C1 June 2012 (MA)

Q1) $\int\left(6x^2+\frac{2}{x^2}+5\right)dx = \int\left(6x^2+2x^{-2}+5\right)dx$

$= \frac{6x^3}{3} + \frac{2x^{-1}}{-1} + 5x + c$

$= 2x^3 - 2x^{-1} + 5x + c$

$= \boxed{2x^3 - \frac{2}{x} + 5x + c}$

Q2a) $(32)^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = 2^3 \boxed{= 8}$

b) $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$

$= \frac{1}{\left(\frac{5x^2}{2}\right)}$

$\boxed{= \frac{2}{5x^2}}$

3) $$\frac{2}{\sqrt{12}-\sqrt{8}} = \frac{2\times(\sqrt{12}+\sqrt{8})}{(\sqrt{12}-\sqrt{8})(\sqrt{12}+\sqrt{8})}$$

$$= \frac{2\sqrt{12}+2\sqrt{8}}{12+\sqrt{12}\sqrt{8}-\sqrt{12}\sqrt{8}-8}$$

$$= \frac{2\sqrt{12}+2\sqrt{8}}{4}$$

$$= \frac{2\sqrt{12}+2\sqrt{8}}{\sqrt{16}}$$

$$= \frac{2\sqrt{4}\sqrt{3}+2\sqrt{4}\sqrt{2}}{\sqrt{4}\sqrt{4}}$$

$$= \frac{4\sqrt{3}+4\sqrt{2}}{4}$$

$$\boxed{=\sqrt{3}+\sqrt{2}}$$

Q4) $y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$

a) $$\frac{dy}{dx} = 15x^2 - 8x^{\frac{1}{3}} + 2$$

$$= \boxed{15x^2 - 8\sqrt[3]{x} + 2}$$

b) $$\frac{d^2y}{dx^2} = 30x - \frac{8x^{-\frac{2}{3}}}{3}$$

$$= \boxed{30x - \frac{8}{3x^{2/3}}}$$

Q5) $a_1 = 3$

$a_{n+1} = 2a_n - c \quad (n \geq 1)$

a) $a_2 = 2a_1 - c$

$\boxed{a_2 = 6 - c}$

b) $a_3 = 2a_2 - c$

$a_3 = 2(6-c) - c$

$a_3 = 12 - 2c - c$

$\boxed{a_3 = 12 - 3c}$

c) $\sum_{i=1}^{4} a_i \geq 23$

$\sum_{i=1}^{4} a_i = a_1 + a_2 + a_3 + a_4$

$a_4 = 2a_3 - c$

$a_4 = 2(12 - 3c) - c$

$a_4 = 24 - 6c - c$

$\underline{a_4 = 24 - 7c}$

$\therefore \; 3 + (6-c) + (12-3c) + (24-7c) \geq 23$

$3 + 6 - c + 12 - 3c + 24 - 7c \geq 23$

$45 - 11c \geq 23$

$$22 - 11c \geq 0$$

$$22 \geq 0 + 11c$$

$$22 \geq 11c$$

$$11c \leq 22$$

$$\therefore \boxed{c \leq 2}$$

Q6) First term, $a = 10p$
common difference, $d = 5p$

a) $U_n = a + (n-1)d$

$$U_{15} = 10 + (15-1)5$$

$$U_{15} = 10 + 70$$

$$\boxed{U_{15} = 80p}$$

b) $S_n = \frac{n}{2}(2a + (n-1)d)$

$$S_{60} = \frac{60}{2}(2(10) + (60-1)5)$$

$$S_{60} = 30(20 + 295)$$

$$S_{60} = 30 \times 315$$

$$\boxed{S_{60} = 9450p}$$

$\therefore$ S_{60} (total savings) are £94.50

c) For the sister, First term, $a = 10p$
Common difference, $d = 10p$

$$S_m = \frac{m}{2}(2a + (m-1)d)$$

$$6300 = \frac{m}{2}(2(10) + (m-1)10)$$

$$6300 = \frac{m}{2}(20 + 10m - 10)$$

$$6300 = \frac{m}{2}(10 + 10m)$$

$$12600 = m(10 + 10m)$$

$$12600 = 10m + 10m^2$$

$$1260 = m + m^2$$

$$m(m+1) = 1260$$

$$\therefore \boxed{m(m+1) = 35 \times 36}$$

d) $\boxed{m = 35}$

Q7) $f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$

when $x = 4$, $f'(x) = \frac{1}{2}(4) - \frac{6}{\sqrt{4}} + 3$

$= 2 - 3 + 3$

$= 2$

$\therefore$ the gradient of the tangent to $y = f(x)$ at point $P(4, -1)$ is 2

Equation of tangent: $y - y_1 = m(x - x_1)$

using $P(4, -1)$ and $m = 2$ $\longrightarrow$ $y - (-1) = 2(x - 4)$

$y + 1 = 2x - 8$

$\boxed{y = 2x - 9}$

b) $f'(x) = \frac{1}{2}x - 6x^{-\frac{1}{2}} + 3$

$f(x) = \int \left(\frac{1}{2}x - 6x^{-\frac{1}{2}} + 3 \right) dx$

$f(x) = \frac{1}{2} \cdot \frac{x^2}{2} - \frac{6x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c$

