PMT



PMT

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Core Mathematics 1 (6663\_01)

#### **Edexcel and BTEC Qualifications**

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at <a href="https://www.edexcel.com">www.edexcel.com</a> or <a href="https://www.edexcel.com">www.btec.co.uk</a>. Alternatively, you can get in touch with us using the details on our contact us page at <a href="https://www.edexcel.com/contactus">www.edexcel.com/contactus</a>.

### Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: <a href="https://www.pearson.com/uk">www.pearson.com/uk</a>

Summer 2014
Publications Code UA038449
All the material in this publication is copyright
© Pearson Education Ltd 2014

# **General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL GCE MATHEMATICS

## **General Instructions for Marking**

- 1. The total number of marks for the paper is 75
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{}$  will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper or ag- answer given
- L or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

# General Principles for Core Mathematics Marking

PMT

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

### 1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where  $|pq| = |c|$ , leading to  $x = ...$ 

$$(ax^2 + bx + c) = (mx + p)(nx + q)$$
, where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = ...$ 

#### 2. Formula

Attempt to use the correct formula (with values for a, b and c).

## 3. Completing the square

Solving 
$$x^2 + bx + c = 0$$
:  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$ 

## Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1.  $(x^n \rightarrow x^{n-1})$ 

## 2. Integration

Power of at least one term increased by 1.  $(x^n \rightarrow x^{n+1})$ 

# Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

# **Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question<br>Number | Scheme   | Marks     |
|--------------------|--|-----------|
| 1.                 | $\int (8x^3 + 4)  \mathrm{d}x = \frac{8x^4}{4} + 4x$ | M1, A1    |
|                    | $=2x^4+4x+c$   | A1        |
|                    |  | (3 marks) |
|                    |  |           |

## Notes

M1 
$$x^n \rightarrow x^{n+1}$$
 so  $x^3 \rightarrow x^4$  or  $4 \rightarrow 4x$  or  $4x^1$ 

- A1 This is for either term with coefficient unsimplified (power must be simplified)—so  $\frac{8}{4}x^4$  or 4x (accept  $4x^1$ )
- A1 Fully correct simplified solution with c i.e.  $2x^4 + 4x + c$  [ allow  $2x^4 + 4x + cx^0$  ]

If the answer is given as  $\int 2x^4 + 4x + c$ , with an integral sign – having never been seen as the fully correct simplified answer without an integral sign – then give M1A1A0 but allow anything before the = sign e.g.  $y = 2x^4 + 4x + c$ ,  $f(x) = 2x^4 + 4x + c$ ,  $f(x) = 2x^4 + 4x + c$ , etc....

If this answer is followed by (for example)  $x^4 + 2x + k$  then treat this as **isw** (ignore subsequent work) If they follow it by finding a value for c, also **isw**, provided correct answer with c has been seen and credited

| Question<br>Number | Scheme   | Marks   |     |
|--------------------|--|---------|-----|
| 2.                 | (a) $32^{\frac{1}{5}} = 2$   | B1      | (1) |
|                    | (a) $32^{\frac{1}{5}} = 2$<br>(b) For $2^{-2}$ or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 as coefficient of $x^k$ , for any value of $k$ including $k = 0$ | M1      |     |
|                    | Correct index for x so $A x^{-2}$ or $\frac{A}{x^2}$ o.e. for any value of A   | B1      |     |
|                    | $= \frac{1}{4x^2} \text{ or } 0.25 \ x^{-2}$   | A1 cao  | (3) |
|                    |  | 4 Marks |     |

- (a) B1 Answer 2 must be in part (a) for this mark
- (b) Look at their final answer

M1 For  $2^{-2}$  or  $\frac{1}{4}$  or  $\left(\frac{1}{2}\right)^2$  or 0.25 in their answer as coefficient of  $x^k$  for numerical value of k (including k = 0) so final answer  $\frac{1}{4}$  is M1 B0 A0

B1  $Ax^{-2}$  or  $\frac{A}{x^2}$  or equivalent e.g.  $Ax^{\frac{10}{5}}$  or  $Ax^{\frac{50}{25}}$  i.e. correct power of x seen in final answer May have a bracket provided it is  $(Ax)^{-2}$  or  $\left(\frac{A}{x}\right)^2$ 

A1  $\frac{1}{4x^2}$  or  $\frac{1}{4}x^{-2}$  or  $0.25 x^{-2}$  oe but must be correct power **and** coefficient combined correctly and must not be followed by a different wrong answer.

**Poor bracketing**:  $2x^{-2}$  earns M0 B1 A0 as correct power of x is seen in this solution (They can recover if they follow this with  $\frac{1}{4x^2}$  etc.)

