions: $x+3=\frac{1}{y}-\frac{2x}{y}$;	each mark is for carrying out the operation correctly; ft earlier errors for equivalent steps if error does not simplify problem; some common errors:
	$yx + 2x = 1 - 3y$ oe or ft	**M1**	for collecting terms; dep on having an ax term and an xy term, oe after division by y,	$y(x + 3) = 1 - 2x$ $yx + 3x = 1 - 2x$ **M0** $yx + 5x = 1$ **M1** ft $x(y + 5) = 1$ **M1** ft $x=\frac{1}{y+5}$ **M1** ft $yx + 3 = 1 - 2x$ **M0** $yx + 2x = -2$ **M1** ft $x(y + 2) = -2$ **M1** ft $x=\frac{-2}{y+2}$ **M1** ft
	$x(y + 2) = 1 - 3y$ oe or ft	**M1**	for taking out x factor; dep on having an ax term and an xy term, oe after division by y,	
	$[x=]\frac{1-3y}{y+2}$ oe or ft as final answer	**M1**	for division with no wrong work after; dep on dividing by a two-term expression; last M not earned for triple-decker fraction as final answer	for **M4**, must be completely correct;

9	$x + 2y = k$ ($k \neq 6$) or $y = -\frac{1}{2}x + c$ ($c \neq 3$)	**M1**	for attempt to use gradients of parallel lines the same; **M0** if just given line used;	eg following an error in manipulation, getting original line as $y = \frac{1}{2}x + 3$ then using $y = \frac{1}{2}x + c$ earns **M1** and can then go on to get **A0** for $y = \frac{1}{2}x - 4$, **M1** for (0, −4) **M1** for (8, 0) and **A0** for area of 16;
	$x + 2y = 12$ or [$y =$]$- \frac{1}{2}x + 6$ oe	**A1**	or **B2**; must be simplified; or evidence of correct 'stepping' using (10, 1) eg may be on diagram;	allow bod **B2** for a candidate who goes straight to $y = -\frac{1}{2}x + 6$ from $2y = -x + 6$; NB the equation of the line is not required; correct intercepts obtained will imply this **A1;**
	(12, 0) or ft	**M1**	or 'when $y = 0$, $x = 12$' etc or using 12 or ft as a limit of integration; intersections must ft from their line or 'stepping' diagram using their gradient	NB for intersections with axes, if both Ms are not gained, it must be clear which coord is being found eg **M0** for intn with x axis = 6 from correct eqn;; if the intersections are not explicit, they may be implied by the area calculation eg use of ht = 6 or the correct ft area found;
	(0, 6)or ft	**M1**	or integrating to give $-\frac{1}{4}x^2 + 6x$ or ft their line	allow ft from the given line as well as others for both these intersection Ms;
	36 [sq units] cao	**A1**	or **B3** www	NB **A0** if 36 is incorrectly obtained eg after intersection $x = -12$ seen (which earns **M0** from correct line);

10	*n* (*n* + 1)(*n* + 2) argument from general consecutive numbers leading to: at least one must be even [exactly] one must be multiple of 3	**M1** **A1** **A1**	condone division by *n* and then (*n* + 1)(*n* + 2) seen, or separate factors shown after factor theorem used; or divisible by 2; if M0: allow **SC1** for showing given expression always even	ignore ‘ = 0’; an induction approach using the factors may also be used eg by those doing paper FP1 as well; **A0** for just substituting numbers for *n* and stating results; allow **SC2** for a correct induction approach using the original cubic (**SC1** for each of showing even and showing divisible by 3)

SECTION B

11	(i) $x + 4x^2 + 24x + 31 = 10$ oe	**M1**	for subst of x or y or subtraction to eliminate variable; condone one error;	
	$4x^2 + 25x + 21$ [= 0]	**M1**	for collection of terms and rearrangement to zero; condone one error;	or $4y^2 - 105y + 671$ [= 0]; eg condone spurious $y = 4x^2 + 25x + 21$ as one error (and then count as eligible for 3rd **M1**);
	$(4x + 21)(x + 1)$	**M1**	for factors giving at least two terms of their quadratic correct or for subst into formula with no more than two errors [dependent on attempt to rearrange to zero];	or $(y - 11)(4y - 61)$; [for full use of completing square with no more than two errors allow 2nd and 3rd **M1**s simultaneously];
	$x = -1$ or $-21/4$ oe isw	**A1**	or **A1** for (−1, 11) and **A1** for (−21/4, 61/4) oe	from formula: accept $x = -1$ or $-42/8$ oe isw
	$y = 11$ or $61/4$ oe isw	**A1**		
11	(ii) $4(x + 3)^2 - 5$ isw	**4**	**B1** for $a = 4$, **B1** for $b = 3$, **B2** for $c = -5$ or **M1** for $31 - 4 \times$ their b^2 soi or for −5/4 or for 31/4 − their b^2 soi	eg an answer of $(x + 3)^2 - {}^5/_4$ earns **B0 B1 M1**; $1(2x + 6)^2 - 5$ earns **B0 B0 B2**; 4(earns first **B1**; condone omission of square symbol
11	(iii)(*A*) $x = -3$ or ft (–their b) from (ii)	**1**		**0** for just –3 or ft; **0** for $x = -3$, $y = -5$ or ft
11	(iii)(*B*) −5 or ft their c from (ii)	**1**	allow $y = -5$ or ft	**0** for just (−3, −5); bod **1** for $x = -3$ stated then $y = -5$ or ft

12	(i) $y = 2x + 5$ drawn -2, -1.4 to -1.2, 0.7 to 0.85	**M1** **A2**	 **A1** for two of these correct	condone unruled and some doubling; tolerance: must pass within/touch at least two circles on overlay; the line must be drawn long enough to intersect curve at least twice; condone coordinates or factors
12	(ii) $4 = 2x^3 + 5x^2$ or $2x + 5 - \frac{4}{x^2} = 0$ and completion to given answer $f(-2) = -16 + 20 - 4 = 0$ use of $x + 2$ as factor in long division of given cubic as far as $2x^3 + 4x^2$ in working $2x^2 + x - 2$ obtained $[x =]\frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times -2}}{2 \times 2}$ oe $\frac{-1 \pm \sqrt{17}}{4}$ oe isw	**B1** **B1** **M1** **A1** **M1** **A1**	 or correct division / inspection showing that $x + 2$ is factor; or inspection or equating coefficients, with at least two terms correct; dep on previous M1 earned; for attempt at formula or full attempt at completing square, using their other factor	condone omission of final '= 0'; may be set out in grid format condone omission of + sign (eg in grid format) not more than two errors in formula / substitution / completing square; allow even if their 'factor' has a remainder shown in working; **M0** for just an attempt to factorise

12	(iii) $\frac{4}{x^2} = x + 2$ or $y = x + 2$ soi	**M1**	eg is earned by correct line drawn	condone intent for line; allow slightly out of tolerance;
	$y = x + 2$ drawn	**A1**		condone unruled; need drawn for $-1.5 \le x \le 1.2$; to pass through/touch relevant circle(s) on overlay
	1 real root	**A1**		
13	(i) [radius =] 4	**B1**	**B0** for ± 4	
	[centre] (4, 2)	**B1**		condone omission of brackets

