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GCE

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Mathematics

Core Mathematics C1 (6663)

June 2006

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Mark Scheme (Results)

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Mathematics

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6663 Core Mathematics C1
Mark Scheme

Question number	Scheme	Marks
1.	$\frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \quad (+c)$	M1
		A1
	$= 2x^3 + 2x + 2x^{\frac{1}{2}}$	A1
	$+c$	B1
		4
	M1 for some attempt to integrate $x^n \to x^{n+1}$	
	1st A1 for either $\frac{6}{3}x^3$ or $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ or better	
	2nd A1 for all terms in x correct. Allow $2\sqrt{x}$ and $2x^1$.	
	B1 for + c, when first seen with a changed expression.	

Question number	Scheme	Marks
2.	<u>Critical Values</u>	
	$(x \pm a)(x \pm b)$ with ab=18 or $x = \frac{7 \pm \sqrt{49 - -72}}{2}$ or $\left(x - \frac{7}{2}\right)^2 \pm \left(\frac{7}{2}\right)^2 - 18$	M1
	$(x - 9)(x + 2)$ or $x = \frac{7 \pm 11}{2}$ or $x = \frac{7}{2} \pm \frac{11}{2}$	A1
	<u>Solving Inequality</u> $x > 9$ or $x < -2$ Choosing "outside"	M1
		A1 **4**

1st M1 For attempting to find critical values.
Factors alone are OK for M1, x = appearing somewhere for the formula and as written for completing the square

1st A1. Factors alone are OK . Formula or completing the square need x = as written.

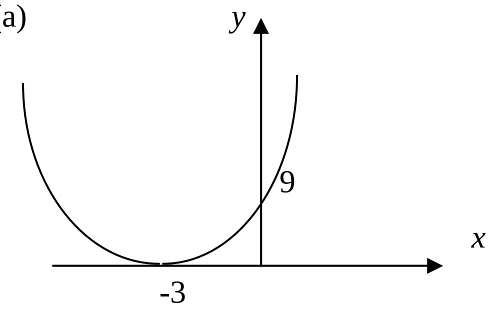
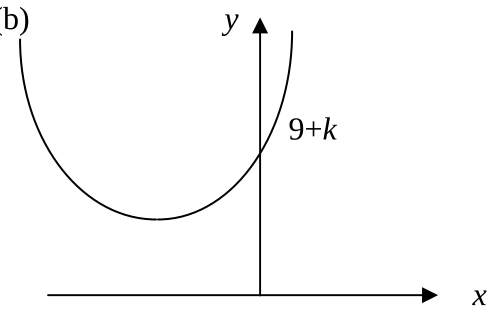
2nd M1 For choosing outside region. Can f.t. their critical values.
They must have two different critical values.

$-2 > x > 9$ is M1A0 but ignore if it follows a correct version

$-2 < x < 9$ is M0A0 whatever the diagram looks like.

2nd A1 Use of $\geq$ in final answer gets A0

 PMT

Question number	Scheme	Marks
3.	(a)  U shape touching x-axis	B1
	(- 3, 0)	B1
	(0, 9)	B1
		(3)
	(b)  Translated parallel to y-axis up	M1
	$(0, 9+k)$	B1f.t.
		(2)
		5
(a)	2nd B1 — They can score this even if other intersections with the x-axis are given.	
	2nd B1 & 3rd B1 — The -3 and 9 can appear on the sketch as shown	
(b)	M1 — Follow their curve in (a) up only. If it is not obvious do not give it. e.g. if it cuts y-axis in (a) but doesn't in (b) then it is M0.	
	B1f.t. — Follow through their 9	

Question number	Scheme	Marks
4. (a)	$a_2 = 4$ $a_3 = 3\times a_2 - 5 = 7$	B1 B1f.t. (2)
(b)	$a_4 = 3a_3 - 5(=16)$ and $a_5 = 3a_4 - 5(=43)$ 3 + 4 + 7 + 16 + 43 = 73	M1 M1 A1c.a.o. (3) **5**
(a)	2nd B1f.t. Follow through their a_2 but it must be a value. $3\times 4-5$ is B0 Give wherever it is first seen.	
(b)	1st M1 For two further attempts to use of $a_{n+1} = 3a_n - 5$, wherever seen. Condone arithmetic slips 2nd M1 For attempting to add 5 relevant terms (i.e. terms derived from an attempt to use the recurrence formula) or an expression. Follow through their values for $a_2 - a_5$ Use of formulae for arithmetic series is M0A0 but could get 1st M1 if a_4 and a_5 are correctly attempted.	