$f(x) = \frac{x^2}{4} - 12\sqrt{x} + 3x + c$

Since $y=f(x)$ passes through $(4,-1)$,
Substitute in $x=4$ and $y=-1$

$$-1 = \frac{4^2}{4} - 12\sqrt{4} + 3(4) + c$$

$$-1 = 4 - 24 + 12 + c$$

$$-1 = -8 + c$$

$$\therefore \underline{c = 7}$$

$$\boxed{f(x) = \frac{x^2}{4} - 12\sqrt{x} + 3x + 7}$$

Q8a) $4x-5-x^2 \equiv -(x^2-4x+5)$

$$= -[(x-2)^2 - 2\cancel{2}+5]$$

$$= -[(x-2)^2 + 1]$$

$$= \boxed{-1 - (x-2)^2}$$

b) $b^2-4ac = (4)^2 - (4)(-1)(-5)$

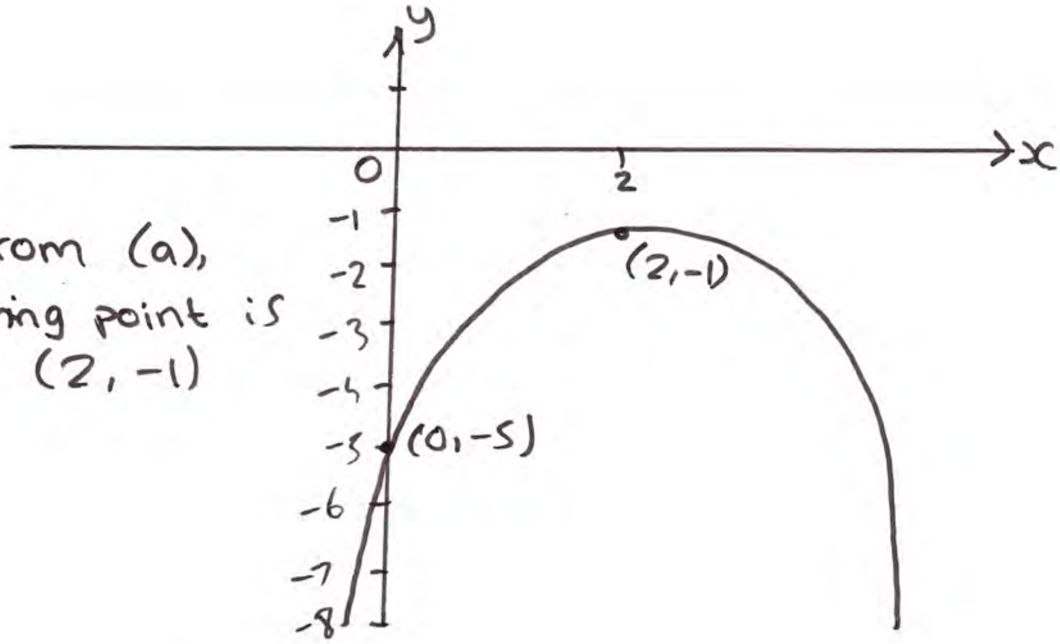
$$b^2-4ac = 16 - 20$$

$$\boxed{b^2-4ac = -4}$$

c) $y = 4x-5-x^2$

when $\underline{x=0, y=-5}$

From (a), turning point is at (2, −1)



Q9) $L_1 : 4y + 3 = 2x$

a) When $y = 4$, $4(4) + 3 = 2x$

$$2(8) + 3 = 2x$$

$$19 = 2x$$

$$x = \frac{19}{2}$$

$$\therefore \boxed{p = \frac{19}{2}}$$

b) $4y + 3 = 2x$

$$4y = 2x - 3$$

$$y = \frac{1}{2}x - \frac{3}{4}$$

Gradient of L_1 is $\frac{1}{2}$

If the line L_2 is perpendicular to L_1, then its gradient is $\underline{-2.}$

Equation of L_2: $y-y_1=m(x-x_1)$

Using point C (2, 4) and $m=-2$ $\rightarrow y-4=-2(x-2)$

$$y-4=-2x+4$$

$$\boxed{2x+y-8=0}$$

c) To find point of intersection, solve simultaneous equations:

$$y=\frac{1}{2}x-\frac{3}{4} \quad (1)$$

$$2x+y-8=0 \quad (2)$$

Substitute (1) into (2):