13	(ii) $(x-4)^2 + (-2)^2 = 16$ oe	**M1**	for subst $y = 0$ in circle eqn;	NB candidates may expand and rearrange eqn first, making errors – they can still earn this **M1** when they subst $y = 0$ in their circle eqn; condone omission of $(-2)^2$ for this first **M1** only; not for second and third **M1**s; do not allow substitution of $x = 0$ for any Ms in this part
	$(x-4)^2 = 12$ or $x^2 - 8x + 4$ [= 0]	**M1**	putting in form ready to solve by comp sq, or for rearrangement to zero; condone one error;	eg allow **M1** for $x^2 + 4 = 0$ [but this two-term quadratic is not eligible for 3rd **M1**];
	$x-4 = \pm\sqrt{12}$ or $[x=]\dfrac{8\pm\sqrt{8^2-4\times1\times4}}{2\times1}$	**M1**	for attempt at comp square or formula; dep on previous M2 earned and on three-term quadratic;	not more than two errors in formula / substitution; allow **M1** for $x-4=\sqrt{12}$; **M0** for just an attempt to factorise
	$[x=]4\pm\sqrt{12}$ or $4\pm2\sqrt{3}$ or $\dfrac{8\pm\sqrt{48}}{2}$ oe isw	**A1**		
	or	**or**		
	sketch showing centre (4, 2) and triangle with hyp 4 and ht 2	**M1**		
	$4^2 - 2^2 = 12$	**M1**	or the square root of this; implies previous M1 if no sketch seen;	
	$[x=]4\pm\sqrt{12}$ oe	**A2**	**A1** for one solution	

13	(iii) subst $\left(4+2\sqrt{2},\ 2+2\sqrt{2}\right)$ into circle eqn and showing at least one step in correct completion	**B1**	or showing sketch of centre C and A and using Pythag: $\left(2\sqrt{2}\right)^2+\left(2\sqrt{2}\right)^2=8+8=16$;	or subst the value for one coord in circle eqn and correctly working out the other as a possible value;
	Sketch of both tangents	**M1**		need not be ruled; must have negative gradients with tangents intended to be parallel and one touching above and to right of centre; mark intent to touch – allow just missing or just crossing circle twice; condone A not labelled
	grad tgt = −1 or −1/their grad CA	**M1**	allow ft after correct method seen for grad CA = $\frac{2+2\sqrt{2}-2}{4+2\sqrt{2}-4}$ oe (may be on/ near sketch);	allow ft from wrong centre found in (i);
	$y-(2+2\sqrt{2})=\text{their } m(x-(4+2\sqrt{2}))$	**M1**	or y = their $mx + c$ and subst of $\left(4+2\sqrt{2},\ 2+2\sqrt{2}\right)$;	for intent; condone lack of brackets for **M1**; independent of previous Ms; condone grad of CA used;
	$y=-x+6+4\sqrt{2}$ oe isw	**A1**	accept simplified equivs eg $x+y=6+4\sqrt{2}$;	**A0** if obtained as eqn of other tangent instead of the tangent at A (eg after omission of brackets);
	parallel tgt goes through $\left(4-2\sqrt{2},\ 2-2\sqrt{2}\right)$	**M1**	or ft wrong centre; may be shown on diagram; may be implied by correct equation for the tangent (allow ft their gradient);	no bod for just $y-2-2\sqrt{2}=-1(x-4-2\sqrt{2})$ without first seeing correct coordinates;
	eqn is $y=-x+6-4\sqrt{2}$ oe isw	**A1**	accept simplified equivs eg $x+y=6-4\sqrt{2}$	**A0** if this is given as eqn of the tangent at A instead of other tangent (eg after omission of brackets)

Section B Total: 36

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

Question			Answer	Marks	Guidance	
1			$y = -2x + 7$ isw (0, 7) and (3.5, 0) oe or ft their $y = -2x + c$	2 1 **[3]**	M1 for $y - 1 = -2(x - 3)$ or $1 = -2 \times 3 + c$ oe	condone lack of brackets and eg $y = 7$, $x = 3.5$ or ft isw but 0 for poor notation such as (3.5, 7) and no better answers seen
2			$[b =] \pm\sqrt{\frac{3a}{2c}}$ oe www	3 **[3]**	M2 for $[b^2 =]\frac{3a}{2c}$ soi or M1 for other $[b^2 =]\frac{ka}{c}$ or $[b^2 =]\frac{a}{kc}$ oe and M1 for correctly taking the square root of their b^2, including the ± sign;	eg M2 for $[b =]\sqrt{\frac{3a}{2c}}$ allow M1 for a triple-decker or quadruple-decker fraction or decimals eg $\frac{1.5a}{c}$, if no recovery later square root must extend below the fraction line
3	(i)		25	2 **[2]**	M1 for $\frac{1}{\frac{1}{25}}$ or $\left(\frac{1}{25}\right)^{-1}$ or 5^2 or $\frac{25}{1}$	
3	(ii)		$\frac{4}{9}$	2 **[2]**	M1 for 4 or 9 or $\frac{1}{9}$ or $\frac{2}{3}$ or $\left(\frac{2}{3}\right)^2$ or $\sqrt[3]{\frac{64}{729}}$ seen	0 for just $\left(\frac{64}{729}\right)^{\frac{1}{3}}$
4			$\frac{x-3}{x+2}$ or $1-\frac{5}{x+2}$ as final answer www	3 **[3]**	B2 for correct answer seen and then spoilt M1 for $(x + 3)(x - 3)$ and M1 for $(x + 2)(x + 3)$	

Question			Answer	Marks	Guidance	
5	(i)		30	3 **[3]**	M1 for $\left(\sqrt{6}\right)^3 = 6\sqrt{6}$ soi and M1 for $\sqrt{24} = 2\sqrt{6}$ soi or allow SC2 for final answer of $5\left(\sqrt{6}\right)^2$ or $5\sqrt{36}$ or $10\sqrt{9}$ etc	M0 for $6000\sqrt{6}$ ie cubing 10 as well for those using indices: M1 for both $10 \times 6^{3/2}$ and $2 \times 6^{1/2}$ oe then M1 for 5×6 oe award SC2 for similar correct answer with no denominator
5	(ii)		$\frac{8}{11}$	2 **[2]**	M1 for common denominator $\left(4+\sqrt{5}\right)\left(4-\sqrt{5}\right)$ soi - may be in separate fractions or for a final answer with denominator 11, even if worked with only one fraction	condone lack of brackets
6	(i)		10 cao	1 **[1]**		
6	(ii)		$-720\ [x^3]$	4 **[4]**	B3 for 720 $[x^3]$ or for $10 \times 9 \times -8\ [x^3]$ or M2 for $10 \times 3^2 \times (-2)^3$ oe or ft from (i) or M1 for two of these three elements correct or ft; condone x still included	condone $-720x$ etc allow equivalent marks for the x^3 term as part of a longer expansion eg M2 for $3^5\left(...10\times\left(\frac{-2}{3}\right)^3...\right)$ or M1 for $10\times\left(\frac{-2}{3}\right)^3$ etc

