

$$1) 10 + x^2 > x(x-2)$$

$$10 + x^2 > x^2 - 2x$$

$$10 > -2x$$

$$\frac{10}{-2} < x \quad \leftarrow \text{flip inequality because } \div (-)$$

$$-5 < x$$

$$2) \int x^2 - \frac{1}{x^2} + \sqrt[3]{x} \, dx$$

$$= \int x^2 - x^{-2} + x^{\frac{1}{3}} \, dx$$

$$= \frac{1}{3} x^3 + x^{-1} + \frac{3}{4} x^{\frac{4}{3}} + C$$

$$3) a) 81^{\frac{1}{2}} = \sqrt{81} = 9$$

$$b) 81^{\frac{3}{4}} = (\sqrt[4]{81})^3 = 3^3 = 27$$

~~$$81^{\frac{1}{4}} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$$~~

$$c) 81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}} = \frac{1}{27}$$

4)

$$a_1 = k \quad a_{n+1} = 4a_n - 7$$

$$a) a_2 = a_{1+1} = 4a_1 - 7 = \underline{4k - 7}$$

$$b) a_3 = a_{2+1} = 4a_2 - 7 = 4(4k - 7) - 7$$

$$= 16k - 28 - 7$$

$$= \underline{\underline{16k - 35}}$$

$$c) a_3 = 13 \Rightarrow 16k - 35 = 13$$

$$16k = 48$$

$$k = \frac{48}{16} = \frac{24}{8} = \frac{12}{4} = \frac{6}{2} = \underline{\underline{3}}$$

$$5a) 2x + y = 8 \Rightarrow y = 8 - 2x \dots \textcircled{2}$$

$$3x^2 + xy = 1 \dots \textcircled{1}$$

sub $\textcircled{2}$ into $\textcircled{1}$

$$3x^2 + x(8 - 2x) = 1$$

$$3x^2 + 8x - 2x^2 = 1$$

$$x^2 + 8x - 1 = 0$$

$$b) x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{64 - 4(-1)}}{2} = \frac{-8 \pm \sqrt{68}}{2}$$

$$\frac{-8 \pm \sqrt{4 \cdot 17}}{2} = \frac{-8 \pm 2\sqrt{17}}{2} = -4 \pm \sqrt{17}$$

~~2x~~ $y = 8 - 2x$

when $x = -4 + \sqrt{17}$

$$y = 8 - 2(-4 + \sqrt{17})$$

$$y = 8 + 8 - 2\sqrt{17}$$

$$y = 16 - 2\sqrt{17}$$

$$x = -4 + \sqrt{17}$$

$$y = 16 - 2\sqrt{17}$$

when $x = -4 - \sqrt{17}$

$$y = 8 - 2(-4 - \sqrt{17})$$

$$y = 8 + 8 + 2\sqrt{17}$$

$$y = 16 + 2\sqrt{17}$$

6a)

$$f(x) = \frac{(2x+1)(x+4)}{\sqrt{x}} = \frac{2x^2 + 9x + 4}{x^{1/2}}$$

$$= 2x^{(2-\frac{1}{2})} + 9x^{(1-\frac{1}{2})} + 4x^{-\frac{1}{2}}$$

$$= 2x^{\frac{3}{2}} + 9x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$$

$$\underline{P=2} \quad \underline{Q=9} \quad \underline{R=4}$$

$$\begin{aligned} \text{b) } f'(x) &= 2\left(\frac{3}{2}\right)x^{\frac{1}{2}} + 9\left(\frac{1}{2}\right)x^{-\frac{1}{2}} - 4\left(\frac{1}{2}\right)x^{-\frac{3}{2}} \\ &= 3x^{\frac{1}{2}} + \frac{9}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}} \end{aligned}$$

$$\text{c) } f'(x) = \frac{dy}{dx} \quad \text{when } x=1 \quad \frac{dy}{dx} = 3 + \frac{9}{2} - 2$$

$$= 1 + \frac{9}{2} = \frac{11}{2}$$

$$2y = 11x + 3$$

$$y = \frac{11}{2}x + \frac{3}{2}$$

$$\frac{dy}{dx} = \frac{11}{2}$$

equal gradients $\therefore$ parallel

7a)