**Special case**  $(2x)^{-2}$  as a **final** answer and  $\left(\frac{1}{2x}\right)^2$  can have M0 B1 A0 if the correct expanded answer is not seen The correct answer  $\frac{1}{4x^2}$  etc. followed by  $\left(\frac{1}{2x}\right)^2$  or  $(2x)^{-2}$ , treat  $\frac{1}{4x^2}$  as final answer so M1 B1 A1 isw

But the correct answer  $\frac{1}{4x^2}$  etc clearly followed by the wrong  $2x^{-2}$  or  $4x^{-2}$ , gets M1 B1 A0 do not ignore subsequent wrong work here

PMT

| Question<br>Number | Scheme  | Marks                  |                   |
|--------------------|---|------------------------|-------------------|
| 3.                 | (a) $3x-7>3-x$<br>4x>10<br>$x>2.5$ , $x>\frac{5}{2}$ , $\frac{5}{2}< x$ o.e.<br>(b) Obtain $x^2-9x-36$ and attempt to solve $x^2-9x-36=0$<br>e.g. $(x-12)(x+3)=0$ so $x=$ , or $x=\frac{9\pm\sqrt{81+144}}{2}$<br>12, $-3-3 \le x \le 12(c) 2.5 < x \le 12$ | M1 A1 M1 A1 M1A1 A1cso | (2)<br>(4)<br>(1) |
|                    |   | (7 marks)              |                   |

### **Notes**

(a) M1 Reaching px > q with one or both of p or q correct. Also give for -4x < -10

A1 Cao x > 2.5 o.e. Accept alternatives to 2.5 like  $2\frac{1}{2}$  and  $\frac{5}{2}$  even allow  $\frac{10}{4}$  and allow  $\frac{5}{2} < x$  o.e. This answer must occur and be credited as part (a) A correct answer implies M1A1

Mark parts (b) and (c) together.

- (b) M1 Rearrange  $3TQ \le 0$  or 3TQ = 0 or even 3TQ > 0 Do not worry about the inequality at this stage AND attempt to solve by factorising, formula or completion of the square with the usual rules (see notes)
  - A1 12 and –3 seen as critical values
  - M1 Inside region for their critical values must be stated not just a table or a graph
  - A1  $-3 \le x \le 12$  Accept  $x \ge -3$  and  $x \le 12$  or [-3, 12]

For the A mark: Do not accept  $x \ge -3$  or  $x \le 12$  nor -3 < x < 12 nor (-3, 12) nor  $x \ge -3$ ,  $x \le 12$  However allow recovery if they follow these statements by a correct statement, either in (b) or as they start part (c)

N.B.  $-3 \le 0 \le 12$  and  $x \ge -3$ ,  $x \le 12$  are poor notation and get M1A0 here.

(c) A1 cso  $2.5 < x \le 12$  Accept x > 2.5 and  $x \le 12$  Allow  $\frac{10}{4}$  Do not accept x > 2.5 or  $x \le 12$ 

Accept (2.5, 12] A graph or table is not sufficient. **Must follow correct earlier work** – except for special case

**Special case** (c) x > 2.5,  $x \le 12$ ;  $2.5 < 0 \le 12$  are poor notation – but if this poor notation has been penalised in (b) then allow A1 here. Any other errors are penalised in both (b) and (c).

| Question<br>Number | Scheme                                       | Marks               |   |
|--------------------|--|---------------------|---|
| 4.                 | (a) -1 accept (-1, 0)<br>(b)                 | B1 (1)              | ) |
|                    | Shape Touches at (0,0) Crosses at (2,0) only | B1<br>B1<br>B1      | ) |
|                    | (c) 2 solutions as <b>curves</b> cross twice | B1 ft (1) (5 marks) | ) |

#### **Notes**

N.B. Check original diagram as answer may appear there.

- (a) B1 The x coordinate of A is -1. Accept -1 or (-1,0) on the diagram or stated with or without diagram Allow (0, -1) on the diagram if it is on the correct axis.
- (b) If no graph is drawn then no marks are available in part (b)
  - Correct shape. The position is not important for this mark but the curve must have two clear turning points and be a +ve  $x^3$  curve ( with a maximum and minimum)
  - B1 The graph touches the origin. Accept touching as a maximum or minimum. There must be a sketch for this mark but sketch may be wrong and this mark is independent of previous mark. Origin is where axes cross and may not be labelled. This may be a quadratic or quartic curve for this mark.
  - B1 The graph crosses the x-axis at the point (2,0) only. If it crosses at (2,0) and (0,0) this is B0. Accept (0,2) or 2 marked on the correct axis. Accept (2,0) in the text of the answer provided that the curve crosses the positive x axis. There must be a sketch for this mark. Do not give credit if (2,0) appears only in a table with no indication that this is the intersection point. (If in doubt send to review) Graph takes precedence over text for third B mark.
- (c) B1ft Two (solutions) **as there are two intersections** (**of the curves**) N.B. Just states 2 with no reason is B0 If the answer states 2 roots and two intersections or crosses twice this is enough for B1 BUT B0 If there is any wrong **reason** given e.g. crosses *x* axis twice, or crosses asymptote twice Isw is not used for this mark so any wrong statement listed to follow a correct statement will result in B0