Question			Answer	Marks	Guidance	
7			$4k^2 - 4 \times 1 \times 5$ or $k^2 - 5$ [< 0] oe or $[(x + k)^2 +]\ 5 - k^2$ [> 0] oe	M2	allow =, > , ≤ etc instead of < or M1 for $b^2 - 4ac$ soi (may be in formula) or for attempt at completing square	allow M2 for $2k^2 < 20$, $2k^2 - 20 = 0$ etc but M1 only for just $2k^2 - 20$ ignore rest of quadratic formula ignore $\sqrt{b^2 - 4ac} < 0$ seen if $b^2 - 4ac < 0$ then used, otherwise just M1 for $\sqrt{b^2 - 4ac} < 0$
			$-\sqrt{5} < k < \sqrt{5}$	A2 **[4]**	may be two separate inequalities or A1 for one 'end' correct or B1 for 'endpoint' = $\sqrt{5}$	allow SC1 for $-\sqrt{10} < k < \sqrt{10}$ following at least M1 for $2k^2 - 20$ oe
8			$16 + 2b + c = 0$ oe	M1	need not be simplified; condone 8 or 32 as first term if 2^4 not seen	in this question use annotation to indicate where part marks are earned
			$81 - 3b + c = 85$ oe	B2	M1 for f(–3) seen or used, condoning one error except $+3b$ – need not be simplified or for long division as far as obtaining $x^3 - 3x^2$ in quotient	eg M1 for $81 - 3b + c = 0$ 'long division' may be seen in grid or a mixture of methods may be used eg B2 for $c - 3(b - 27) = 85$
			$20 + 5b = 0$ oe	M1	for elimination of one variable, ft their equations in b and c, condoning one error in rearrangement of their original equations or in one term in the elimination	correct operation must be used in elimination
			$b = -4$ and $c = -8$	A1 **[5]**	allow correct answers to imply last M1 after correct earlier equations	for misread of x^4 as x^3 or x^2 or higher powers, allow all 3 Ms equivalently

Question			Answer	Marks	Guidance	
9			$6n + 9$ isw or $3(2n + 3)$	B1		
			$6n$ is even [but 9 is odd], even + odd = odd or $2n + 3$ is odd since even + odd = odd and odd × odd = odd	B1 dep	this mark is dependent on the previous B1 accept equiv. general statements using either $6n + 9$ or $3(2n + 3)$	
			'n is a multiple of 3' or 'n is divisible by 3' without additional incorrect statement(s)	B2	B2 for 'it is divisible by 9, so n is divisible by 3' M1 for '$6n$ is divisible by 9' or '$2n + 3$ is divisible by 3' or for 'n is a multiple of 3' oe with additional incorrect statement(s)	B2 for just 'it is divisible by 3' but M1 for 'it is divisible by 9, so it is divisible by 3' eg M1 for 'n is divisible by 9, so n is divisible by 3' N.B. 0 for 'n is a factor of 3' (but M1 may be earned earlier)
				[4]		

Question			Answer	Marks	Guidance	
10	(i)		$AB^2 = (1-(-1))^2 + (5-1)^2$	M1	oe, or square root of this; condone poor notation re roots; condone $(1 + 1)^2$ instead of $(1-(-1))^2$ allow M1 for vector AB = $\begin{pmatrix}-2\\-4\end{pmatrix}$, condoning poor notation, or triangle with hyp AB and lengths 2 and 4 correctly marked	
			$BC^2 = (3-(-1))^2 + (-1-1)^2$	M1	oe, or square root of this; condone poor notation re roots; condone $(3 + 1)^2$ instead of $(3-(-1))^2$ oe allow M1 for vector BC = $\begin{pmatrix}4\\-2\end{pmatrix}$, condoning poor notation, or triangle with hyp BC and lengths 4 and 2 correctly marked	
			shown equal eg $AB^2 = 2^2 + 4^2$ [=20] and $BC^2 = 4^2 + 2^2$ [=20] with correct notation for final comparison	A1	or statement that AB and BC are each the hypotenuse of a right-angled triangle with sides 2 and 4 so are equal SC2 for just $AB^2 = 2^2 + 4^2$ and $BC^2 = 4^2 + 2^2$ (or roots of these) with no clearer earlier working; condone poor notation	eg A0 for AB = 20 etc
				[3]		

Question			Answer	Marks	Guidance	
10	(ii)		[grad. of AC =] $\frac{5-(-1)}{1-3}$ or $\frac{6}{-2}$ oe	M1	award at first step shown even if errors after	
			[grad. of BD =] $\frac{5-1}{11-(-1)}$ or $\frac{4}{12}$ oe	M1		if one or both of grad AC = -3 and grad BD = 1/3 seen without better working for both gradients, award one M1 only. For M1M1 it must be clear that they are obtained independently
			showing or stating product of gradients = –1 or that one gradient is the negative reciprocal of the other oe	B1	eg accept $m_1 \times m_2 = -1$ or 'one gradient is negative reciprocal of the other' B0 for 'opposite' used instead of 'negative' or 'reciprocal'	may be earned independently of correct gradients, but for all 3 marks to be earned the work must be fully correct
				[3]		

Question			Answer	Marks	Guidance	
10	(iii)		midpoint E of AC = (2, 2) www	B1	condone missing brackets for both B1s	0 for ((5+ –1)/2, (1+3)/2) = (2, 2)
			eqn BD is $y = \frac{1}{3}x + \frac{4}{3}$ oe	M1	accept any correct form isw or correct ft their gradients or their midpt F of BD this mark will often be gained on the first line of their working for BD	may be earned using (2, 2) but then must independently show that B or D or (5, 3) is on this line to be eligible for A1
			eqn AC is $y = –3x + 8$ oe	M1	accept any correct form isw or correct ft their gradients or their midpt E of AC this mark will often be gained on the first line of their working for AC [see appendix for alternative methods instead showing E is on BD for this M1]	if equation(s) of lines are seen in part ii, allow the M1s if seen/used in this part
			using both lines and obtaining intersection E is (2, 2) (NB must be independently obtained from midpt of AC)	A1		[see appendix for alternative ways of gaining these last two marks in different methods]
			midpoint F of BD = (5,3)	B1	this mark is often earned earlier	
					see the appendix for some common alternative methods for this question; for all methods, for A1 to be earned, all work for the 5 marks must be correct	for all methods show annotations M1 B1 etc then omission mark or A0 if that mark has not been earned
				[5]		

Question			Answer	Marks	Guidance	
11	**(i)**		$(2x + 1)(x + 2)(x – 5)$	M1	or $(x + 1/2)(x + 2)(x – 5)$; need not be written as product	throughout, ignore ‘=0’
			correct expansion of two linear factors of their product of three linear factors	M1		for all Ms in this part condone missing brackets if used correctly
			expansion of their linear and quadratic factors	M1	dep on first M1; ft one error in previous expansion; condone one error in this expansion	
					or for direct expansion of all three factors, allow M2 for $2x^3 – 10x^2 + 4x^2 + x^2 – 20x – 5x + 2x – 10$ [or half all these], or M1 if one or two errors,	dep on first M1
			[y =] $2x^3 – 5x^2 – 23x – 10$ or $a = –5$, $b = – 23$ and $c = – 10$	A1		condone poor notation when ‘doubling’ to reach expression with $2x^3$...
					for an attempt at setting up three simultaneous equations in a, b, and c: M1 for at least two of the three equations	$250 + 25a + 5b + c = 0$ $–16 + 4a –2b + c = 0$ $–¼+ ¼\ a – ½\ b + c = 0$ oe
					then M2 for correctly eliminating any two variables or M1 for correctly eliminating one variable to get two equations in two unknowns	
				[4]	and then A1 for values.	