Question number	Scheme	Marks
5. (a)	$(y = x^4 + 6x^{\frac{1}{2}} \Rightarrow y' =)\ 4x^3 + 3x^{-\frac{1}{2}}$ or $4x^3 + \dfrac{3}{\sqrt{x}}$	M1A1A1 (3)
(b)	$(x+4)^2 = x^2 + 8x + 16$	M1
	$\dfrac{(x+4)^2}{x} = x + 8 + 16x^{-1}$ (allow 4+4 for 8)	A1
	$(y = \dfrac{(x+4)^2}{x} \Rightarrow y' =)\ 1 - 16x^{-2}$ o.e.	M1A1 (4)
		7

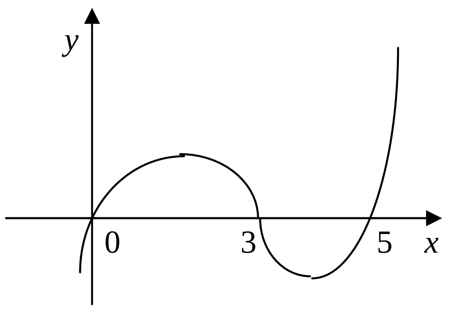
(a)	M1	For some attempt to differentiate $x^n \to x^{n-1}$
	1st A1	For one correct term as printed.
	2nd A1	For both terms correct as printed.
		$4x^3 + 3x^{-\frac{1}{2}} + c$ scores M1A1A0
(b)	1st M1	For attempt to expand $(x+4)^2$, must have x^2, x, x^0 terms and at least 2 correct
		e.g. $x^2 + 8x + 8$ or $x^2 + 2x + 16$
	1st A1	Correct expression for $\dfrac{(x+4)^2}{x}$. As printed but allow $\dfrac{16}{x}$ and $8x^0$.
	2nd M1	For some correct differentiation, any term. Can follow through their simplification.
		N.B. $\dfrac{x^2 + 8x + 16}{x}$ giving rise to $(2x + 8)/1$ is M0A0
ALT	Product or Quotient rule (If in doubt send to review)	
	M2	For correct use of product or quotient rule. Apply usual rules on formulae.
	1st A1	For $\dfrac{2(x+4)}{x}$ or $\dfrac{2x(x+4)}{x^2}$
	2nd A1	for $-\dfrac{(x+4)^2}{x^2}$

Question number	Scheme	Marks
6. (a)	$16+4\sqrt{3}-4\sqrt{3}-\left(\sqrt{3}\right)^2$ or $16-3$	M1
	$= 13$	A1c.a.o (2)
(b)	$\frac{26}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}$	M1
	$= \frac{26(4-\sqrt{3})}{13} = \underline{8-2\sqrt{3}}$ or $8+(-2)\sqrt{3}$ or *a*= 8 and *b* = -2	A1 (2)
		4
(a)	M1 For 4 terms, at least 3 correct e.g. $8+4\sqrt{3}-4\sqrt{3}-\left(\sqrt{3}\right)^2$ or $16\pm8\sqrt{3}-\left(\sqrt{3}\right)^2$ or 16 + 3 4^2 instead of 16 is OK $(4+\sqrt{3})(4+\sqrt{3})$ scores M0A0	
(b)	M1 For a correct attempt to rationalise the denominator Can be implied NB $\frac{-4+\sqrt{3}}{-4+\sqrt{3}}$ is OK	

Question number	Scheme	Marks
7.	$a+(n-1)d=k$ $\quad$ $k = 9$ or 11	M1
	$(u_{11}=)\ a+10d=9$	A1c.a.o.
	$\frac{n}{2}[2a+(n-1)d]=77$ or $\frac{(a+l)}{2}\times n=77$ $\quad$ $l = 9$ or 11	M1
	$(S_{11}=)\ \frac{11}{2}(2a+10d)=77$ or $\frac{(a+9)}{2}\times 11=77$	A1
	e.g. $a+10d=9$, $a+5d=7$ or $a+9=14$	M1
	$a = 5$ and $d = 0.4$ or exact equivalent	A1 A1
		7

