

Mark Scheme (Results) Summer 2009

GCE

GCE Mathematics (6663/01)

June 2009
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Marks
Q1 (a) (b)	$(3\sqrt{7})^2 = 63$ $(8 + \sqrt{5})(2 - \sqrt{5}) = 16 - 5 + 2\sqrt{5} - 8\sqrt{5}$ $= 11, -6\sqrt{5}$	B1 (1) M1 A1, A1 (3) [4]
(a) (b)	<p>B1 for 63 only</p> <p>M1 for an attempt to expand <u>their</u> brackets with ≥ 3 terms correct.</p> <p>They may collect the $\sqrt{5}$ terms to get $16 - 5 - 6\sqrt{5}$</p> <p>Allow $-\sqrt{5} \times \sqrt{5}$ or $-(\sqrt{5})^2$ or $-\sqrt{25}$ instead of the -5</p> <p>These 4 values may appear in a list or table but they should have minus signs included</p> <p>The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule</p> <p>1st A1 for 11 from $16 - 5$ <u>or</u> $-6\sqrt{5}$ from $-8\sqrt{5} + 2\sqrt{5}$</p> <p>2nd A1 for <u>both</u> 11 and $-6\sqrt{5}$.</p> <p><u>S.C - Double sign error in expansion</u></p> <p>For $16 - 5 - 2\sqrt{5} + 8\sqrt{5}$ leading to $11 + \dots$ allow <u>one</u> mark</p>	

Question Number	Scheme	Marks
Q2	$32 = 2^5 \text{ or } 2048 = 2^{11}, \quad \sqrt{2} = 2^{1/2} \text{ or } \sqrt{2048} = (2048)^{1/2}$ $a = \frac{11}{2} \quad \left(\text{or } 5\frac{1}{2} \text{ or } 5.5 \right)$	B1, B1 B1 [3]
	<p>1st B1 for $32 = 2^5$ or $2048 = 2^{11}$ This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32 \times 2 = 2^5 \times 2 (= 2^6)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT</p> <p>2nd B1 for $2^{1/2}$ or $(2048)^{1/2}$ seen. This mark may be implied</p> <p>3rd B1 for answer as written. Need $a = \dots$ so $2^{1/2}$ is B0</p> <p style="text-align: center;">$a = \frac{11}{2} \left(\text{or } 5\frac{1}{2} \text{ or } 5.5 \right)$ with no working scores full marks.</p> <p style="text-align: center;">If $a = 5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work.</p> <p style="text-align: center;">Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1.</p> <p><u>Special case:</u> If $\sqrt{2} = 2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a = 2\frac{1}{2}$, $a = 4\frac{1}{2}$, the second B1 is given by implication.</p>	

Question Number	Scheme	Marks
Q3 (a)	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3)
(b)	$\frac{2x^4}{4} + \frac{3x^{-1}}{-1} (+C)$	M1 A1
	$\frac{x^4}{2} - 3x^{-1} + C$	A1
		(3) [6]
(a)	<p>M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$</p> <p>1st A1 for $6x^2$</p> <p>2nd A1 for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone $+ -6x^{-3}$ here. Inclusion of $+c$ scores A0 here.</p>	
(b)	<p>M1 for some attempt to integrate an x term of the given y. $x^n \rightarrow x^{n+1}$</p> <p>1st A1 for both x terms correct but unsimplified- as printed or better. Ignore $+c$ here</p> <p>2nd A1 for both x terms correct and simplified and $+c$. Accept $-\frac{3}{x}$ but <u>NOT</u> $+ -3x^{-1}$</p> <p>Condone the $+c$ appearing on the first (unsimplified) line but missing on the final (simplified) line</p> <p>Apply ISW if a correct answer is seen</p> <p>If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).</p>	