Allow ft – so if their graph crosses the hyperbola once – allow "one solution as there is one intersection" And if it crosses three times – allow "three solutions as there are three intersections" or four etc.. If it does not cross at all (e.g.negative cubic) – allow "no solutions as there are no intersections" However in (c) if they have sketched a curve (even a fully correct one) but not extended it to intersect the hyperbola and they put "no points of intersection so no solutions" then this scores B0. Accept "lines or curves cross over twice, or touch twice, or meet twice...etc as explanation, but need some form of words)

| Question<br>Number | Scheme   | Marks           |
|--------------------|--|-----------------|
| 5.                 | (a) $7 = 5a_1 - 3 \implies a_1 =$<br>$a_1 = 2$ | M1<br>A1<br>(2) |
|                    | (b) $a_3 = "32"$ and $a_4 = "157"$             | M1              |
|                    | $\sum_{r=1}^{r=4} a_r = a_1 + a_2 + a_3 + a_4$ |                 |
|                    | = "2"+ "7"+ "32"+ "157"                        | dM1             |
|                    | = 198  | A1              |
|                    |  | (3)             |
|                    |  | (5 marks)       |
|                    |  |                 |

#### **Notes**

(a) M1 Writes  $7 = 5a_1 - 3$  and attempts to solve leading to an answer for  $a_1$ . If they rearrange wrongly before any substitution this is M0

A1 Cao  $a_1 = 2$ 

Special case: Substitutes n = 1 into 5n - 3 and obtains answer 2. This is fortuitous and gets M0A0 but full marks are available on (b).

- (b) M1 Attempts to find either their  $a_3$  or their  $a_4$  using  $a_{n+1} = 5a_n 3$ ,  $a_2 = 7$ Needs clear attempt to use formula or is implied by correct answers or correct follow through of their  $a_3$ 
  - dM1 (Depends on previous M mark) Sum of their four adjacent terms from the given sequence. n.b May be given for  $9 + a_3 + a_4$  as they may add 2 + 7 to give 9 (dM0 for sum of an Arithmetic series)
  - A1 cao 198

Special case

- (a)  $a_1 = 32$  is M0 A0
- (b) Adds for example 7+32+157+782 = or 32+157+782+3907 is M1 M1 A0 Total mark possible is 2/5

(This is not treated as a misread – as it changes the question)

| Question<br>Number | Scheme  |  | Marks     |
|--------------------|---|--|-----------|
| 6.                 | (a) $80 = 5 \times 16$<br>$\sqrt{80} = 4\sqrt{5}$   |  | B1 (1)    |
|                    | Method 1  | Method 2                               | (1)       |
|                    | (b) $\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$  | $(p+q\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$  | B1ft      |
|                    | $= \frac{\sqrt{80}}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}  \text{or}  \frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$ | $p\sqrt{5+q}\sqrt{5+p+5}q = 4\sqrt{5}$ | M1        |
|                    | $= \frac{20 - 4\sqrt{5}}{4} \qquad \text{or} \qquad \frac{4\sqrt{5} - 20}{-4}$  | p + 5 q = 0 $p + q = 4$                | A1        |
|                    | $=5-\sqrt{5}$   | p = 5, q = -1                          | A1cao     |
|                    |   |  |           |
|                    |   |  | (4)       |
|                    |   |  | (5 marks) |

(a) B1 Accept  $4\sqrt{5}$  or c = 4 – no working necessary

(b) (Method 1)

B1ft Only ft on 
$$c$$
 See  $\frac{\sqrt{80}}{\sqrt{5}+1}$  or  $\frac{c\sqrt{5}}{\sqrt{5}+1}$ 

- M1 State intention to multiply by  $\sqrt{5} 1$  or  $1 \sqrt{5}$  in the numerator **and** the denominator
- A1 Obtain denominator of 4 ( for  $\sqrt{5} 1$  ) or -4 (for  $1 \sqrt{5}$ ) or correct simplified numerator of  $20 4\sqrt{5}$  or  $4(5 \sqrt{5})$  or  $4\sqrt{5} 20$  or  $4(\sqrt{5} 5)$  So either numerator or denominator must be correct
- A1 Correct answer only. Both **numerator and denominator must have been correct and d**ivision of numerator and denominator by 4 has been performed.