Question			Answer	Marks	Guidance	
11	(ii)		graph of cubic correct way up	B1		must not be ruled; no curving back; condone slight 'flicking out' at ends; allow min on y axis or in 3rd or 4th quadrants; condone some 'doubling' or 'feathering' (deleted work still may show in scans)
			crossing x axis at –2, –1/2 and 5	B1	B0 if stops at x-axis on graph or nearby in this part mark intent for intersections with both axes	allow if no graph, but marked on x-axis
			crossing y axis at –10 or ft their cubic in (i)	B1 **[3]**	or $x = 0$, $y = -10$ or ft in this part if consistent with graph drawn;	allow if no graph, but eg B0 for graph nowhere near their indicated –10 or ft
11	(iii)		(0, –18); accept –18 or ft their constant – 8	1 **[1]**	or ft their intn on y-axis – 8	
11	(iv)		roots at 2.5, 1, 8	M1	or attempt to substitute $(x - 3)$ in $(2x + 1)(x + 2)(x - 5)$ or in $(x + 1/2)(x + 2)(x - 5)$ or in their unfactorised form of f(x)– attempt need not be simplified	
			$(2x - 5)(x - 1)(x - 8)$	A1	accept $2(x - 2.5)$ oe instead of $(2x - 5)$	M0 for use of $(x + 3)$ or roots –3.5, –5, 2 but then allow SC1 for $(2x + 7)(x + 5)(x - 2)$
			(0, –40); accept –40	B2 **[4]**	M1 for $-5 \times -1 \times -8$ or ft or for f(–3) attempted or g(0) attempted or for their answer ft from their factorised form	eg M1 for (0, –70) or –70 after $(2x + 7)(x + 5)(x - 2)$ after M0, allow SC1 for f(3) = –70

Question			Answer	Marks	Guidance	
12	(i)		(–1, 6) (0,1) (1,–2) (2,–3) (3,–2) (4, 1) (5,6) seen plotted	B2	or for a curve within 2 mm of these points; B1 for 3 correct plots or for at least 3 of the pairs of values seen eg in table	use overlay; scroll down to spare copy of graph to see if used [or click 'fit height' also allow B1 for ($2 \pm \sqrt{3}$, 0) and (2, −3) seen or plotted and curve not through other correct points
			smooth curve through all 7 points	B1 dep	dep on correct points; tolerance 2 mm;	condone some feathering/ doubling (deleted work still may show in scans); curve should not be flat-bottomed or go to a point at min. or curve back in at top;
			(0.3 to 0.5, –0.3 to –0.5) and (2.5 to 2.7, –2.5 to –2.7) and (4, 1)	B2 **[5]**	may be given in form x = ..., y = ... B1 for two intersections correct or for all the x values given correctly	
12	(ii)		$\frac{1}{x-3} = x^2 - 4x + 1$	M1		
			$1 = (x - 3)(x^2 - 4x + 1)$	M1	condone omission of brackets only if used correctly afterwards, with at most one error;	condone omission of '=1' for this M1 only if it reappears allow for terms expanded correctly with at most one error
			at least one further correct interim step with '=1' or '=0' ,as appropriate, leading to given answer, which must be stated correctly	A1 **[3]**	there may also be a previous step of expansion of terms without an equation, eg in grid if M0, allow SC1 for correct division of given cubic by quadratic to gain $(x - 3)$ with remainder –1, or vice-versa	NB mark method not answer - given answer is $x^3 - 7x^2 + 13x - 4 = 0$

Question			Answer	Marks	Guidance	
12	(iii)		quadratic factor is $x^2 - 3x + 1$	B2	found by division or inspection; allow M1 for division by $x - 4$ as far as $x^3 - 4x^2$ in the working, or for inspection with two terms correct	
			substitution into quadratic formula or for completing the square used as far as $\left(x-\frac{3}{2}\right)^2=\frac{5}{4}$	M1	condone one error	no ft from a wrong 'factor';
			$\frac{3\pm\sqrt{5}}{2}$ oe	A2	A1 if one error in final numerical expression, but only if roots are real	isw factors
				[5]		

<u>Appendix: alternative methods for 10(iii)</u> [details of equations etc are in main scheme]
for a mixture of methods, look for the method which gives most benefit to candidate, but take care not to award the second M1 twice
the final A1 is not earned if there is wrong work leading to the required statements
ignore wrong working which has not been used for the required statements
for full marks to be earned in this part, there must be enough to show both the required statements

find midpt E of AC find eqn BD	B1 M1	find midpt E of AC find eqn BD	B1 M1	find midpt E of AC find eqn BD	B1 M1	find midpt E of AC use gradients or vectors to show E is on BD eg grad BE = $\frac{2-1}{2--1}=\frac{1}{3}$ and grad ED = $\frac{5-2}{11-2}=\frac{1}{3}$ [condone poor vector notation]	B1 M2
show E on BD find midpt F of BD	M1 B1	show E on BD find midpt F of BD	M1 B1	show E on BD show $BE^2 = 10$ and $DE^2 = 90$ oe	M1 B1	find midpt F of BD	B1
state so not E	A1	find eqn of AC and correctly show F not on AC (the correct eqn for AC earns the second M1 as per the main scheme, if not already earned)	A1	showing $BE^2 = 10$ and $DE^2 = 90$ oe earns this A mark as well as the B1 if there are no errors elsewhere	A1	state so not E or show F not on AC	A1
	[5]						**5]**

Question			Answer	Marks	Guidance	
1	(i)		$\frac{9}{25}$ or 0.36 isw	2 **[2]**	M1 for numerator or denominator correct or for squaring correctly or for inverting correctly	M1 for eg $\frac{1}{\left(\frac{25}{9}\right)}$ or $\left(\frac{25}{9}\right)^{-1}$ or $\frac{25}{9}$ or for $\left(\frac{3}{5}\right)^2$ or $\frac{3}{5}$ M0 for just $\frac{1}{\left(\frac{5}{3}\right)^2}$
1	(ii)		27	2 **[2]**	M1 for $81^{\frac{1}{4}}=3$ soi	eg M1 for 3^3 M0 for $81^3 = 531441$ (true but not helpful)
2			$4x^4y^{-3}$ or $\frac{4x^4}{y^3}$ as final answer	3 **[3]**	B1 each 'term'; or M1 for numerator = $64x^{15}y^3$ and M1 for denominator = $16x^{11}y^6$	B0 if obtained fortuitously mark B scheme or M scheme to advantage of candidate, but not a mixture of both schemes

