C1 m

1) $10 + x^2 > x(x - 2)$

$10 + x^2 > x^2 - 2x$

$10 > -2x$

$\frac{10}{-2} < x$ ← flip inequality because $\div (-)$

$-5 < x$

2) $\int x^2 - \frac{1}{x^2} + \sqrt[3]{x} \, dx$

$= \int x^2 - x^{-2} + x^{\frac{1}{3}} \, dx$

$= \frac{1}{3}x^3 + x^{-1} + \frac{3}{4}x^{\frac{4}{3}} + c$

3) a) $81^{\frac{1}{2}} = \sqrt{81} = 9$

b) $81^{\frac{3}{4}} = (\sqrt[4]{81})^3 = 3^3 = 27$

c) $81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}} = \frac{1}{27}$

4) $a_1 = k \qquad a_{n+1} = 4a_n - 7$

a) $a_2 = a_{1+1} = 4a_1 - 7 = \underline{4k - 7}$

b) $a_3 = a_{2+1} = 4a_2 - 7 = 4(4k - 7) - 7$

$= 16k - 28 - 7$

$= \underline{\underline{16k - 35}}$

c) $a_3 = 13 \Rightarrow 16k - 35 = 13$

$16k = 48$

$k = \frac{48}{16} = \frac{24}{8} = \frac{12}{4} = \frac{6}{2} = \underline{\underline{3}}$

5a) $2x + y = 8 \Rightarrow y = 8 - 2x \quad \ldots ②$

$3x^2 + xy = 1 \quad \ldots ①$

sub ② into ①

$3x^2 + x(8 - 2x) = 1$

$3x^2 + 8x - 2x^2 = 1$

$x^2 + 8x - 1 = 0$

b) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{64 - 4(-1)}}{2} = \frac{-8 \pm \sqrt{68}}{2}$

$\frac{-8 \pm \sqrt{4}\sqrt{17}}{2} = \frac{-8 \pm 2\sqrt{17}}{2} = -4 \pm \sqrt{17}$

$y = 8 - 2x$

when $x = -4 + \sqrt{17}$

$y = 8 - 2(-4 + \sqrt{17})$

$y = 8 + 8 - 2\sqrt{17}$

$y = 16 - 2\sqrt{17}$

when $x = -4 - \sqrt{17}$

$y = 8 - 2(-4 - \sqrt{17})$

$y = 8 + 8 + 2\sqrt{17}$

$y = 16 + 2\sqrt{17}$

$x = -4 \pm \sqrt{17}$

$y = 16 \mp 2\sqrt{17}$

6a)

$$f(x) = \frac{(2x+1)(x+4)}{\sqrt{x}} = \frac{2x^2+9x+4}{x^{1/2}}$$

$$= 2x^{(2-\frac{1}{2})} + 9x^{(1-\frac{1}{2})} + 4x^{-1/2}$$

$$= 2x^{3/2} + 9x^{1/2} + 4x^{-1/2}$$

$\underline{P=2}$ $\underline{Q=9}$ $\underline{R=4}$

b) $f'(x) = 2\left(\frac{3}{2}\right)x^{1/2} + 9\left(\frac{1}{2}\right)x^{-1/2} - 4\left(\frac{1}{2}\right)x^{-3/2}$