Accept 
$$p=5$$
,  $q=-1$  or accept  $5-\sqrt{5}$  or  $-\sqrt{5}+5$  Also accept  $5-1\sqrt{5}$ 

(Method 2)

- B1ft Only ft on c  $(p+q\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$  or  $c\sqrt{5}$
- M1 Multiply out the lhs and replace  $\sqrt{80}$  by  $c\sqrt{5}$
- A1 Compare rational and irrational parts to give p + q = 4, and p + 5q = 0
- A1 Solve equations to give p = 5, q = -1

Common error:

$$\frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{4\sqrt{5}-20}{4} = \sqrt{5}-5 \text{ gets B1 M1 A1 (for correct numerator – denominator is wrong for their product) then A0}$$

Correct answer with no working – send to review – have they used a calculator? Correct answer after trial and improvement with evidence that  $(5 - \sqrt{5})(\sqrt{5} + 1) = \sqrt{80}$  could earn all four marks

| Question<br>Number | Scheme   | Marks      |
|--------------------|--|------------|
| 7.                 | (a) $(1-2x)^2 = 1-4x+4x^2$   | M1         |
|                    | $\frac{d}{dx}(1-2x)^2 = \frac{d}{dx}(1-4x+4x^2) = -4+8x \text{ o.e.}$  | M1A1       |
|                    |  | (3)        |
|                    | Alternative method using chain rule: Answer of -4 ( $1-2x$ )   | M1M1A1 (3) |
|                    | (b) $\frac{x^5 + 6\sqrt{x}}{2x^2} = \frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}, = \frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ | M1,A1      |
|                    | Attempts to differentiate $x^{-\frac{3}{2}}$ to give $k x^{-\frac{5}{2}}$  | M1         |
|                    | $=\frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}} \text{ o.e.}$   | A1         |
|                    | Quotient Rule (May rarely appear) – See note below   | (4)        |
|                    |  | (7 marks)  |

- (a) M1 Attempt to multiply out bracket. Must be 3 or 4 term quadratic and must have constant term 1
  - M1  $x^n \to x^{n-1}$ . Follow through on any term in an incorrect expression. Accept a constant  $\to 0$
  - A1 -4+8x Accept -4 (1-2x) or equivalent. This is not cso and may follow error in the constant term Following correct answer by -2 + 4x apply isw

Correct answer with no working – assume chain rule and give M1M1A1 i.e. 3/3

Common errors:  $(1-2x)^2 = 2-4x+4x^2$  is M0, then allow M1A1 for -4 + 8x

$$(1-2x)^2 = 1-4x^2$$
 is M0 then -8x earns M1A0 or  $(1-2x)^2 = 1-2x^2$  is M0 then -4x earns M1A0

### **Use of Chain Rule**:

M1M1: first M1 for complete method so  $2 \times (\pm 2)(1-2x)$  second M1 for (1-2x) (as power reduced)

Then A1 for -4 (1-2x) or for -4 + 8x

So (i) 2(1-2x) gets M0 M1A0 for reducing power and (ii)  $2\times2(1-2x)$  gets M1 M1A0

(b) M1 An attempt to divide by  $2x^2$  first. This can be implied by the sight of the following

Some correct working e.g.  $\frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}$  or  $(x^5 + 6\sqrt{x})(2x^2)^{-1}$  leading to  $ax^p + bx^q$  in either case

or can be implied by  $\frac{1}{2}x^3 + 3x^p$  (after no working) i.e. both coefficients correct and power 3 correct

Common error:  $(x^5 + 6\sqrt{x})2x^{-2}$  is M0 (may earn next M mark for the differentiation  $x^{-\frac{3}{2}} \to x^{-\frac{5}{2}}$ )

A1 Writing the given expression as  $\frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$  or  $0.5x^3 + \frac{6}{2}x^{-\frac{3}{2}}$  or  $0.5x^3 + \frac{6}{2}x^{-\frac{1}{2}}$  or etc...

M1  $x^{-\frac{3}{2}} \rightarrow x^{-\frac{5}{2}}$  A1 Cao  $\frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$  o.e. e.g.  $\frac{3}{2}x^2 - \frac{9}{2x^2\sqrt{x}}$  then isw. Allow factorised form. Do not penalise  $+-\frac{9}{2}x^{-\frac{5}{2}}$  used instead of  $-\frac{9}{2}x^{-\frac{5}{2}}$ 

Use of Quotient Rule: M1,A1:Reaching  $\frac{2x^2(5x^4+3x^{-\frac{1}{2}})-4x(x^5+6x^{\frac{1}{2}})}{4x^4}, = \frac{6x^6-18x^{\frac{3}{2}}}{4x^4}$ 

Send to review if doubtful M1A1: Simplifying (e.g. dividing numerator and denominator by 2) to reach  $\frac{3x^6 - 9x^{\frac{7}{2}}}{2x^4}$  o.e.