Question			Answer	Marks	Guidance	
3			obtaining a correct relationship in any 3 of C, d, r and A or obtaining a correct relationship in k and no more than 2 other variables	M2	may substitute into given relationship; or M1 for at least two of $A = \pi r^2$, $C = \pi d$, $C = 2\pi r$, $d = 2r$ or $r = \frac{d}{2}$ seen or used	eg M2 for $Cd = 4\pi r^2$ or $\pi d^2 = k\pi r^2$ seen/obtained condone eg Area $= \pi r^2$; allow $A = \pi\left(\frac{d}{2}\right)^2$ to imply $A = \pi r^2$ and $r = \frac{d}{2}$ and so earn M1, if M2 not earned
			convincing argument leading to $k = 4$	A1 **[3]**	must be from general argument, not just substituting values for r or d; may start from given relationship and derive $k = 4$	eg M1only for eg $A = \pi r^2$ and $C = \pi d$ and so $k = 4$ with no further evidence
4			$(5x + 2)(x - 6)$	M1	for factors giving at least two out of three terms correct when expanded and collected	or use of formula or completing the square with at most one error (comp square must reach $[5](x - a)^2 \leq b$ oe or $(5x - c)^2 \leq d$ oe stage) if correct: $5(x - 2.8)^2 \leq 51.2$ or $(x - 2.8)^2 \leq 10.24$ or $(5x - 14)^2 \leq 256$
			boundary values –0.4 oe and 6 soi	A1	A0 for just $\frac{28 \pm \sqrt{1024}}{10}$	
			$-0.4 \leq x \leq 6$ oe	A2 **[4]**	may be separate inequalities; mark final answer A1 for one end correct eg $x \leq 6$ or for $-0.4 < x < 6$ oe or B1 for $a \leq x \leq b$ ft their boundary values	condone unsimplified but correct $\frac{28 - \sqrt{1024}}{10} \leq x \leq \frac{28 + \sqrt{1024}}{10}$ etc allow A1 for $-0.4 \leq 0 \leq 6$ condone errors in the inequality signs during working towards final answer

Question			Answer	Marks	Guidance	
5			$4 + 2k + c = 0$ or $2^2 + 2k + c = 0$	B1	may be rearranged	
			$9 - 3k + c = 35$	B1	may be rearranged; the $(-3)^2$ must be evaluated / used as 9	condone -3^2 seen if used as 9
			correct method to eliminate one variable from their eqns	M1	eg subtraction or substitution for c; condone one error	M0 for addition of eqns unless also multiplied appropriately
			$k = -6$, $c = 8$	A1	from fully correct method, allowing recovery from slips	if no errors and no method seen, allow correct answers to imply M1 provided B1B1 has been earned
			or	**or**		
			$[x^2 + kx + c =]\ (x - 2)(x - a)$	M1	or $(x - 2)(x + b)$	
			$-5 \times (-3 - a) = 35$ oe	M1		
			$a = 4$	A1		
			$k = -6$, $c = 8$	A1		
				[4]		

Question			Answer	Marks	Guidance	
6			identifying term as $20(2x)^3\left(\frac{5}{x}\right)^3$ oe	M3	condone lack of brackets; M1 for $[k](2x)^3\left(\frac{5}{x}\right)^3$ soi (eg in list or table), condoning lack of brackets and M1 for $k = 20$ or eg $\frac{6\times5\times4}{3\times2\times1}$ or for 1 6 15 20 15 6 1 seen (eg Pascal's triangle seen, even if no attempt at expansion) and M1 for selecting the appropriate term (eg may be implied by use of only $k = 20$, but this M1 is not dependent on the correct k used)	xs may be omitted; eg M3 for $20 \times 8 \times 125$ first M1 not earned if elements added not multiplied; otherwise, if in list or table bod intent to multiply M0 for binomial coefficient if it still has factorial notation may be gained even if elements added
			20 000	A1 **[4]**	or B4 for 20 000 obtained from multiplying out $\left(2x+\frac{5}{x}\right)^6$ allow SC3 for 20000 as part of an expansion	
7	(i)		$9\sqrt{3}$ www oe as final answer	2 **[2]**	M1 for $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{75} = 5\sqrt{3}$ soi	
7	(ii)		$\frac{39+7\sqrt{5}}{44}$ www as final answer	3 **[3]**	M1 for attempt to multiply numerator and denominator by $7-\sqrt{5}$ B1 for each of numerator and denominator correct (must be simplified)	condone $\frac{39}{44}+\frac{7\sqrt{5}}{44}$ for 3 marks eg M0B1 if denominator correctly rationalised to 44 but numerator not multiplied

Question			Answer	Marks	Guidance	
8			$5c + 9t = 2ac + at$	M1	for correct expansion of brackets	
			$5c - 2ac = at - 9t$ oe	M1	for correct collection of terms, ft eg after M0 for $5c + 9t = 2ac + t$ allow this M1 for $5c - 2ac = -8t$ oe	for each M, ft previous errors if their eqn is of similar difficulty;
			$c(5 - 2a) = at - 9t$ oe	M1	for correctly factorising, ft; must be $c \times$ a two-term factor	may be earned before t terms collected
			$[c =]\dfrac{at-9t}{5-2a}$ or $\dfrac{t(a-9)}{5-2a}$ oe as final answer	M1 **[4]**	for correct division, ft their two-term factor	treat as MR if t is the subject, with a penalty of 1 mark from those gained, marking similarly
9	(i)		sketch of cubic the right way up, with two tps	B1		No section to be ruled; no curving back; condone some curving out at ends but not approaching another turning point; condone some doubling (eg erased curves may continue to show); ignore position of turning points for this mark
			their graph touching the x-axis at –2 and crossing it at 3 and no other places	B1	if intns are not labelled, they must be shown nearby	mark intent if 'daylight' between curve and axis at $x = -2$
			intersection of y-axis at –12	B1 **[3]**		if no graph but –12 marked on y-axis, or in table, allow this 3rd mark
9	(ii)		–5 and 0	B2 **[2]**	B1 each; allow B2 for –5, –5, 0; or B1 for both correct with one extra value or for (–5, 0) and (0, 0) or SC1 for both of 1 and 6	if their graph wrong, allow –5 and 0 from starting again with eqn, or ft their graph with two intns with x-axis

Question			Answer	Marks	Guidance	
10	(i)		midpt of AB = $\left(\frac{1}{2}, \frac{5}{2}\right)$ oe www	B2	allow unsimplified B1 for one coordinate correct	if working shown, should come from $\left(\frac{3+-2}{2}, \frac{4+1}{2}\right)$ oe NB B0 for x coord. = $\frac{5}{2}$, (obtained from subtraction instead of addition)
			grad AB = $\frac{4-1}{3-(-2)}$ oe	M1	must be obtained independently of given line; accept 3 and 5 correctly shown eg in a sketch, followed by 3/5 M1 for rise/run = 3/5 etc M0 for just 3/5 with no evidence	for those who find eqn of AB first, M0 for just $\frac{y-4}{1-4} = \frac{x-3}{-2-3}$ oe, but M1 for $y-4 = \frac{1-4}{-2-3}(x-3)$ oe ignore their going on to find the eqn of AB after finding grad AB
			using gradient of AB to obtain grad perp bisector	M1	for use of $m_1m_2 = -1$ soi or ft their gradient AB M0 for just $\frac{-5}{3}$ without AB grad found	this second M1 available for starting with given line = $\frac{-5}{3}$ and obtaining grad. of AB from it
			$y - 2.5 = \frac{-5}{3}(x - 0.5)$ oe	M1	eg M1 for $y = \frac{-5}{3}x + c$ and subst of midpt; ft their gradient of perp bisector and midpt; M0 for just rearranging given equation	no ft for gradient of AB used

