MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

INTRODUCTION TO ADVANCED MATHEMATICS, C1

Practice Paper C1-D

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used..
- Answer **all** the questions.
- You **not** permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You re advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 **(i)** Statement P is $a + b = 4$.
Statement Q is $a = 1$ and $b = 3$.

Which one of the following is correct?

$$P \Rightarrow Q, \quad P \Leftrightarrow Q, \quad P \Leftarrow Q$$ [1]

(ii) Statement R is $x = 2$.
Statement S is $x^2 = 4$.

Which one of the following is correct?

$$R \Rightarrow S, \quad R \Leftrightarrow S, \quad R \Leftarrow S$$ [1]

2 Find the equation of the straight line which is parallel to the line $y = 3x + 5$ and which goes through the point (2, 12). [3]

3 Find the term which has the highest coefficient in the expansion of $(1 + x)^8$. [3]

4 The surface area of the surface of a cylinder is given by the formula

$$A = 2\pi r(r + h)$$

Rearrange this formula so that h is the subject. [3]

5 Solve the following equations.

(a) $2^x = \frac{1}{8}$. [1]

(b) $x^{-\frac{1}{2}} = \frac{1}{4}$ [2]

6 Find the positive integer values of x for which

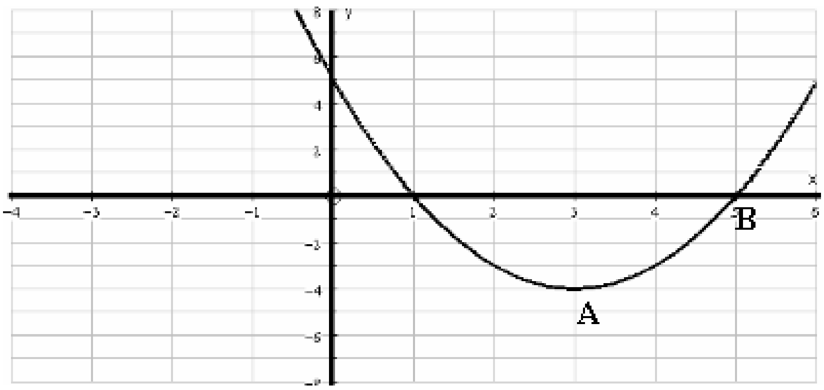
$$\frac{1}{2}(26 - 2x) \geq 2(3 + x).$$ [3]

7 The remainder when $x^3 - 2x + 4$ is divided by $(x - 2)$ is twice the remainder when $x^2 + x + k$ is divided by $(x + 1)$.
Find the value of k. [5]

8 Find the values of a and b for which $\dfrac{4}{(2\sqrt{3}-1)} = a + b\sqrt{3}$. [5]

9 Find the coordinates of the points where the curve $y = x^2 - 2x - 8$ meets the line $y = x + 2$. [4]

10 The diagram shows the graph of $y = \text{f}(x)$.



A is the minimum point of the curve at (3, -4) and B is the point (5, 0).

On separate diagrams on graph paper, draw the graphs of the following.
In each case give the coordinates of the images of the points A and B.

(i) $y = \text{f}(x) + 2$, [3]

(ii) $y = \text{f}(x + 2)$. [2]

Section B (36 marks)

11 Fig. 11 shows the graph of $y = ax^2 + bx + c$.

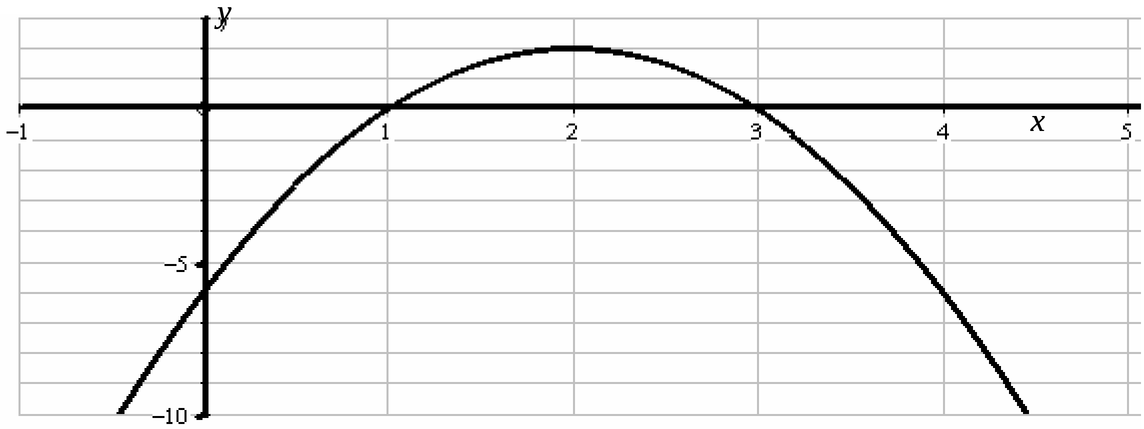


Fig. 11

(i) Explain why a must be negative. [1]

(ii) State two factors of $y = ax^2 + bx + c$. [2]

(iii) Hence, or otherwise, find the values of a, b and c. [4]

Another function is given by $y = x^2 - 4x + 10$.

(iv) Write this in completed square form. [3]

(v) Explain why the graphs of these two functions never meet. [2]

12 The function f(x) is given by $f(x) = x^3 + 6x^2 + 5x - 12$.

(i) Show that $(x + 3)$ is a factor of f(x). [1]

(ii) Find the other factors of f(x). [3]

(iii) State the coordinates where the graph of $y = f(x)$ cuts the x axis.
Hence sketch the graph of $y = f(x)$. [3]

(iv) On the same graph sketch also $y = x(x - 1)(x - 2)$ Label the two points of intersection of the two curves A and B. [2]

(v) By equating the two curves, show that the x coordinates of A and B satisfy the equation $3x^2 + x - 4 = 0$.
Solve this equation to find the x-coordinates of A and B. [3]

13 In Fig.13, XP and XQ are the perpendicular bisectors of AC and BC respectively.

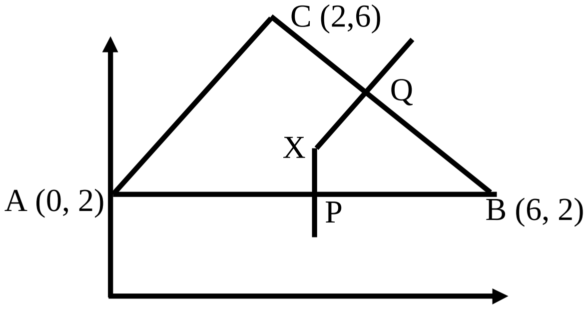


Fig. 13

(i) Find the coordinates of X. [5]

(ii) Hence show that AX = BX = CX. [3]

(iii) The circumcircle of a triangle is the circle which passes through the vertices of the triangle.
Write down the equation of the circumcircle of the triangle ABC. [2]

(iv) Find the coordinates of the points where the circle cuts the x axis. [2]