| Question<br>Number | Scheme   | Marks        |     |
|--------------------|--|--------------|-----|
| 8.                 | (a) Use $n^{th}$ term = $a + (n-1)d$ with $d = 10$ ; $a = 150$ and $n = 8$ , or $a = 160$ and $n = 7$ , or $a = 170$ and $n = 6$ : = $150 + 7 \times 10$ or $160 + 6 \times 10$ or $170 + 5 \times 10$ = $220*$ (Or gives clear list – see note) | M1<br>A1*    | (2) |
| Or                 | If answer 220 is assumed and $150 + (n - 1)$ $10 = 220$ or variation is solved for $n = 10$ Then $n = 10$ , so 2007 is the year (must conclude the year)   | M1<br>A1*    | (2) |
|                    | (b) Use $S_n = \frac{n}{2} \{ 2a + (n-1)10 \}$ Or $S_n = \frac{n}{2} \{ a + l \}$ and $l = a + (n-1)10$  | M1           |     |
|                    | $= 7(300+13\times10) \qquad \text{or } 7(150+280)$ $= 7\times430$  | A1           |     |
|                    | $= 7 \wedge 430$<br>= 3010   | A1           | (2) |
|                    | (c) Cost in year $n = 900+(n-1)\times-20$<br>Sales in year $n = 150+(n-1)\times10$   | M1           | (3) |
|                    | Cost =3×Sales $\Rightarrow$ 900+(n-1)×-20 = 3×(150+(n-1)×10)<br>900-20n+20 = 450+30n-30<br>500 = 50n   | M1           |     |
|                    | n = 10   | M1           |     |
|                    | Year is 2009 As $n$ is not defined they may work correctly from another base year to get the answer 2009 and their $n$ may not equal 10. If doubtful – send to review.   | A1 (9 marks) | (4) |

#### **Notes**

- (a) M1 Attempt to use  $n^{th}$  term = a + (n-1)d with d = 10, and correct combination of a and n i.e. a = 150 and n = 8 or a = 160 and n = 7, or a = 170 and n = 6
  - A1 \* Shows that 220 computers are sold in 2007 with no errors

Note that this is a given solution, so needed 150+7×10 or 160+6×10 or 170+5×10 or equivalent.

Accept a correct list showing all values and years for both marks Just 150,160,170,180,190,200,210,220 is M1A0 Need some reference to years as well as the list of numbers of computers for A1.

(b) M1 Attempts to use  $S_n = \frac{n}{2} \{2a + (n-1)d\}$  with d = 10, and correct combination of a and n i.e. a = 150 and n = 14 or a = 160 and n = 13 or a = 170 and n = 12

and 
$$n = 14$$
, or  $a = 160$  and  $n = 13$ , or  $a = 170$  and  $n = 12$ 

A1 Uses  $S_n = \frac{n}{2} \{ 2a + (n-1)d \}$  with  $a = 150$ ,  $d = 10$  and  $n = 14$  [N.B.  $S_n = \frac{n}{2} \{ a + l \}$  needs  $l = a + (n-1)d$  as well

NB A0 for a = 160 and n = 13 or a = 170 and n = 12 unless they then add the first, or first two terms respectively.

A1 Cao 3010. This answer (with no working) implies correct method M1A1A1.

Special case: If a complete list 150+160+170+180+190+200+210+220+230+240+250+260+270+280 is seen, then there is an error finding the sum then score M1A1A0, but incomplete or wrong lists score M0A0A0

- (c) M1 Writes down an expression for the cost =  $900+(n-1)\times-20$  or writes 900+(n-1) d and states d = -20 Allow  $900+n\times-20$ . Allow recovery from invisible brackets.
  - M1 Attempts to write down an equation in n for statement 'cost =3×sales'  $900+(n-1)\times-20 = 3\times(150+(n-1)\times10)$ . Accept the 3 on the wrong side and allow use of 20 instead of -20 and allow n (consistently) instead of n-1 for this mark. Ignore £ signs in equation.
  - M1 Solves the correct linear equation in n to achieve n = 10 (for those using n 1) or n = 9 (for those using n). Ignore £ signs.
  - A1 Cso Year 2009 (A0 for the answer Year 10 if 2009 is not given)