Question			Answer	Marks	Guidance	
			completion to given answer $3y + 5x = 10$, showing at least one interim step	M1 **[6]**	condone a slight slip if they recover quickly and general steps are correct (eg sometimes a slip in working with the c in $y = \frac{-5}{3}x + c$ - condone $3y = -5x + c$ followed by substitution and consistent working) M0 if clearly 'fudging'	NB answer given; mark process not answer; annotate if full marks not earned eg with a tick for each mark earned scores such as B2M0M0M1M1 are possible after B2, allow full marks for complete method of showing given line has gradient perp to AB (grad AB must be found independently at some stage) and passes through midpt of AB
10	**(ii)**		$3y + 5(4y - 21) = 10$	M1	or other valid strategy for eliminating one variable attempted eg $\frac{-5}{3}x + \frac{10}{3} = \frac{x}{4} + \frac{21}{4}$; condone one error	or eg $20y = 5x + 105$ and subtraction of two eqns attempted no ft from wrong perp bisector eqn, since given allow M1 for candidates who reach $y = 115/23$ and then make a worse attempt, thinking they have gone wrong
			$(-1, 5)$ or $y = 5, x = -1$ isw	A2 **[3]**	A1 for each value; if AO allow SC1 for both values correct but unsimplified fractions, eg $\left(\frac{-23}{23}, \frac{115}{23}\right)$	NB M0A0 in this part for finding E using info from (iii) that implies E is midpt of CD

Question			Answer	Marks	Guidance	
10	(iii)		$(x-a)^2+(y-b)^2=r^2$ seen or used	M1	or for $(x+1)^2+(y-5)^2=k$, or ft their E, where $k>0$	
			1^2+4^2 oe (may be unsimplified), from clear use of A or B	M1	for calculating AE or BE or their squares, or for subst coords of A or B into circle eqn to find r or r^2, ft their E;	this M not earned for use of CE or DE or ½ CD NB some cands finding $AB^2=34$ then obtaining 17 erroneously so M0
			$(x+1)^2+(y-5)^2=17$	A1	for eqn of circle centre E, through A and B; allow A1 for $r^2=17$ found after $(x+1)^2+(y-5)^2=r^2$ stated and second M1 clearly earned if $(x+1)^2+(y-5)^2=17$ appears without clear evidence of using A or B, allow the first M1 then M0 SC1	SC also earned if circle comes from C or D and E, but may recover and earn the second M1 later by using A or B
			showing midpt of CD = (–1, 5)	M1		
			showing CE or DE = $\sqrt{17}$ oe or showing one of C and D on circle	M1	alt M1 for showing $CD^2=68$ oe allow to be earned earlier as an invalid attempt to find r	

Question			Answer	Marks	Guidance	
				[5]	showing that both C and D are on circle and commenting that E is on CD is enough for last M1M1; similarly showing $CD^2 = 68$ and both C and D are on circle oe earns last M1M1	other methods exist, eg: may find eqn of circle with centre E and through C or D and then show that A and B and other of C/D are on this circle – the marks are then earned in a different order; award M1 for first fact shown and then final M1 for completing the argument; if part-marks earned, annotate with a tick for each mark earned beside where earned
11	(i)		$\left(x-\frac{5}{2}\right)^2-\frac{1}{4}$ oe	B3	B1 for $a = 5/2$ oe and M1 for 6 – *their* a^2 soi;	condone $\left(x-\frac{5}{2}\right)^2-\frac{1}{4}$ oe $= 0$ condone omission of index –can earn all marks bod M1 for 6 – 4.25 or 6 – 25/2 etc, if bearing some relation to an attempt at 6 – *their* 2.5^2; M0 for just 1.75 etc without further evidence
			$\left(\frac{5}{2},-\frac{1}{4}\right)$ oe or ft	B1 [4]	accept $x = 2.5$, $y = -0.25$ oe	condone starting again and finding using calculus

Question			Answer	Marks	Guidance	
11	(ii)		(2, 0) and (3, 0)	B2	B1 each or B1 for both correct plus an extra or M1 for $(x-2)(x-3)$ or correct use of formula or for $their\ a \pm \sqrt{their\ b}$ ft from (i)	condone not expressed as coordinates, for both x and y values; accept eg in table or marked on graph
			(0, 6)	B1		
			graph of quadratic the correct way up and crossing both axes	B1 **[4]**	ignore label of their tp; condone stopping at y-axis	condone 'U' shape or slight curving back in/out; condone some doubling / feathering – deleted work sometimes still shows up in scoris; must not be ruled; condone fairly straight with clear attempt at curve at minimum; be reasonably generous on attempt at symmetry
11	(iii)		$x^2 - 5x + 6 = 2 - x$	M1	for attempt to equate or subtract eqns or attempt at rearrangement and elimination of x	accept calculus approach: $y' = 2x - 5$
			$x^2 - 4x + 4$ [= 0]	M1	for rearrangement to zero ft and collection of terms; condone one error; if using completing the square, need to get as far as $(x-k)^2 = c$, with at most one error [$(x-2)^2 = 0$ if correct]	use of $y' = -1$ M1

Question			Answer	Marks	Guidance	
			$x = 2$, [$y = 0$]	A1	condone omission of $y = 0$ since already found in (ii) if they have eliminated x, $y = 0$ is not sufft for A1 – need to get $x = 2$ A0 for $x = 2$ and another root	$x = 2$ A1
			'double root at $x = 2$ so tangent' oe; www;	A1 **[4]**	eg 'only one point of contact, so tangent'; or showing $b^2 - 4ac = 0$, and concluding 'so tangent'; www	tgt is y [– 0] = –(x – 2) and obtaining given line A1
12	**(i)**		f(1) = 1–1 +1 + 9 – 10 [= 0]	B1	allow for correct division of f(x) by (x – 1) showing there is no remainder, or for (x − 1)($x^3 + x + 10$) found, showing it 'works' by multiplying it out	condone $1^4 - 1^3 + 1^2 + 9 - 10$
			attempt at division by (x – 1) as far as $x^4 - x^3$ in working	M1	allow equiv for (x + 2) as far as $x^4 + 2x^3$ in working or for inspection with at least two terms of cubic factor correct	eg for inspection, M1 for two terms right and two wrong
			correctly obtaining $x^3 + x + 10$	A1 **[3]**	or $x^3 - 3x^2 + 7x - 5$	if M0 and this division / factorising is done in part (ii) or (iii), allow SC1 if correct cubic obtained there; attach the relevant part to (i) with a formal chain link if not already seen in the image zone for (i)

Question			Answer	Marks	Guidance	
12	(ii)		[g(–2) =] –8 – 2 + 10 or f(–2) = 16 + 8 + 4 – 18 – 10	M1	[in this scheme $g(x) = x^3 + x + 10$] allow M1 for correct trials with at least two values of x (other than 1) using $g(x)$ or $f(x)$ or $x^3 - 3x^2 + 7x - 5$ (may allow similar correct trials using division or inspection)	eg f(2) = 16 – 8 + 4 + 18 – 10 or 20 f(3) = 81 – 27 + 9 + 27 – 10 or 80 f(0) = – 10 f(–1) = 1 + 1 + 1 – 9 – 10 or –16 No ft from wrong cubic 'factors' from (i)
			$x = -2$ isw	A1	allow these marks if already earned in (i)	
						NB factorising of $x^3 + x + 10$ or $x^3 - 3x^2 + 7x - 5$ in (ii) earns credit for (iii) [annotate with a yellow line in both parts to alert you – the image zone for (iii) includes part (ii)]
				[2]		