$$= 3x^{1/2} + \frac{9}{2}x^{-1/2} - 2x^{-3/2}$$

c) $f'(x) = \frac{dy}{dx}$ when $x=1$ $\frac{dy}{dx} = 3 + \frac{9}{2} - 2$

$$= 1 + \frac{9}{2} = \frac{11}{2}$$

$2y = 11x + 3$

$y = \frac{11}{2}x + \frac{3}{2}$

$\frac{dy}{dx} = \frac{11}{2}$ equal gradients $\therefore$ parallel

7a) $x^3 - 4x$

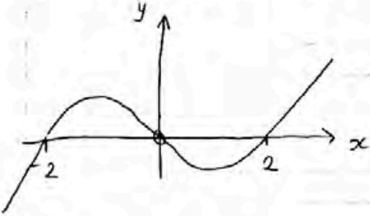
$= x(x^2-4)$

$= x(x+2)(x-2)$

b) $y = x^3 - 4x \quad = x(x+2)(x-2)$

when $\quad y = 0$

$x = 0 \quad x = -2 \quad x = 2$



as $x \to \overset{+VE}{\infty}$

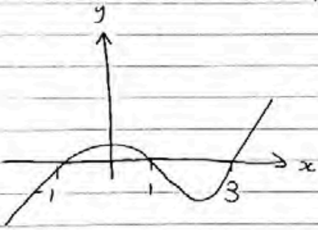
$y = \infty(\infty+2)(\infty-2)$

$y = \to \overset{+VE}{\infty}$

c) $y = (x-1)^3 - 4(x-1)$

is $x^3 - 4x$ with a transformation

+1 in +VE x direction



8 a) $l_1: y = 3x - 6$

$\frac{dy}{dx} = 3$

$(6, 2)$

$\frac{dy}{dx}$ $l_2 : -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{3}(x - 6)$

$y = -\frac{1}{3}x + 2 + 2$

$y = -\frac{1}{3}x + 4$

b) intersect $l_1 = l_2$

$-\frac{1}{3}x + 4 = 3x - 6$

$10 = 3x + \frac{1}{3}x$

$10 = \frac{10}{3}x$

$\frac{30}{10} = x = 3$

when $x = 3$ $y = 3(3) - 6 = 9 - 6 = 3$

$(3, 3)$

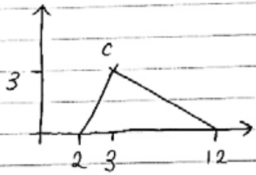
c) l_1 : $y = 3x - 6$

when $y = 0$ $x = 2$

l_2 $y = -\frac{1}{3}x + 4$

when $y = 0$ $x = 12$

$C = (3, 3)$



$\text{Area} = \frac{1}{2} b \times h$

$= \frac{1}{2} 10 \times 3 = 15$

9)

$S_n = (a) + (a+d) \ldots \boxed{a+(n-2)d} + \boxed{a+(n-1)d} \ldots ②$

$S_n = \boxed{a+(n-1)d} + \boxed{a+(n-2)d} \ldots (a+d) + (a) \ldots ①$

① + ②

$2S_n = [2a+(n-1)d] + [2a+(n-1)d] \ldots + [2a+(n-1)d] \ldots$

$2S_n = n[2a+(n-1)d]$

$S_n = \frac{n}{2}[2a+(n-1)d]$

b) longest side = n^{th} value = $a+(n-1)d=6$

$a+(16-1)d=6$

$a+15d=6 \quad \Rightarrow \quad a=6-15d \quad \dots ①$

parameter = $S_n = 72$

$\frac{1}{2}(16)[2a+(16-1)d]=72$

$8[2a+15d]=72$

$2a+15d=9 \quad \dots ②$

$2(6-15d)+15d=9$

$12-30d+15d=9$

$3=15d$

$\boxed{d=\frac{1}{5}} \quad \dots ③$

③ into ① $\quad a=6-15\left(\frac{1}{5}\right)$

$a=6-3$

$\underline{a=3}$

a = length of shortest

10) $\frac{dy}{dx} = x^3 + 2x - 7$

$\frac{d^2y}{dx^2} = 3x^2 + 2$

b) x^2 is always (+) $\therefore$ $x^2 \geq 0$

$\therefore$ $3x^2 \geq 3(0)$

$\therefore$

$3x^2 + 2 \geq 0 + 2$

$\frac{d^2y}{dx^2} \geq 2$

c) $\int \frac{dy}{dx} = y$

$\int x^3 + 2x - 7 \, dx = y$

$= \frac{1}{4}x^4 + x^2 - 7x + c = y$

When $x = 2$ $y = 4$

$\frac{1}{4}(2)^4 + 2^2 - 7(2) + c = (4)$

$4 + 4 - 14 - 4 = -c$

$\underline{c = 10}$

$y = \frac{1}{4}x^4 + x^2 - 7x + 10$

10d) at P $x = 2$

$\frac{dy}{dx}$ at P $= (2)^3 + 2(2) - 7$

$\frac{dy}{dx} = 8 + 4 - 7 = 5$

at normal $\frac{dy}{dx} = -\frac{1}{5}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{5}(x - 2)$

$y - 4 = -\frac{1}{5}x + \frac{2}{5}$

$5y - 20 = -x + 2$

$x + 5y - 22 = 0$