

June 2013 MA - C1

1)

$$\frac{7 + \sqrt{5}}{\sqrt{5} - 1}$$

	$\sqrt{5}$	1
$\sqrt{5}$	5	$\sqrt{5}$
7	$7\sqrt{5}$	7

$$\frac{7 + \sqrt{5}}{\sqrt{5} - 1} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1} = \frac{5 + 7 + 7\sqrt{5} + \sqrt{5}}{\sqrt{5}^2 - 1^2}$$

Use difference of 2 squares to find denominator

$$= \frac{12 + 8\sqrt{5}}{4}$$

$$= 3 + 2\sqrt{5}$$

2)

$$\int (10x^4 - 4x - 3x^{\frac{1}{2}}) dx$$

$$= 10 \frac{x^5}{5} - 4 \frac{x^2}{2} - 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= 2x^5 - 2x^2 - 6\sqrt{x} + C$$

3)

$$a) \quad 8^{\frac{5}{3}} = (\sqrt[3]{8})^5$$

$$= 2^5$$

$$= 32$$

b)

$$\frac{(2x^{\frac{1}{2}})^3}{4x^2} = \frac{2^3 x^{\frac{3}{2}}}{2^2 x^2}$$

$$= 2^{3-2} x^{\frac{3}{2}-2}$$

$$= 2x^{-\frac{1}{2}}$$

$$= \frac{2}{\sqrt{x}}$$

$$= \frac{2\sqrt{x}}{x}$$

4)

$$a_1 = 4$$

a)

$$\begin{aligned} a_2 &= k(4+2) \\ &= 6k \end{aligned}$$

b)

$$\begin{aligned} a_3 &= k(6k+2) \\ &= 6k^2 + 2k \end{aligned}$$

$$\sum [a_1, a_2, a_3] = 2$$

$$4 + 6k + (6k^2 + 2k) = 2$$

$$6k^2 + 8k + 4 = 2$$

$$3k^2 + 4k + 2 = 1$$

$$3k^2 + 4k + 1 = 0$$

$$\frac{-4 \pm \sqrt{16 - 12}}{6} = \frac{-4 \pm 2}{6}$$

$$k = -\frac{1}{3}, -1$$

5)

a) $2(3x+4) > 1-x$

$6x + 8 > 1-x$

$7x > -7$

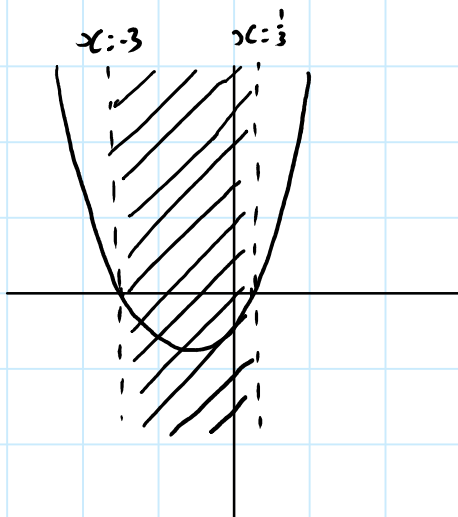
$x > -1$

b) $3x^2 + 8x - 3 < 0$

$$\frac{-8 \pm \sqrt{64 - 36}}{6} = \frac{-8 \pm 10}{6}$$

$$x = \frac{1}{3}, -3$$

$$(3x-1)(x+3) < 0$$



$$-3 < x < \frac{1}{3}$$

6)