Question Number	Scheme	Marks
Q4 (a)	$5x > 10, x > 2$ [Condone $x > \frac{10}{2} = 2$ for M1A1]	M1, A1 (2)
(b)	$(2x + 3)(x - 4) = 0$, ‘Critical values’ are $-\frac{3}{2}$ and 4 $-\frac{3}{2} < x < 4$	M1, A1 M1 A1ft (4)
(c)	$2 < x < 4$	B1ft (1) [7]
(a)	M1 for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$ Must have a or b correct so eg $3x > 4$ scores M0	
(b)	1 st M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values 1 st A1 for $-\frac{3}{2}$ and 4 seen. They may write $x < -\frac{3}{2}$, $x < 4$ and still get this A1 2 nd M1 for choosing the “inside region” for their critical values 2 nd A1ft follow through their 2 distinct critical values Allow $x > -\frac{3}{2}$ with “or” “,” “ \cup ” “ \cap ” “ $x < 4$ to score M1A0 but “and” or “ \cap ” score M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only	
(c)	B1ft Allow if a correct answer is seen or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) <u>must be regions</u> . Do not follow through single values. If their follow through answer is the empty set accept \emptyset or $\{\}$ or equivalent in words If (a) or (b) are not given then score this mark for cao NB You may see $x < 4$ (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft Do not award marks for part (b) if only seen in part (c) Use of \leq instead of $<$ (or \geq instead of $>$) loses one accuracy mark only, at first occurrence.	


Question Number	Scheme	Marks
Q5 (a) (b) (c)	$a + 9d = 2400 \quad a + 39d = 600$ $d = \frac{-1800}{30} \quad d = -60 \quad (\text{accept } \pm 60 \text{ for A1})$ $a - 540 = 2400 \quad a = 2940$ $\text{Total} = \frac{1}{2}n\{2a + (n-1)d\} = \frac{1}{2} \times 40 \times (5880 + 39 \times -60) \quad (\text{ft values of } a \text{ and } d)$ $= \underline{70\,800}$	M1 M1 A1 (3) M1 A1 (2) M1 A1ft A1cao (3) [8]
(a)	<p><u>Note:</u> If the sequence is considered ‘backwards’, an equivalent solution may be given using $d = 60$ with $a = 600$ and $l = 2940$ for part (b). This can still score full marks. Ignore labelling of (a) and (b)</p> <p>1st M1 for an attempt to use 2400 and 600 in $a + (n-1)d$ formula. Must use both values i.e. need $a + pd = 2400$ <u>and</u> $a + qd = 600$ where $p = 8$ or 9 and $q = 38$ or 39 (any combination) 2nd M1 for an attempt to solve <u>their</u> 2 linear equations in a and d as far as $d = \dots$ A1 for $d = \pm 60$. Condone correct equations leading to $d = 60$ or $a + 8d = 2400$ and $a + 38d = 600$ leading to $d = -60$. They should get penalised in (b) and (c). NB This is a “one off” ruling for A1. Usually an A mark must follow from their work. ALT 1st M1 for $(30d) = \pm (2400 - 600)$ 2nd M1 for $(d =) \pm \frac{(2400 - 600)}{30}$ A1 for $d = \pm 60$ $a + 9d = 600, a + 39d = 2400$ only scores M0 BUT if they solve to find $d = \pm 60$ then use ALT scheme above.</p> <p>(b) M1 for use of <u>their</u> d in a correct linear equation to find a leading to $a = \dots$ A1 their a must be compatible with their d so $d = 60$ must have $a = 600$ and $d = -60, a = 2940$ So for example they can have $2400 = a + 9(60)$ leading to $a = \dots$ for M1 but it scores A0 Any approach using a list scores M1A1 for a correct a but M0A0 otherwise</p> <p>(c) M1 for use of a correct S_n formula with $n = 40$ and at least one of a, d or l correct or correct ft. 1st A1ft for use of a correct S_{40} formula and both a, d or a, l correct or correct follow through ALT Total = $\frac{1}{2}n\{a + l\} = \frac{1}{2} \times 40 \times (2940 + 600)$ (ft value of a) M1 A1ft 2nd A1 for 70800 only</p>	