Special case. **Just answer or trial and improvement** with no equation leading to answer scores SC 0,0,1,1 Equations satisfying the method mark descriptors followed by trial and improvement could get all four marks

| Question<br>Number | Scheme  | Marks      |
|--------------------|---|------------|
| 9.                 | (a) $2x + 3y = 26 \Rightarrow 3y = 26 \pm 2x$ and attempt to find m from $y = mx + c$       | M1         |
|                    | $(\Rightarrow y = \frac{26}{3} - \frac{2}{3}x)$ so gradient = $-\frac{2}{3}$                | A1         |
|                    | Gradient of perpendicular = $\frac{-1}{\text{their gradient}}$ (= $\frac{3}{2}$ )           | M1         |
|                    | Line goes through (0,0) so $y = \frac{3}{2}x$   | A1         |
|                    | 3   | (4)        |
|                    | (b) Solves their $y = \frac{3}{2}x$ with their $2x + 3y = 26$ to form equation in x or in y | M1         |
|                    | Solves their equation in x or in y to obtain $x = \mathbf{or} y =$                          | dM1        |
|                    | x=4 or any equivalent e.g. 156/39 or $y=6$ o.a.e  | A1         |
|                    | $B=(0,\frac{26}{3})$ used or stated in (b)  | B1         |
|                    | Method 1 ( see other methods in notes below)  |            |
|                    | Area = $\frac{1}{2}$ ×"4"× $\frac{"26"}{3}$   | dM1        |
|                    | $= \frac{52}{3}$ (oe with integer numerator and denominator)                                | A1         |
|                    |   | (6)        |
|                    |   | (10 marks) |

(a) M1 Complete method for finding gradient. (This may be implied by later correct answers.) e.g. Rearranges  $2x + 3y = 26 \Rightarrow y = mx + c$  so m =

Or finds coordinates of two points on line and finds gradient e.g. (13,0) and (1,8) so  $m = \frac{8-0}{1-13}$ 

- A1 States or implies that gradient =  $-\frac{2}{3}$  condone  $-\frac{2}{3}x$  if they continue correctly. Ignore errors in constant term in straight line equation
- M1 Uses  $m_1 \times m_2 = -1$  to find the gradient of  $l_2$ . This can be implied by the use of  $\frac{-1}{\text{their gradient}}$
- A1  $y = \frac{3}{2}x$  or 2y 3x = 0 Allow  $y = \frac{3}{2}x + 0$  Also accept 2y = 3x, y = 39/26x or even  $y 0 = \frac{3}{2}(x 0)$  and isw

### **Notes Continued**

- (b) M1 Eliminates variable between their  $y = \frac{3}{2}x$  and their (possibly rearranged) 2x + 3y = 26 to form an equation in x or y. (They may have made errors in their rearrangement)
  - dM1 (Depends on previous M mark) Attempts to solve their equation to find the value of x or y
  - A1 x = 4 or equivalent or y = 6 or equivalent
  - B1 y coordinate of B is  $\frac{26}{3}$  (stated or implied) isw if written as  $(\frac{26}{3}, 0)$ . Must be used or stated in (b)
  - dM1 (Depends on previous M mark) Complete method to find area of triangle OBC (using their values of x and/or y at point C and their 26/3)
  - A1 Cao  $\frac{52}{3}$  or  $\frac{104}{6}$  or  $\frac{1352}{78}$  o.e

### Method 1:

Uses the area of a triangle formula  $\frac{1}{2} \times OB \times (x \text{ coordinate of } C)$ 

**Alternative methods**: Several Methods are shown below. The only mark which differs from Method 1 is the last M mark and its use in each case is described below:

**Method 2** in 9(b) using 
$$\frac{1}{2} \times BC \times OC$$

dM1 Uses the area of a triangle formula  $\frac{1}{2} \times BC \times OC$  Also finds OC (= $\sqrt{52}$ ) and BC= ( $\frac{4}{3}\sqrt{13}$ )

**Method 3** in 9(b) using 
$$\frac{1}{2} \begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$$

dM1 States the area of a triangle formula  $\frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$  or equivalent with their values

Method 4 in 9(b) using area of triangle OBX – area of triangle OCX where X is point (13, 0)

dM1 Uses the correct subtraction 
$$\frac{1}{2} \times 13 \times "\frac{26}{3}" - \frac{1}{2} \times 13 \times "6"$$

**Method 5** in 9(b) using area =  $\frac{1}{2}$  (6 × 4) +  $\frac{1}{2}$  (4 × 8/3) drawing a line from C parallel to the x axis and dividing triangle into two right angled triangles

dM1 for correct method area = 
$$\frac{1}{2}$$
 ("6" × "4") +  $\frac{1}{2}$  ("4" × ["26/3"-"6"])