Gradient of the line

a)

$$\frac{\Delta y}{\Delta x} = \frac{12-3}{11-1}$$

$$= \frac{9}{10}$$

$$= \frac{3}{4}$$

Using A as (x, y)

$$y-3 = \frac{3}{4}(x-1)$$

$$4y-12 = 3x+3$$

$$3x-4y+15 = 0$$

$$a=3, b=-4, c=15$$

6)

b)

$$3x - 4y + 15 = 0, \quad 3y + 4x - 30 = 0$$

$$9x - 12y + 45 = 0, \quad 12y + 16x - 120 = 0$$

$$12y = 9x + 45 \quad \therefore (9x + 45) + 16x - 120 = 0$$

$$25x - 75 = 0$$

$$x - 3 = 0$$

$$x = 3$$

$$3(3) - 4y + 15 = 0$$

$$4y = 24$$

$$y = 6$$

$$(3, 6)$$

7)

 $n = \text{Week No.}$

a)

$$\therefore \text{Phones per Week } n = 200 + (n-1)20$$

$$600 = 200 + (N-1)20$$

$$400 = 20N - 20$$

$$20 = N - 1$$

$$N = 21$$

b)

$$S_{21} = \frac{1}{2} \times 21 (200 + 600)$$

$$= 400 \times 21$$

$$= 8400$$

$$\begin{array}{r} 21 \\ \times 400 \\ \hline 8400 \end{array}$$

$$600 \times (52 - 21) = 600 \times 31$$

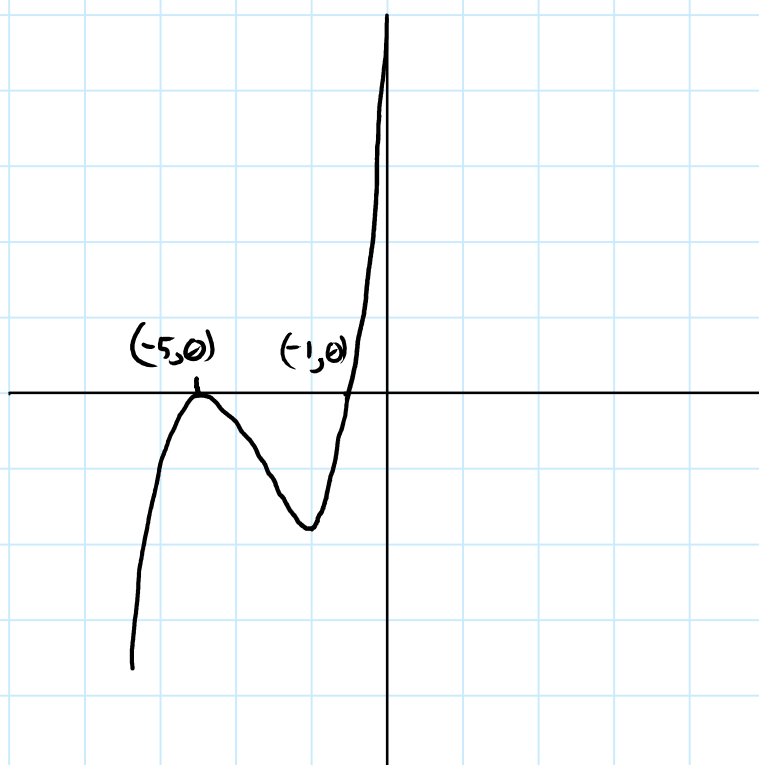
$$= 18600$$

$$\begin{array}{r} 31 \\ \times 600 \\ \hline 8600 \\ + 10000 \\ \hline 18600 \end{array}$$

$$8400 + 18600 = 27000 \text{ phones}$$

g)

a)



b)

$$f(x+2) = (x+2+3)^2(x+2-1)$$
$$= (x+5)^2(x+1)$$

c)

$$5^2 \times 1 = 25$$

$$y = 25 \text{ when } x = 0$$

9)

a)

$$f'(x) = \frac{(3-x^2)^2}{x^2}$$

$$= \frac{3^2 + 2(3)(-x^2) + (-x^2)^2}{x^2}$$

$$= \frac{9}{x^2} - \frac{6x^2}{x^2} + \frac{x^4}{x^2}$$

$$= 9x^{-2} - 6 + x^2$$

$$A = -6, B = 1$$

b)