1st M1	Use of u_n to form a linear equation in *a* and *d*. *a* + *nd* =9 is M0A0
1st A1	For *a* + 10*d* = 9.
2nd M1	Use of S_n to form an equation for *a* and *d* (LHS) or in *a* (RHS)
2nd A1	A correct equation based on S_n.
	For 1st 2 Ms they must write *n* or use *n* = 11.
3rd M1	Solving (LHS simultaneously) or (RHS a linear equation in *a*)
	Must lead to *a* = … or *d* = …. and depends on one previous M
3rd A1	for *a* = 5
4th A1	for *d* = 0.4 (o.e.)
ALT	Uses $\frac{(a+l)}{2}\times n=77$ to get *a* = 5, gets second and third M1A1 i.e. 4/7
	Then uses $\frac{n}{2}[2a+(n-1)d]=77$ to get *d*, gets 1st M1A1 and 4th A1
MR	Consistent MR of 11 for 9 leading to *a* = 3, *d* = 0.8 scores M1A0M1A0M1A1ftA1ft

Question number	Scheme	Marks
8. (a)	$b^2 - 4ac = 4p^2 - 4(3p+4) = 4p^2 - 12p - 16\ (=0)$	M1, A1
	or $(x+p)^2 - p^2 + (3p+4) = 0 \Rightarrow p^2 - 3p - 4 (=0)$	
	$(p - 4)(p + 1) = 0$	M1
	p = (-1 or) 4	A1c.s.o. (4)
(b)	$x = \frac{-b}{2a}$ or $(x+p)(x+p) = 0 \Rightarrow x = ...$	M1
	x (= -p) = <u>- 4</u>	A1f.t. (2)
		6
(a)	1st M1 — For use of $b^2 - 4ac$ or a full attempt to complete the square leading to a 3TQ in p. May use $b^2 = 4ac$. One of b or c must be correct.	
	1st A1 — For a correct 3TQ in p. Condone missing "=0" but all 3 terms must be on one side.	
	2nd M1 — For attempt to solve their 3TQ leading to p = …	
	2nd A1 — For p = 4 (ignore p = -1). $b^2 = 4ac$ leading to $p^2 = 4(3p+4)$ and then "spotting" $p = 4$ scores 4/4.	
(b)	M1 — For a full method leading to a repeated root x = …	
	A1f.t. — For x = -4 (- their p)	
	<u>Trial and Improvement</u>	
	M2 — For substituting values of p into the equation and attempting to factorize. (Really need to get to p = 4 or -1)	
	A2c.s.o. — Achieve p = 4. Don't give without valid method being seen.	

Question number	Scheme	Marks
9. (a)	$f(x) = x[(x-6)(x-2)+3]$ or $x^3-6x^2-2x^2+12x+3x = x($	M1
	$f(x) = x(x^2-8x+15)$ — b= - 8 or c = 15	A1
	both and a = 1	A1 (3)
(b)	$(x^2-8x+15) = (x-5)(x-3)$	M1
	$f(x) = x(x-5)(x-3)$	A1 (2)
(c)	 Shape	B1
	their 3 or their 5	B1f.t.
	both their 3 and their 5 and (0,0) by implication	B1f.t. (3)
		8
(a)	M1 for a correct method to get the factor of x. $x($ as printed is the minimum. 1st A1 for b = -8 or c = 15. -8 comes from -6-2 and must be coefficient of x, and 15 from 6x2+3 and must have no xs. 2nd A1 for a =1, b = -8 and c = 15. Must have $x(x^2-8x+15)$.	
(b)	M1 for attempt to factorise their 3TQ from part (a). A1 for all 3 terms correct. They must include the x. For part (c) they must have at most 2 non-zero roots of their f(x) =0 to ft their 3 and their 5.	
(c)	1st B1 for correct shape (i.e. from bottom left to top right and two turning points.) 2nd B1f.t. for crossing at their 3 or their 5 indicated on graph or in text. 3rd B1f.t. if graph passes through (0, 0) [needn't be marked] and both their 3 and their 5.	