#### Method 6 Uses calculus

$$dM1 \int_{0}^{4} \frac{26}{3} - \frac{2x}{3} - \frac{3x}{2} dx = \left[ \frac{26}{3} x - \frac{x^{2}}{3} - \frac{3x^{2}}{4} \right]_{0}^{4}$$

| Question<br>Number | Scheme  | Marks                |
|--------------------|---|----------------------|
| 10.                | (a) $f(x) = \int \left(\frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1\right) dx$   |                      |
|                    | $x^{n} \to x^{n+1} \Rightarrow f(x) = \frac{3}{8} \times \frac{x^{3}}{3} - 10 \frac{x^{\frac{1}{2}}}{\underline{1}} + x(+c)$                          | M1, A1, A1           |
|                    | Substitute $x = 4$ , $y = 25 \implies 25 = 8 - 40 + 4 + c \implies c =$   | M1                   |
|                    | $(f(x)) = \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$   | A1                   |
|                    | (b) Sub $x=4$ into $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1$<br>$\Rightarrow f'(4) = \frac{3}{8} \times 4^2 - 10 \times 4^{-\frac{1}{2}} + 1$ | (5)<br>M1            |
|                    | $\Rightarrow f'(4) = 2$ Gradient of tangent = 2 \Rightarrow Gradient of normal is -1/2  | A1<br>dM1            |
|                    | Substitute $x = 4$ , $y = 25$ into line equation with their changed gradient e.g. $y - 25 = -\frac{1}{2}(x - 4)$                                      | dM1                  |
|                    | $\pm k(2y + x - 54) = 0$ o.e. (but must have integer coefficients)  | A1cso (5) (10 Marks) |

### **Notes**

- (a) M1 Attempt to integrate  $x^n \to x^{n+1}$ 
  - A1 Term in  $x^3$  or term in  $x^{\frac{1}{2}}$  correct, coefficient need not be simplified, no need for +x nor +c
  - Al ALL three terms correct, coefficients need not be simplified, no need for +c
  - M1 For using x = 4, y = 25 in their f(x) to form a linear equation in c and attempt to find c
  - A1 =  $\frac{x^3}{8} 20x^{\frac{1}{2}} + x + 53$  cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be f(x) or y). Need full expression with 53 These marks need to be scored in part (a)
- (b) M1 Attempt to substitute x = 4 into f'(x) must be in part (b)
  - A1 f'(x) = 2 at x = 4
  - dM1 (Dependent on first method mark in part (b)) Using  $m_1 \times m_2 = -1$  to find the gradient of the normal from their tangent gradient (Give mark if gradient of 1 becomes -1 as they will lose accuracy)
  - dM1 (Dependent on first method mark in part (b)) Attempt to find the equation of the normal (not tangent). Eg use x=4, y=25 in  $y=\frac{-1}{2}$ ' x+c to find a value of c or use  $\frac{1}{2} = \frac{y-25}{x-4}$  with their adapted gradient.
  - A1 cso  $\pm k(2y+x-54) = 0$  (where k is any integer)

| Question<br>Number | Scheme  | Marks  |
|--------------------|---|--|
| 11.                | (a) Discriminant = $b^2 - 4ac = 8^2 - 4 \times 2 \times 3 = 40$   | M1, A1 (2)   |
|                    | (b) $2x^2 + 8x + 3 = 2(x^2 + \dots)$ or $p=2$   | B1   |
|                    | $=2((x+2)^2 \pm)$ or $q=2$  | M1   |
|                    | $=2(x+2)^2-5$ or $p=2$ , $q=2$ and $r=-5$   | A1   |
|                    | (c) Method 1A: Sets derivative " $4x + 8$ " = $4 \Rightarrow x =$ , $x = -1$<br>Substitute $x = -1$ in $y = 2x^2 + 8x + 3$ ( $\Rightarrow y = -3$ )<br>Substitute $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y + 3) = 4(x + 1)$ and expand $c = 1$ or writing $y = 4x + 1$ | (3)<br>M1, A1<br>dM1<br>dM1<br>dM1<br>A1cso          |
|                    | Method 1B: Sets derivative " $4x+8$ " = $4 \Rightarrow x = $ , $x = -1$<br>Substitute $x = -1$ in $2x^2 + 8x + 3 = 4x + c$<br>Attempts to find value of $c$   | (5)<br>M1, A1<br>dM1<br>dM1<br>A1cso                 |
|                    | c = 1 or writing $y = 4x + 1$   | (5)  |
|                    | Method 2: Sets $2x^2 + 8x + 3 = 4x + c$ and collects $x$ terms together<br>Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent<br>States that $b^2 - 4ac = 0$<br>$4^2 - 4 \times 2 \times (3 - c) = 0$ and so $c = -1$  | M1 A1 dM1 dM1 A1cso                                  |
|                    | $c = 1$ Method 3: Sets $2x^2 + 8x + 3 = 4x + c$ and collects $x$ terms together Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent Uses $2(x+1)^2 - 2 + 3 - c = 0$ or equivalent Writes $-2 + 3 - c = 0$ So $c = 1$  | (5)<br>M1<br>A1<br>dM1<br>dM1<br>dM1<br>A1cso<br>(5) |
|                    | Also see special case for using a perpendicular gradient (overleaf)   | (10 marks)   |

- (a) M1 Attempts to calculate  $b^2 4ac$  using  $8^2 4 \times 2 \times 3$  must be correct not just part of a quadratic formula Cao 40
- (b) B1 See 2(...) or p = 2

M1 ... $((x+2)^2 \pm ...)$  is sufficient evidence or obtaining q=2

A1 Fully correct values.  $2(x+2)^2 - 5$  or p = 2, q = 2, r = -5 cso. Ignore inclusion of "=0".