$$f''(x) = \frac{d}{dx} [9x^{-2} - 6 + x^2]$$

$$= -18x^{-3} - 0 + 2x$$

$$= 2x - 18x^{-3}$$

9)

$$\begin{aligned} \text{c) } \int f'(x) dx &= \int 4x^{-2} - 6 + x^2 dx \\ &= \left[4 \frac{x^{-1}}{-1} - 6 \frac{x^1}{1} + \frac{x^3}{3} \right] + C \\ f(x) &= -4x^{-1} - 6x + \frac{x^3}{3} + C \end{aligned}$$

given $f(-3) = 10$

$$\frac{-4}{-3} - 6(-3) + \frac{-3^3}{3} + C = 10$$

$$3 + 18 - 9 + C = 10$$

$$C = -2$$

$$\therefore f(x) = -4x^{-1} - 6x + \frac{1}{3}x^3 - 2$$

10)

$$2x + y = 1$$

$$x^2 - 4ky + 5k = 0$$

a)

$$y = 1 - 2x$$

$$\therefore x^2 - 4k[1 - 2x] + 5k = 0$$

$$x^2 + 8kx + k = 0$$

b)

if $ax^2 + bx + c$ has equal roots

$$b^2 - 4ac = 0$$

$$(8k)^2 - 4k = 0$$

$$16k^2 - k = 0$$

$k \neq 0$ given in question

$$\therefore k = \frac{1}{16}$$

$$10) \quad x^2 + \frac{1}{2}x + \frac{1}{16} = 0$$

$$c) \quad 16x^2 + 8x + 1 = 0$$

$$\frac{-8 \pm \sqrt{64 - 64}}{32} = \frac{-8}{32}$$

$$x = -\frac{1}{4}$$

$$y = 1 - 2x$$

$$y = 1 - 2\left(-\frac{1}{4}\right) \\ = \frac{3}{2}$$

$$x = -\frac{1}{4}, \quad y = \frac{3}{2}$$

11)

$$a) \quad \theta = \frac{3}{x} + 4$$

$$\theta = 3 + 4x$$

$$\frac{-3}{4} = x$$

$$b) \quad y = \frac{3}{(x+\theta)} + 4$$

$$\therefore x = \theta, \quad y = 4$$

$$c) \quad \frac{dy}{dx} = 3(-1)x^{-2} + 4(\theta)$$

$$= \frac{-3}{x^2}$$

$$\text{gradient at P} \quad \frac{-3}{(-3)^2}$$

$$= -\frac{1}{3}$$

$$\text{gradient of Normal} \quad +3$$

$$\therefore 3 \times \frac{-1}{3} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 3(x - -3)$$

$$y - 3 = 3x + 9$$

$$y = 3x + 12$$

11)

d)

$$y = 3x + 12$$

y intercept:

$$y = 3(0) + 12$$

$$y = 12$$

x intercept:

$$0 = 3x + 12$$

$$-12 = 3x$$

$$x = -4$$

OR

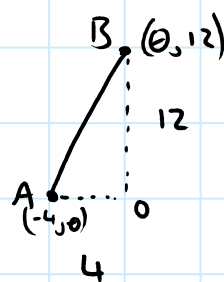
$$\begin{aligned} \vec{AB} &= \begin{pmatrix} 0 \\ 12 \end{pmatrix} - \begin{pmatrix} -4 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ 12 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 4 \\ 12 \end{vmatrix} &= \sqrt{12^2 + 4^2} \\ &= \sqrt{(3 \times 4)^2 + 4^2} \\ &= \sqrt{4^2 \sqrt{3^2 + 1^2}} \\ &= 4\sqrt{10} \end{aligned}$$

2 points are:

$$B(0, 12)$$

$$A(-4, 0)$$



$$\begin{aligned} |AB| &= \sqrt{4^2 + 12^2} \\ &= \sqrt{16 + 144} \\ &= \sqrt{160} \\ &= 4\sqrt{10} \end{aligned}$$