Question Number	Scheme	Marks
Q6	<p>$b^2 - 4ac$ attempted, in terms of p.</p> <p>$(3p)^2 - 4p = 0$ o.e.</p> <p>Attempt to solve for p e.g. $p(9p - 4) = 0$ Must potentially lead to $p = k, k \neq 0$</p> <p>$p = \frac{4}{9}$ (Ignore $p = 0$, if seen)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1cso</p> <p>[4]</p>
	<p>1st M1 for an attempt to substitute into $b^2 - 4ac$ or $b^2 = 4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. Use of inequalities can score the M marks only</p> <p>1st A1 for any correct equation: $(3p)^2 - 4 \times 1 \times p = 0$ or better</p> <p>2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p = \frac{4}{9}$.</p> <p>Accept factors or use of quadratic formula or $(p \pm \frac{2}{9})^2 = k^2$ (o.e. eg) $(3p \pm \frac{2}{3})^2 = k^2$ or equivalent work on <u>their</u> eqn.</p> <p>$9p^2 = 4p \Rightarrow \frac{9p^2}{p} = 4$ which would lead to $9p = 4$ is OK for this 2nd M1</p> <p>ALT <u>Comparing coefficients</u></p> <p>M1 for $(x + \alpha)^2 = x^2 + \alpha^2 + 2\alpha x$ and A1 for a correct equation eg $3p = 2\sqrt{p}$</p> <p>M1 for forming solving leading to $\sqrt{p} = \frac{2}{3}$ or better</p> <p><u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.</p>	

Question Number	Scheme	Marks
Q7	<p>(a) $(a_2 =)2k - 7$</p> <p>(b) $(a_3 =)2(2k - 7) - 7$ or $4k - 14 - 7, = 4k - 21$ (*)</p> <p>(c) $(a_4 =)2(4k - 21) - 7$ ($= 8k - 49$)</p> $\sum_{r=1}^4 a_r = k + "(2k - 7)" + (4k - 21) + "(8k - 49)"$ $k + (2k - 7) + (4k - 21) + (8k - 49) = 15k - 77 = 43 \quad k = 8$	<p>B1 (1)</p> <p>M1, A1cso (2)</p> <p>M1</p> <p>M1</p> <p>M1 A1 (4)</p> <p>[7]</p>
	<p>(b) M1 must see $2(\text{their } a_2) - 7$ or $2(2k - 7) - 7$ or $4k - 14 - 7$. Their a_2 must be a function of k. A1cso must see the $2(2k - 7) - 7$ or $4k - 14 - 7$ expression and the $4k - 21$ with no incorrect working</p> <p>(c) 1st M1 for an attempt to find a_4 using the given rule. Can be awarded for $8k - 49$ seen. Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks. 2nd M1 for attempting the sum of the 1st 4 terms. Must have "+" not just , or clear attempt to sum. Follow through their a_2 and a_4 provided they are linear functions of k. Must lead to linear expression in k. Condone use of their linear $a_3 \neq 4k - 21$ here too. 3rd M1 for forming a linear equation in k using their sum and the 43 and attempt to solve for k as far as $pk = q$ A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0</p> <p><u>Answer Only</u> (e.g. trial improvement) Accept $k = 8$ <u>only if</u> $8 + 9 + 11 + 15 = 43$ is seen as well</p> <p><u>Sum</u> $a_2 + a_3 + a_4 + a_5$ or $a_2 + a_3 + a_4$</p> <p>Allow: M1 if $8k - 49$ is seen, M0 for the sum (since they are not adding the 1st 4 terms) then M1 if they use their sum along with the 43 to form a linear equation and attempt to solve but A0</p>	