[In many respects these marks are similar to three B marks.

p=2 is B1; q=2 is B1 and p=2, q=2 and r=-5 is final B1 but they must be entered on epen as **B1 M1 A1**]

**Special case**: Obtains  $2x^2 + 8x + 3 = 2(x+2) - 1$  This may have first B1, for p = 2 then M0A0

- (c) Method 1A (Differentiates and puts gradient equal to 4. Needs both x and y to find c)
  - Attempts to solve their  $\frac{dy}{dx} = 4$ . They must reach x = ... (Just differentiating is M0 A0) M1
  - x = -1 (If this follows  $\frac{dy}{dx} = 4x + 8$ , then give M1 A1 by implication) **A**1
  - (Depends on previous M mark) Substitutes **their** x = -1 into f(x) or into "their f(x) from (b)" to find ydM1
- (Depends on both previous M marks) Substitutes **their** x = -1 and **their** y = -3 values into y = 4x + c to find cdM1 or uses equation of line is (y + "3")=4(x + "1") and rearranges to y = mx + c

**PMT** 

- **A**1 c = 1 or allow for y = 4x + 1 cso
- (c) Method 1B (Differentiates and puts gradient equal to 4. Also equates equations and uses x to find c)
  - Exactly as in Method 1A above M1A1
    - (Depends on previous M mark) Substitutes **their** x = -1 into  $2x^2 + 8x + 3 = 4x + c$ dM1
  - Attempts to find value of c then A1 as before dM1
- (c) Method 2 (uses repeated root to find c by discriminant)
  - Sets  $2x^2 + 8x + 3 = 4x + c$  and tries to collect x terms together M1
  - Collects terms e.g.  $2x^2 + 4x + 3 c = 0$  or  $-2x^2 4x 3 + c = 0$  or  $2x^2 + 4x + 3 = c$  or even **A**1  $2x^2 + 4x = c - 3$  Allow "=0" to be missing on RHS.
  - (If the line is a tangent it meets the curve at just one point so repeated root and  $b^2 4ac = 0$ ) dM1 Stating that  $b^2 - 4ac = 0$  is enough
  - Using  $b^2 4ac = 0$  to obtain equation in terms of c dM1 (Eg.  $4^2 - 4 \times 2 \times (3 - c) = 0$ ) AND leading to a solution for c
  - c = 1 or allow for y = 4x + 1 cso **A**1
- (c) Method 3 (Similar to method 2 but uses completion of the square on the quadratic to find repeated root)
  - Sets  $2x^2 + 8x + 3 = 4x + c$  and tries to collect x terms together. May be implied by  $2x^2 + 8x + 3 4x \pm c$  on M1one side
  - Collects terms e.g.  $2x^2 + 4x + 3 c = 0$  or  $-2x^2 4x 3 + c = 0$  or  $2x^2 + 4x + 3 = c$ **A**1 or even  $2x^2 + 4x = c - 3$ Allow "=0" to be missing on RHS.
  - Then use completion of square  $2(x+1)^2 2 + 3 c = 0$  (Allow  $2(x+1)^2 k + 3 c = 0$ ) dM1 where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square
  - dM1 -2 + 3 - c = 0 AND leading to a solution for c (Allow -1 + 3 - c = 0) (x = -1 has been used) **A**1  $c = 1 \cos \theta$

In Method 1 they may use part (b) and differentiate their f(x) and put it equal to 4 They can earn M1, but do not follow through errors.

In Methods 2 and 3 they may use part (b) to write

their  $2(x+2)^2 - 5 = 4x + c$ . They need to expand and collect x terms together for M1

Then expanding gives  $2x^2 + 4x + 3 - c = 0$  for A1 – do not follow through errors

Then the scheme is as before

If they just state c = 1 with little or no working – please send to review,

## PTO for special case

Special case uses perpendicular gradient (maximum of 2/5)

Sets 
$$4x + 8 = -\frac{1}{4} \Rightarrow x = , \qquad x = -\frac{33}{16}$$
 M1 A0

Substitute 
$$x = -\frac{33}{16}$$
 in  $y = 2x^2 + 8x + 3$  ( $\Rightarrow y = -\frac{639}{128}$ )

Substitute 
$$x = -\frac{33}{16}$$
 and  $y = -\frac{639}{128}$  into  $y = 4x + c$  or into  $(y + \frac{639}{128}) = 4(x + \frac{33}{16})$  and expand M1 A0

PMT