C1 S16 UK

1. Find

$$\int \left(2x^4 - \frac{4}{\sqrt{x}} + 3 \right) \mathrm{d}x$$

giving each term in its simplest form. $4x^{-\frac{1}{2}}$

(4)

$$\frac{2x^5}{5} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c = \frac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c$$

2. Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

(2)

$(3^2)^{(3x+1)} = 3^{6x+2}$ $\therefore$ $y = 6x + 2$

3. (a) Simplify

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

a) $\sqrt{25}\sqrt{2} - \sqrt{9}\sqrt{2} = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

b) $\dfrac{12\sqrt{3}}{2\sqrt{2}} = \dfrac{6\sqrt{3}\sqrt{2}}{\sqrt{2}\sqrt{2}} = 3\sqrt{6}$

4.

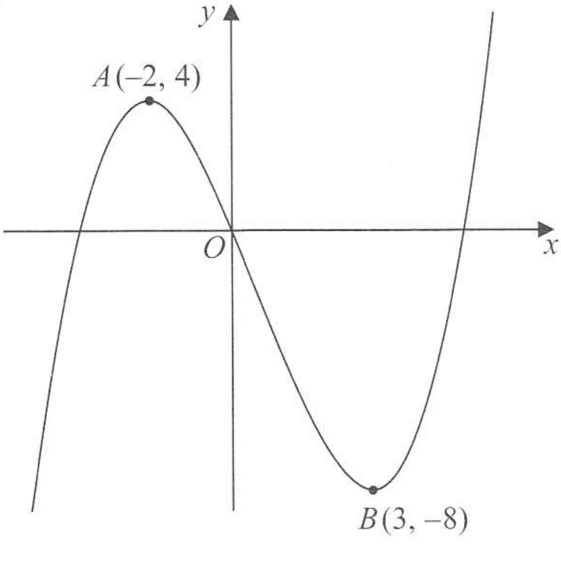


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \text{f}(x)$. The curve has a maximum point A at $(-2, 4)$ and a minimum point B at $(3, -8)$ and passes through the origin O.

On separate diagrams, sketch the curve with equation

(a) $y = 3\text{f}(x)$,

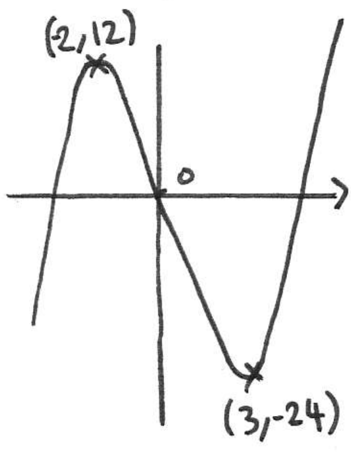
(2)

(b) $y = \text{f}(x) - 4$

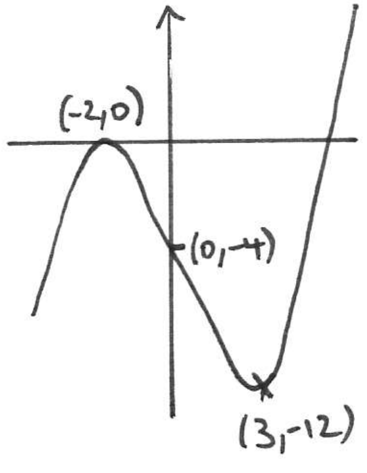
(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.

a) $3f(x)$ $\updownarrow \times 3$



b) $f(x)-4$ $\downarrow 4$



5. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

$y = -4x - 1 \quad \Rightarrow \quad y^2 = (-4x-1)^2 = 16x^2 + 8x + 1$

$(16x^2 + 8x + 1) + 5x^2 + 2x = 0 \quad \Rightarrow \quad 21x^2 + 10x + 1 = 0$

$(7x+1)(3x+1) = 0 \quad \Rightarrow \quad x = -\frac{1}{7}, \; -\frac{1}{3}$

$y = \frac{4}{7} - 1 = \frac{-3}{7} \quad y = \frac{4}{3} - 1 = \frac{1}{3}$

$\left(-\frac{1}{7}, -\frac{3}{7}\right) \; ; \left(-\frac{1}{3}, \frac{1}{3}\right)$

6. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geqslant 1$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k. **(2)**

Find

(b) $\sum_{r=1}^{3}(1 + a_r)$ in terms of k, giving your answer in its simplest form, **(3)**

(c) $\sum_{r=1}^{100}(a_{r+1} + ka_r)$ **(1)**

a) $a_1 = 4 \qquad a_2 = 5 - 4k \qquad a_3 = 5 - k(5-4k)$

$= 5 - 5k + 4k^2$

b) $(1+4) + (1+5-4k) + (1+5-5k+4k^2)$

$= 17 - 9k + 4k^2$

c) $= (a_2 + ka_1) \quad = 5 - \cancel{4k} + \cancel{4k}$

$+ (a_3 + ka_2) \quad = 5 - \cancel{5k} + \cancel{4k^2} + 5k - \cancel{4k^2}$

$+ (a_4 + ka_3) \quad \vdots$

$+$

$\vdots$

$+ (a_{101} + ka_{100})$

$= 5 \times 100 = 500$

2

7. Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0$$

find $\frac{dy}{dx}$. Give each term in your answer in its simplified form.

(6)

$$y' = 6x + 2x^{-\frac{2}{3}} + \frac{5}{3}x^{\frac{3}{2}} + \frac{7}{6}x^{-\frac{3}{2}}$$

$$\frac{2x^3}{3x^{\frac{1}{2}}} - \frac{7}{3\sqrt{x}}$$

$$= \frac{2}{3}x^{\frac{5}{2}} - \frac{7}{3}x^{-\frac{1}{2}}$$

8. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

(a) Show that $4p^2 - 20p + 9 < 0$

(4)

(b) Hence find the set of possible values of p.

(4)

$3x - 7 = 2px^2 - 6px + 4p$

$\Rightarrow 2px^2 - 6px - 3x + 4p + 7 = 2px^2 - (6p+3)x + (4p+7) = 0$

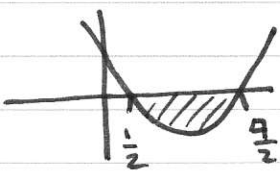
If line does not cross or touch curve $\Rightarrow b^2 - 4ac < 0$

$(6p+3)^2 - 4(2p)(4p+7) < 0$

$(36p^2 + 36p + 9) - 32p^2 - 56p < 0 \Rightarrow 4p^2 - 20p + 9 < 0$

b) $(2p-9)(2p-1) < 0$

$p = \frac{9}{2}$ $p = \frac{1}{2}$



$p > \frac{1}{2}$ and $p < \frac{9}{2}$

(or) $\frac{1}{2} < p < \frac{9}{2}$

9. On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(a) Show that, immediately after his 12th birthday, the total of these gifts was £225 **(1)**

(b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday. **(2)**

(c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday. **(3)**

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375

(d) Show that $n^2 + 7n = 25 \times 18$ **(3)**

(e) Find the value of n, when he had received £3375 in total, and so determine John's age at this time. **(2)**

a) 10^{th} $u_1 = 60$ $\quad S_3 = 60+75+90 = £225$

11^{th} $u_2 = 75$

12^{th} $u_3 = 90$ $\quad a=60 \quad d=15$

-9

b) $18^{th} = u_9 = a+8d = 60+8\times 15 = £180$

c) $21^{st} = u_{12} = a+11d = 60+11\times 15 = £225$

$S_{12} = \frac{n}{2}[a+L] = 6[60+225) = 6\times 285 = £1710$

d) $\frac{n}{2}[2a+(n-1)d] = 3375 \Rightarrow n[120+(n-1)\times 15] = 6750$

$\Rightarrow 120n + 15n^2 - 15n = 6750 \Rightarrow 15n^2 + 105n = 6750$

$\div 15 \Rightarrow n^2 + 7n = 450 \quad \therefore n^2+7n = 25\times 18$ #

c) $n^2+7n-25\times 18 = 0 \Rightarrow (n+25)(n-18) = 0$

$\therefore n = \cancel{-25} \quad \underline{n=18} \quad +9 \quad$ he was 27

10.