Question Number	Scheme	Marks
Q8 (a)	$AB: m = \frac{2-7}{8-6}, \left(= -\frac{5}{2} \right)$ <p>Using $m_1 m_2 = -1$: $m_2 = \frac{2}{5}$</p> $y - 7 = \frac{2}{5}(x - 6), \quad 2x - 5y + 23 = 0 \quad (\text{o.e. with integer coefficients})$	B1 M1 M1, A1 (4)
(b)	Using $x = 0$ in the answer to (a), $y = \frac{23}{5}$ or 4.6	M1, A1ft (2)
(c)	Area of triangle = $\frac{1}{2} \times 8 \times \frac{23}{5} = \frac{92}{5}$ (o.e.) e.g. $\left(18\frac{2}{5}, 18.4, \frac{184}{10} \right)$	M1 A1 (2) [8]
(a)	<p>B1 for an expression for the gradient of AB. Does not need the $= -2.5$</p> <p>1st M1 for use of the perpendicular gradient rule. Follow through their m</p> <p>2nd M1 for the use of (6, 7) and their changed gradient to form an equation for l.</p> <p>Can be awarded for $\frac{y-7}{x-6} = \frac{2}{5}$ o.e.</p> <p>Alternative is to use (6, 7) in $y = mx + c$ to <u>find a value</u> for c. Score when $c = \dots$ is reached.</p> <p>A1 for a correct equation in the required form and must have “= 0” and integer coefficients</p>	
(b)	<p>M1 for using $x = 0$ in their answer to part (a) e.g. $-5y + 23 = 0$</p> <p>A1ft for $y = \frac{23}{5}$ provided that $x = 0$ clearly seen <u>or</u> $C(0, 4.6)$. Follow through their equation in (a)</p> <p>If $x = 0, y = 4.6$ are clearly seen but C is given as (4.6,0) apply ISW and award the mark.</p> <p>This A mark requires a simplified fraction or an exact decimal</p> <p>Accept their 4.6 marked on diagram next to C for M1A1ft</p>	
(c)	<p>M1 for $\frac{1}{2} \times 8 \times y_C$ so can follow through their y coordinate of C.</p> <p>A1 for 18.4 (o.e.) but their y coordinate of C must be positive</p>	
<p><u>Use of 2 triangles or trapezium and triangle</u></p> <p>Award M1 when an expression for area of OCB only is seen</p>		
<p><u>Determinant approach</u></p> <p>Award M1 when an expression containing $\frac{1}{2} \times 8 \times y_C$ is seen</p>		

Question Number	Scheme	Marks
Q9 (a)	$\left[(3-4\sqrt{x})^2 = \right] 9-12\sqrt{x}-12\sqrt{x}+(-4)^2 x$ $9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$	M1 A1, A1 (3)
(b)	$f'(x) = -\frac{9}{2}x^{-\frac{3}{2}}, + \frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A1ft (3)
(c)	$f'(9) = -\frac{9}{2} \times \frac{1}{27} + \frac{16}{2} \times \frac{1}{3} = -\frac{1}{6} + \frac{16}{6} = \frac{5}{2}$	M1 A1 (2)
	<p>(a) M1 for an attempt to expand $(3-4\sqrt{x})^2$ with at least 3 terms correct- as printed or better Or $9-k\sqrt{x}+16x$ ($k \neq 0$) . See also the MR rule below 1st A1 for their coefficient of $\sqrt{x} = 16$. Condone writing $(\pm)9x^{(\pm)\frac{1}{2}}$ instead of $9x^{-\frac{1}{2}}$ 2nd A1 for $B = -24$ or their constant term = -24</p> <p>(b) M1 for an attempt to differentiate an x term $x^n \rightarrow x^{n-1}$ 1st A1 for $-\frac{9}{2}x^{-\frac{3}{2}}$ <u>and</u> their constant B differentiated to zero. NB $-\frac{1}{2} \times 9x^{-\frac{3}{2}}$ is A0 2nd A1ft follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for A, i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$</p> <p>(c) M1 for some correct substitution of $x = 9$ in <u>their</u> expression for $f'(x)$ including an attempt at $(9)^{\pm\frac{k}{2}}$ (k odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or 3 A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18}, \frac{135}{54}$ or even $\frac{67.5}{27}$ <u>Misread (MR)</u> Only allow MR of the form $\frac{(3-k\sqrt{x})^2}{\sqrt{x}}$ N.B. Leads to answer in (c) of $\frac{k^2-1}{6}$</p> <p>Score as M1A0A0, M1A1A1ft, M1A1ft</p>	[8]