Question			Answer	Marks	Guidance	
1			$y = -0.5x + 3$ oe www isw	3 **[3]**	B2 for $2y = -x + 6$ oe or M1 for gradient = $-\frac{1}{2}$ oe seen or used and M1 for $y - 1 =$ *their m* $(x - 4)$	for 3 marks must be in form $y = ax + b$ or M1 for $y =$ *their* $mx + c$ and (4, 1) substituted
2			substitution to eliminate one variable simplification to $ax = b$ or $ax - b = 0$ form, or equivalent for y (0.7, 0.1) oe or $x = 0.7$, $y = 0.1$ oe isw	M1 M1 A2 **[4]**	or multiplication to make one pair of coefficients the same; condone one error in either method or appropriate subtraction / addition; condone one error in either method A1 each	 independent of first M1
3	(i)		25	2 **[2]**	M1 for $\left(\frac{10}{2}\right)^2$ or $\left(\frac{1}{0.2}\right)^2$ oe soi or for $\frac{1}{0.04}$ oe	ie M1 for one of the two powers used correctly M0 for just $\frac{1}{0.4}$ with no other working
3	(ii)		$8a^9$	3 **[3]**	B2 for 8 or M1 for $16^{\frac{1}{4}} = 2$ soi and B1 for a^9	ignore ± eg M1 for 2^3; M0 for just 2

4			$r = \sqrt{\dfrac{3V}{\pi(a+b)}}$ oe www as final answer	3 **[3]**	M1 for dealing correctly with 3 and M1 for dealing correctly with $\pi(a + b)$, ft and M1 for correctly finding square root, ft *their* 'r^2 ='; square root symbol must extend below the fraction line	M0 if triple-decker fraction, at the stage where it happens, then ft; condone missing bracket at rh end M0 if ±... or r > ... for M3, final answer must be correct
5			f(2) = 18 seen or used 32 + 2k – 20 = 18 oe [k =] 3	M1 A1 A1 **[3]**	or long division oe as far as obtaining a remainder (ie not involving x) and equating that remainder to 18 (there may be errors along the way) after long division: 2(k + 16) – 20 = 18 oe	 A0 for just 2^5 instead of 32 unless 32 implied by further work

6			–2560 www	4 **[4]**	B3 for 2560 from correct term (NB coefficient of x^4 is 2560) or B3 for neg answer following 10 × 4 × –64 and then an error in multiplication or M2 for $10 \times 2^2 \times (-4)^3$ oe; must have multn signs or be followed by a clear attempt at multn; or M1 for $2^2 \times (-4)^3$ oe (condone missing brackets) or for 10 used or for 1 5 10 10 5 1 seen for those who find the coefft of x^2 instead: allow M1 for 10 used or for 1 5 10 10 5 1 seen ; and a further SC1 if they get 1280, similarly for finding coefficient of x^4 as 2560	ignore terms for other powers; condone x^3 included; but eg 10 × 4 × –64 = 40 – 64 = –24 gets M2 only condone missing brackets eg allow M2 for $10 \times 2^2 \times -4x^3$ ${}^5C_3$ or factorial notation is not sufficient but accept $\frac{5\times4\times3\times2\times1}{2\times1\times3\times2\times1}$ oe 10 may be unsimplified, as above M1 only for eg 10, 2^2 and $-4x^3$ seen in table with no multn signs or evidence of attempt at multn [lack of neg sign in the x^2 or x^4 terms means that these are easier and so not eligible for just a 1 mark MR penalty]
7	**(i)**		$5^{3.5}$ oe or $k = 7/2$ oe	2 **[2]**	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ soi	M0 for just answer of 5^3 with no reference to 125

7	**(ii)**		attempting to multiply numerator and denominator of fraction by $1+2\sqrt{5}$	M1		some cands are incorporating the $10+7\sqrt{5}$ into the fraction. The M1s are available even if this is done wrongly or if $10+7\sqrt{5}$ is also multiplied by $1+2\sqrt{5}$
			denominator = –19 soi	M1	must be obtained correctly, but independent of first M1	eg M1 for denominator of 19 with a minus sign in front of whole expression or with attempt to change signs in numerator
			$8+3\sqrt{5}$	A1 **[3]**		
8			$3(x-2)^2-7$ isw or a = 3, b = 2 c = 7 www	4	B1 each for a = 3, b = 2 oe and B2 for c = 7 oe or M1 for $[-]\frac{7}{3}$ or for 5 – *their a*(*their b*)^2 or for $\frac{5}{3}-(\textit{their } b)^2$ soi	condone omission of square symbol; ignore '= 0' may be implied by their answer
			–7 or ft	B1 **[5]**	B0 for (2, –7)	may be obtained by starting again eg with calculus
9	**(i)**		$3n$ isw	1 **[1]**	accept equivalent general explanation	

9	**(ii)**		at least one of $(n-1)^2$ and $(n+1)^2$ correctly expanded	M1	must be seen	M0 for just $n^2 + 1 + n^2 + n^2 + 1$
			$3n^2 + 2$	B1		accept even if no expansions / wrong expansions seen
			comment eg $3n^2$ is always a multiple of 3 so remainder after dividing by 3 is always 2	B1	dep on previous B1 B0 for just saying that 2 is not divisible by 3 – must comment on $3n^2$ term as well allow B1 for $\frac{3n^2+2}{3} = n^2 + \frac{2}{3}$	SC: n, $n + 1$, $n + 2$ used similarly can obtain first M1, and allow final B1 for similar comment on $3n^2 + 6n + 5$
				[3]		
10	**(i)**		[radius =] $\sqrt{20}$ or $2\sqrt{5}$ isw	B1	B0 for $\pm\sqrt{20}$ oe	
			[centre =] (3, 2)	B1		condone lack of brackets with coordinates, here and in other questions
				[2]		

10	(ii)		substitution of $x = 0$ or $y = 0$ into circle equation	M1	or use of Pythagoras with radius and a coordinate of the centre eg $20 - 2^2$ or $h^2 + 3^2 = 20$ ft their centre and/or radius	equation may be expanded first, and may include an error bod intent allow M1 for $(x - 3)^2 = 20$ and/or $(y - 2)^2 = 20$
			$(x - 7)(x + 1)$ [=0]	M1	no ft from wrong quadratic; for factors giving two terms correct, or formula or completing square used with at most one error	completing square attempt must reach at least $(x - a)^2 = b$ following use of Pythagoras allow M1 for attempt to add 3 to [±]4
			(7, 0) and (–1, 0) isw	A1	accept $x = 7$ or -1 (both required)	
			$[y =]\dfrac{4 \pm \sqrt{(-4)^2 - 4\times1\times(-7)}}{2}$ oe	M1	no ft from wrong quadratic; for formula or completing square used with at most one error	completing square attempt must reach at least $(y - a)^2 = b$ following use of Pythagoras allow M1 for attempt to add 2 to $[\pm]\sqrt{11}$
			$\left(0, 2 \pm \sqrt{11}\right)$ or $\left(0, \dfrac{4 \pm \sqrt{44}}{2}\right)$ isw	A1 **[5]**	accept $y = \dfrac{4 \pm \sqrt{44}}{2}$ oe isw	annotation is required if part marks are earned in this part: putting a tick for each mark earned is sufficient