Question number	Scheme		Marks
10.(a)	$f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1} (+c)$	$-\frac{3}{x}$ is OK	M1A1
	$(3, 7\frac{1}{2})$ gives $\frac{15}{2} = 9 - \frac{3}{3} + c$	3^2 or 3^{-1} are OK instead of 9 or $\frac{1}{3}$	M1A1f.t.
	$c = -\frac{1}{2}$		A1 (5)
(b)	$f(-2) = 4 + \frac{3}{2} - \frac{1}{2}$ (*)		B1c.s.o. (1)
(c)	$m = -4 + \frac{3}{4}$, $= -3.25$		M1,A1
	Equation of tangent is: $y - 5 = -3.25(x + 2)$		M1
	<u>$4y + 13x + 6 = 0$</u>	o.e.	A1 (4)
			10

(a)	1st M1	for some attempt to integrate $x^n \to x^{n+1}$
	1st A1	for both x terms as printed or better. Ignore $(+c)$ here.
	2nd M1	for use of $(3, 7\frac{1}{2})$ or (-2, 5) to form an equation for c. There must be some correct substitution. No $+c$ is M0. Some changes in x terms of function needed.
	2nd A1f.t.	for a correct equation for c. Follow through their integration. They must tidy up fraction/fraction and signs (e.g. - - to +).
(b)	B1cso	If (-2, 5) is used to find c in (a) B0 here unless they verify f(3)=7.5.
(c)	1st M1	for attempting $m = f'(\pm 2)$
	1st A1	for $-\frac{13}{4}$ or -3.25
	2nd M1	for attempting equation of tangent at (-2, 5), f.t. their m, based on $\frac{dy}{dx}$.
	2nd A1	o.e. must have a, b and c integers and = 0.
		Treat (a) and (b) together as a batch of 6 marks.

Question number	Scheme	Marks
11.(a)	$m = \frac{8-2}{11+1}$ $(= \frac{1}{2})$	M1 A1
	$y - 2 = \frac{1}{2}(x - -1)$ or $y - 8 = \frac{1}{2}(x - 11)$ o.e.	M1
	$y = \frac{1}{2}x + \frac{5}{2}$ accept exact equivalents e.g. $\frac{6}{12}$	A1c.a.o. (4)
(b)	Gradient of $l_2 = -2$	M1
	Equation of l_2: $y - 0 = -2(x - 10)$ [$y = -2x + 20$]	M1
	$\frac{1}{2}x + \frac{5}{2} = -2x + 20$	M1
	$\underline{x = 7 \text{ and } y = 6}$ depend on all 3 Ms	A1, A1 (5)
(c)	$RS^2 = (10-7)^2 + (0-6)^2 (= 3^2 + 6^2)$	M1
	$RS = \sqrt{45} = 3\sqrt{5}$ (*)	A1c.s.o. (2)
(d)	$PQ = \sqrt{12^2 + 6^2}, = 6\sqrt{5}$ or $\sqrt{180}$ or $PS = 4\sqrt{5}$ and $SQ = 2\sqrt{5}$	M1,A1
	Area = $\frac{1}{2}PQ \times RS = \frac{1}{2}6\sqrt{5} \times 3\sqrt{5}$	dM1
	$\underline{= 45}$	A1 c.a.o. (4)
		15
(a)	1st M1 for attempting $\frac{y_1 - y_2}{x_1 - x_2}$, must be y over x. No formula condone one sign slip, but if formula is quoted then there must be some correct substitution. 1st A1 for a fully correct expression, needn't be simplified. 2nd M1 for attempting to find equation of l_1.	
(b)	1st M1 for using the perpendicular gradient rule 2nd M1 for attempting to find equation of l_2. Follow their gradient provided different. 3rd M1 for forming a suitable equation to find S.	
(c)	M1 for expression for RS or RS^2. Ft their S coordinates	
(d)	1st M1 for expression for PQ or PQ^2. $PQ^2 = 12^2 + 6^2$ is M1 but $PQ = 12^2 + 6^2$ is M0 Allow one numerical slip. 2nd dM1 for a full, correct attempt at area of triangle. Dependent on previous M1.	

GENERAL PRINCIPLES FOR C1 MARKING

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values. There must be some correct substitution.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

A misread must be consistent for the whole question to be interpreted as such.

These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.