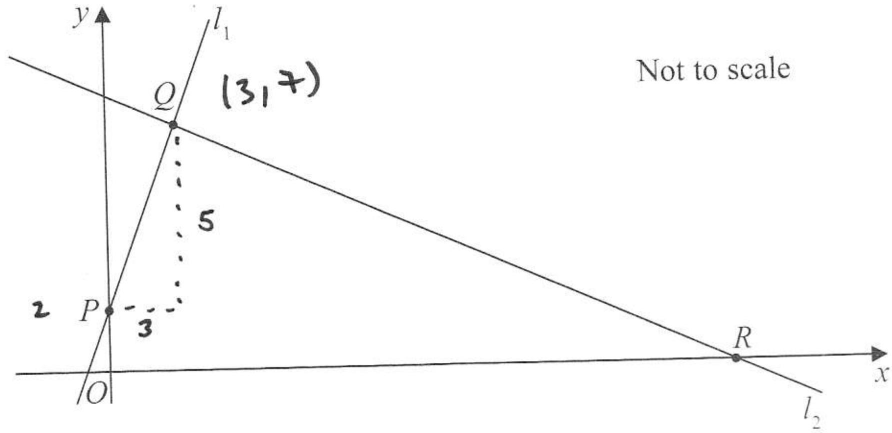


Figure 2

The points P (0, 2) and Q (3, 7) lie on the line l_1, as shown in Figure 2.

The line l_2 is perpendicular to l_1, passes through Q and crosses the x-axis at the point R, as shown in Figure 2.

Find

(a) an equation for l_2, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers, **(5)**

(b) the exact coordinates of R, **(2)**

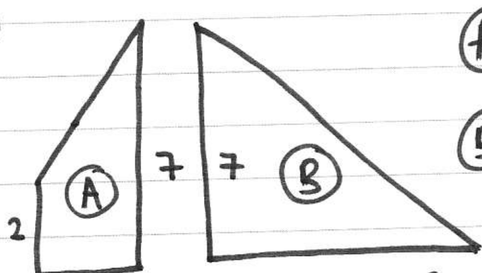
(c) the exact area of the quadrilateral $ORQP$, where O is the origin. **(5)**

a) $m_{l_1} = \frac{5}{3} \therefore m_{l_2} = -\frac{3}{5}$ $\quad y - 7 = -\frac{3}{5}(x-3)$

$\Rightarrow 5y - 35 = -3x + 9 \quad \Rightarrow 3x + 5y - 44 = 0$

b) $y = 0 \Rightarrow 3x = 44 \quad \therefore x = \frac{44}{3} \quad R\left(\frac{44}{3}, 0\right)$

c)



$\frac{44}{3} - 3 = \frac{35}{3}$

$A = \frac{(2+7)}{2} \times 3 = \frac{27}{2} = \frac{81}{6}$

$B = \frac{35}{6} \times 7 = \frac{245}{6}$

$\therefore \text{Area} = \frac{326}{6} = \frac{163}{3}$

11. The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{dy}{dx}$ **(2)**

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of k, **(4)**

(c) the value of the y coordinate of P, **(2)**

(d) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(2)**

a) $y' = 6x^2 + 2kx + 5$ $\quad$ $2y = 17x + 1$

$y = \frac{17}{2}x + \frac{1}{2}$

b) $\therefore M_t = \frac{17}{2}$ as parallel when $x = -2$.

$6x^2 + 2kx + 5 = \frac{17}{2}$ $\Rightarrow$ $12x^2 + 4kx + 10 = 17$

$x = -2 \Rightarrow 48 - 8k = 7$ $\Rightarrow 8k = 41$ $\therefore k = \frac{41}{8}$

c) $y = 2(-2)^3 + \frac{41}{8}(-2)^2 + 5(-2) + 6$

$y = -16 + \frac{41}{2} - 10 + 6 = \frac{41}{2} - 20 = \frac{41}{2} - \frac{40}{2} = \frac{1}{2}$ $\quad P(-2, \frac{1}{2})$

d) $y - \frac{1}{2} = \frac{17}{2}(x+2)$

$2y - 1 = 17x + 34$ $\therefore$ $17x - 2y + 35 = 0$