10	**(iii)**		show both A and B are on circle	B1	explicit substitution in circle equation and at least one stage of interim working required oe	or clear use of Pythagoras to show AC and BC each = $\sqrt{20}$
			(4, 5)	B2	B1 each or M1 for $\left(\frac{7+1}{2}, \frac{6+4}{2}\right)$	
			$\sqrt{10}$	B2	from correct midpoint and centre used; B1 for $\pm\sqrt{10}$ M1 for $(4-3)^2 + (5-2)^2$ or $1^2 + 3^2$ or ft their centre and/or midpoint, or for the square root of this	may be a longer method finding length of ½ AB and using Pythag. with radius; no ft if one coord of midpoint is same as that of centre so that distance formula/Pythag is not required eg centre correct and midpt (3, –1) annotation is required if part marks are earned in this part: putting a tick for each mark earned is sufficient
				[5]		

11	**(i)**		sketch of cubic the right way up, with two tps and clearly crossing the x axis in 3 places	B1		no section to be ruled; no curving back; condone slight 'flicking out' at ends but not approaching another turning point; condone some doubling (eg erased curves may continue to show); accept min tp on y-axis or in 3rd or 4th quadrant; curve must clearly extend beyond the x axis at both 'ends'
			crossing/reaching the x-axis at –4, –2 and 1.5	B1	intersections must be shown correctly labelled or worked out nearby; mark intent	accept curve crossing axis halfway between 1 and 2 if 3/2 not marked
			intersection of y-axis at –24	B1 **[3]**		NB to find –24 some are expanding f(x) here, which gains M1 in iiiA. If this is done, put a yellow line here and by (iii)A to alert you; this image appears again there
11	**(ii)**		–2, 0 and 7/2 oe isw or ft their intersections	2 **[2]**	B1 for 2 correct or ft or for (–2, 0) (0, 0) and (3.5, 0) or M1 for $(x + 2)\,x\,(2x - 7)$ oe or SC1 for –6, –4 and –1/2 oe	

11	**(iii)**	(*A*)	correct expansion of product of 2 brackets of f(x)	M1	need not be simplified; condone lack of brackets for M1 or allow M1 for expansion of all 3 brackets, showing all terms, with at most one error: $2x^3 + 4x^2 + 8x^2 - 3x^2 + 16x - 12x - 6x - 24$	eg $2x^2 + 5x - 12$ or $2x^2 + x - 6$ or $x^2 + 6x + 8$ may be seen in (i) – allow the M1; the part (i) work appears at the foot of the image for (iii)A, so mark this rather than in (i)
			correct expansion of quadratic and linear and completion to given answer	A1 **[2]**	for correct completion if all 3 brackets already expanded, with some reference to show why –24 changes to –9	condone lack of brackets if they have gone on to expand correctly; condone '+15' appearing at some stage NB answer given; mark the whole process

11	**(iii)**	(*B*)	g(1) = 2 + 9 – 2 – 9 [=0]	B1	allow this mark for $(x - 1)$ shown to be a factor and a statement that this means that $x = 1$ is a root [of $g(x) = 0$] oe	B0 for just $g(1) = 2(1)^3 + 9(1)^2 - 2(1) - 9$ [=0]
			attempt at division by $(x - 1)$ as far as $2x^3 - 2x^2$ in working	M1	or inspection with at least two terms of quadratic factor correct	M0 for division by $x + 1$ after $g(1) = 0$ unless further working such as $g(-1) = 0$ shown, but this can go on to gain last M1A1
			correctly obtaining $2x^2 + 11x + 9$	A1	allow B2 for another linear factor found by the factor theorem	NB mixture of methods may be seen in this part – mark equivalently eg three uses of factor theorem, or two uses plus inspection to get last factor;
			factorising a correct quadratic factor	M1	for factors giving two terms correct; eg allow M1 for factorising $2x^2 + 7x - 9$ after division by $x + 1$	allow M1 for $(x + 1)(x + 18/4)$ oe after -1 and $-18/4$ oe correctly found by formula
			$(2x + 9)(x + 1)(x - 1)$ isw	A1 **[5]**	allow $2(x + 9/2)(x + 1)(x - 1)$ oe; dependent on 2nd M1 only; condone omission of first factor found; ignore '= 0' seen	SC alternative method for last 4 marks: allow first M1A1 for $(2x + 9)(x^2 - 1)$ and then second M1A1 for full factorisation

12	(i)		$y = 2x + 3$ drawn accurately (–1.6 to –1.7, –0.2 to –0.3) (2.1 to 2.2, 7.2 to 7.4)	M1 B1 B1 [3]	at least as far as intersecting curve twice intersections may be in form x = ..., y = ...	ruled straight line and within 2mm of (2, 7) and (–1, 1) if marking by parts and you see work relevant to (ii), put a yellow line here and in (ii) to alert you to look
12	(ii)		$\frac{1}{x-2} = 2x+3$ $1 = (2x + 3)(x - 2)$ $1 = 2x^2 - x - 6$ oe $\frac{1 \pm \sqrt{1^2 - 4 \times 2 \times -7}}{2 \times 2}$ oe $\frac{1 \pm \sqrt{57}}{4}$ isw	M1 M1 A1 M1 A1 [5]	or attempt at elimination of x by rearrangement and substitution condone lack of brackets for correct expansion; need not be simplified; NB A0 for $2x^2 - x - 7 = 0$ without expansion seen [given answer] use of formula or completing square on given equation, with at most one error isw eg coordinates; after completing square, accept $\frac{1}{4} \pm \sqrt{\frac{57}{16}}$ or better	may be seen in (i) – allow marks; the part (i) work appears at the foot of the image for (ii) so show marks there rather than in (i) implies first M1 if that step not seen implies second M1 if that step not seen after $\frac{1}{x-2} = 2x+3$ seen completing square attempt must reach at least [2]$(x - a)^2 = b$ or $(2x - c)^2 = d$ stage oe with at most one error

12	**(iii)**		$\frac{1}{x-2}=-x+k$ and attempt at rearrangement	M1		
			$x^2-(k+2)x+2k+1[=0]$	M1	for simplifying and rearranging to zero; condone one error; collection of x terms with bracket not required	eg M1 bod for $x^2-(k+2)x+2k$ or M1 for $x^2-2kx+2k+1[=0]$
			b^2-4ac = 0 oe seen or used	M1		= 0 may not be seen, but may be implied by their final values of k
			[k =] 0 or 4 as final answer, both required	A1 **[4]**	SC1 for 0 and 4 found if 3rd M1 not earned (may or may not have earned first two Ms)	eg obtained graphically or using calculus and/or final answer given as a range

Appendix: revised tolerances for modified papers for visually impaired candidates (graph in 12(i) with 6mm squares)

12	**(i)**		*y* = 2*x* + 3 drawn accurately	M1	at least as far as intersecting curve twice	ruled straight line and within **3** mm of (2, 7) and (–1, 1)
			(–1.6 to –1.**8**, –0.2 to –0.3)	B1	intersections may be in form *x* = ..., *y* = ...	
			(2.1 to 2.**3**, 7.**1** to 7.4)	B1		
				[3]		if marking by parts and you see work relevant to (ii), put a yellow line here and in (ii) to alert you to look