Question Number	Scheme	Marks
Q10 (a) (b) (c)	$x(x^2 - 6x + 9)$ $= x(x - 3)(x - 3)$ Shape  <u>Through</u> origin (<u>not</u> touching) Touching x -axis only once Touching at $(3, 0)$, or 3 on x -axis [Must be on graph not in a table] Moved horizontally (either way) $(2, 0)$ and $(5, 0)$, or 2 and 5 on x -axis	B1 M1 A1 (3) B1 B1 B1 B1ft (4) M1 A1 (2) [9]
(a) S.C. (b) (c)	B1 for correctly taking out a factor of x M1 for an attempt to factorize their 3TQ e.g. $(x + p)(x + q)$ where $ pq = 9$. So $(x - 3)(x + 3)$ will score M1 but A0 A1 for a fully correct factorized expression - accept $x(x - 3)^2$ If they "solve" use ISW If the only correct linear factor is $(x - 3)$, perhaps from factor theorem, award B0M1A0 Do not award marks for factorising in part (b) <u>For the graphs</u> "Sharp points" will lose the 1 st B1 in (b) but otherwise be generous on shape Condone $(0, 3)$ in (b) and $(0, 2)$, $(0, 5)$ in (c) if the points are marked in the correct places. 2 nd B1 for a curve that starts or terminates at $(0, 0)$ score B0 4 th B1ft for a curve that touches (not crossing or terminating) at $(a, 0)$ where their $y = x(x - a)^2$ M1 for their graph moved horizontally (only) <u>or</u> a fully correct graph Condone a partial stretch if ignoring their values looks like a simple translation A1 for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)	

Question Number	Scheme	Marks
Q11 (a) (b) (c)	$x = 2: y = 8 - 8 - 2 + 9 = 7$ (*) $\frac{dy}{dx} = 3x^2 - 4x - 1$ $x = 2: \frac{dy}{dx} = 12 - 8 - 1 (= 3)$ $y - 7 = 3(x - 2), \quad \underline{y = 3x + 1}$ $m = -\frac{1}{3}$ (for $-\frac{1}{m}$ with their m) $3x^2 - 4x - 1 = -\frac{1}{3}, \quad 9x^2 - 12x - 2 = 0$ or $x^2 - \frac{4}{3}x - \frac{2}{9} = 0$ (o.e.) $\left(x = \frac{12 + \sqrt{144 + 72}}{18}\right) (\sqrt{216} = \sqrt{36}\sqrt{6} = 6\sqrt{6})$ or $(3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$ $x = \frac{1}{3}(2 + \sqrt{6})$ (*)	B1 (1) M1 A1 A1ft M1, <u>A1</u> (5) B1ft M1, A1 M1 A1cso (5) [11]
(a) (b) (c) ALT	B1 there must be a clear attempt to substitute $x = 2$ leading to 7 e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$ 1 st M1 for an attempt to differentiate with at least one of the given terms fully correct. 1 st A1 for a fully correct expression 2 nd A1ft for sub. $x = 2$ in <u>their</u> $\frac{dy}{dx}$ ($\neq y$) accept for a correct expression e.g. $3 \times (2)^2 - 4 \times 2 - 1$ 2 nd M1 for use of their “3” (provided it comes from their $\frac{dy}{dx}$ ($\neq y$) and $x=2$) to find equation of tangent. Alternative is to use (2, 7) in $y = mx + c$ to <u>find a value</u> for c . Award when $c = \dots$ is seen. No attempted use of $\frac{dy}{dx}$ in (b) scores 0/5 1 st M1 for forming an equation from their $\frac{dy}{dx}$ ($\neq y$) and their $-\frac{1}{m}$ (must be changed from m) 1 st A1 for a correct 3TQ all terms on LHS (condone missing =0) 2 nd M1 for proceeding to $x = \dots$ or $3x = \dots$ by formula or completing the square for a 3TQ. Not factorising. Condone \pm 2 nd A1 for proceeding to given answer with no incorrect working seen. Can still have \pm . <u>Verify (for M1A1M1A1)</u> 1 st M1 for attempting to square need ≥ 3 correct values in $\frac{4+6+4\sqrt{6}}{9}$, 1 st A1 for $\frac{10+4\sqrt{6}}{9}$ 2 nd M1 Dependent on 1 st M1 in this case for substituting in all terms of their $\frac{dy}{dx}$ 2 nd A1cso for cso <u>with a full comment</u> e.g. “the x co-ord of Q is ...”	