$$x^3 - 4x$$

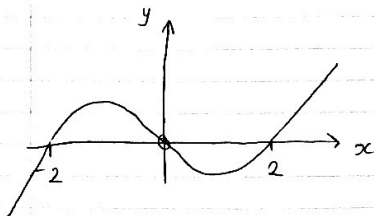
$$= x(x^2 - 4)$$

$$= x(x+2)(x-2)$$

b) $y = x^3 - 4x = x(x+2)(x-2)$

when $y = 0$

$x = 0 \quad x = -2 \quad x = 2$



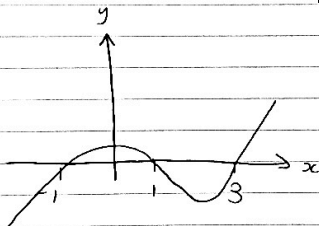
as $x \rightarrow \infty$ $+ve$

$y = \infty (\infty + 2)(\infty - 2)$

$y = \rightarrow +\infty$

c) $y = (x-1)^3 - 4(x-1)$

is $x^3 - 4x$ with a transformation
+1 in +ve x direction



8 a) $l_1: y = 3x - 6$

$$\frac{dy}{dx} = 3$$

$(6, 2)$

$$\frac{dy}{dx} l_2: -\frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{3}(x - 6)$$

$$y = -\frac{1}{3}x + 2 + 2$$

$$y = -\frac{1}{3}x + 4$$

b) intersect $l_1 = l_2$

$$-\frac{1}{3}x + 4 = 3x - 6$$

$$10 = 3x + \frac{1}{3}x$$

$$10 = \frac{10}{3}x$$

$$\frac{30}{10} = x = 3$$

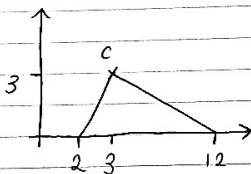
when $x = 3$ $y = 3(3) - 6 = 9 - 6 = 3$

$$(3, 3)$$

c) $l_1: y = 3x - 6$
 when $y = 0$ $x = 2$

$l_2: y = -\frac{1}{3}x + 4$
 when $y = 0$ $x = 12$

$C = (3, 3)$



$\text{Area} = \frac{1}{2} b \times h$

$= \frac{1}{2} 10 \times 3 = 15$

9) $S_n = (a) + (a+d) + \dots + [a+(n-2)d] + [a+(n-1)d] \dots ②$

$S_n = [a+(n-1)d] + [a+(n-2)d] + \dots + (a+d) + (a) \dots ①$

$\text{①} + \text{②}$
 $2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \dots + [2a+(n-1)d] \dots$

$2S_n = n[2a+(n-1)d]$

$S_n = \frac{n}{2}[2a+(n-1)d]$

b) longest side = n^{th} value = $a + (n-1)d = 6$

$$a + (16-1)d = 6$$

$$a + 15d = 6 \Rightarrow a = 6 - 15d \dots \textcircled{1}$$

$$\text{parameter} = S_n = 72$$

$$\frac{1}{2}(16)[2a + (16-1)d] = 72$$

$$8[2a + 15d] = 72$$

$$2a + 15d = 9 \dots \textcircled{2}$$

$$2(6 - 15d) + 15d = 9$$

$$12 - 30d + 15d = 9$$

$$3 = 15d$$

$$\boxed{d = \frac{1}{5}} \dots \textcircled{3}$$

$$\textcircled{3} \text{ into } \textcircled{1} \quad a = 6 - 15\left(\frac{1}{5}\right)$$

$$a = 6 - 3$$

$$\underline{a = 3}$$

$a =$ length of shortest

$$10) \quad \frac{dy}{dx} = x^3 + 2x - 7$$

$$\frac{d^2y}{dx^2} = 3x^2 + 2$$

$$b) \quad x^2 \text{ is always } (+) \therefore x^2 \geq 0$$

$$\therefore \text{ ~~3x^2 \geq 3(0)~~ } \quad 3x^2 \geq 3(0)$$

$\therefore$

$$3x^2 + 2 \geq 0 + 2$$

$$\frac{d^2y}{dx^2} \geq 2$$

$$c) \quad \int \frac{dy}{dx} = y$$

$$\int x^3 + 2x - 7 dx = y$$

$$= \frac{1}{4}x^4 + x^2 - 7x + C = y$$

$$\text{When } x=2 \quad y=4$$

$$\frac{1}{4}(2)^4 + 2^2 - 7(2) + C = 4$$

$$4 + 4 - 14 - 4 = -C$$

$$\underline{C = 10}$$

$$y = \frac{1}{4}x^4 + x^2 - 7x + 10$$

10d) at P $x = 2$

$$\frac{dy}{dx} \text{ at P} = (2)^3 + 2(2) - 7$$

$$\frac{dy}{dx} = 8 + 4 - 7 = 5$$

$$\text{at normal } \frac{dy}{dx} = -\frac{1}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{5}(x - 2)$$

$$y - 4 = -\frac{1}{5}x + \frac{2}{5}$$

$$5y - 20 = -x + 2$$

$$x + 5y - 22 = 0$$