$$2x+\left(\frac{1}{2}x-\frac{3}{4}\right)-8=0$$

$$2x+\frac{1}{2}x-\frac{3}{4}-8=0$$

$$\frac{5}{2}x=\frac{35}{4}$$

$$5x=\frac{35}{2}$$

$$\underline{x=\frac{7}{2}}$$

Substitute into ① for y:

$$y = \frac{1}{2}x - \frac{3}{4}$$

$$y = \frac{1}{2}\left(\frac{7}{2}\right) - \frac{3}{4}$$

$$y = \frac{7}{4} - \frac{3}{4}$$

$$\underline{y = 1}$$

∴ the coordinates of D are $\boxed{\left(\frac{7}{2}, 1\right)}$

d) Length $CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{\left(\frac{7}{2} - 2\right)^2 + (1 - 4)^2}$$

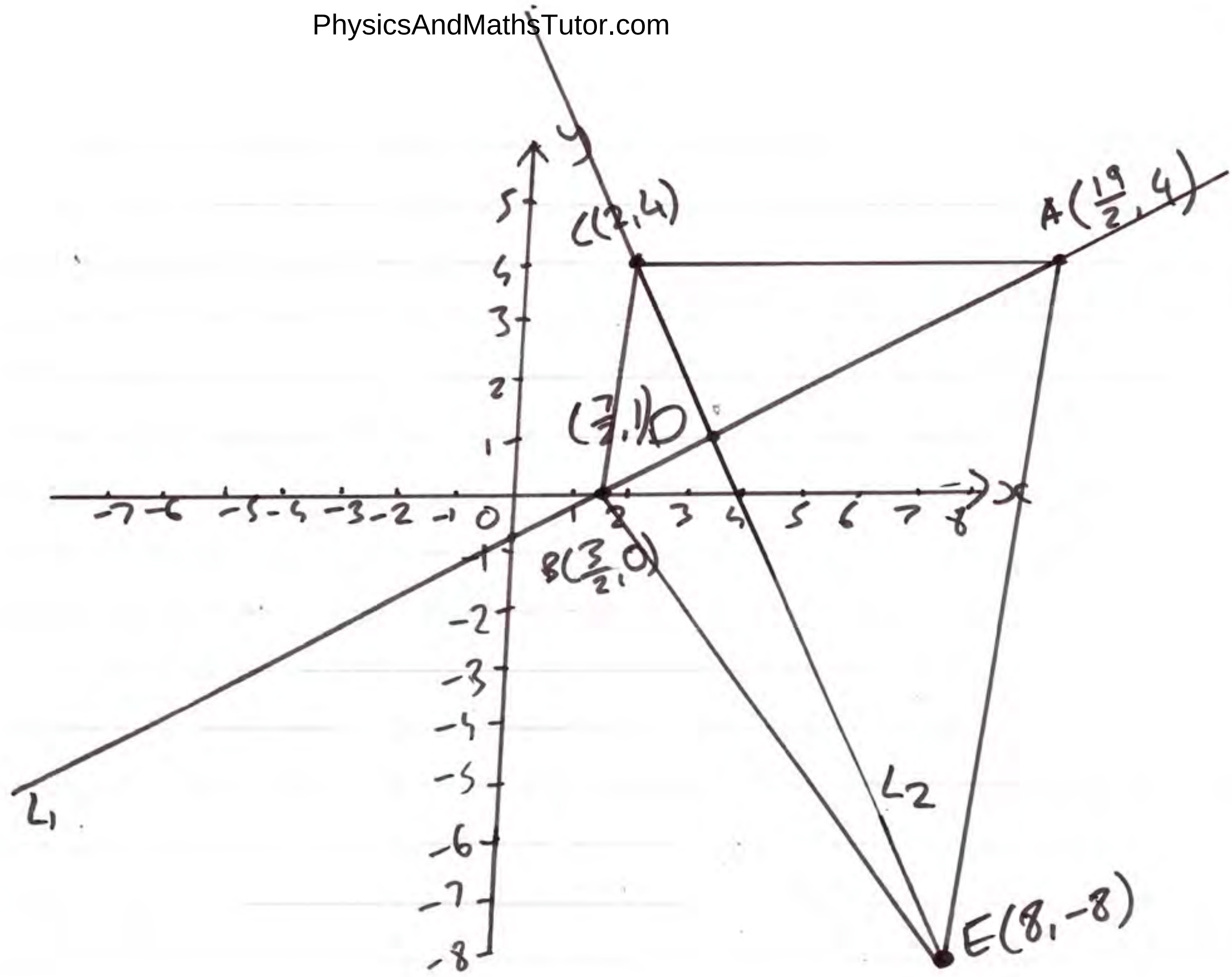
$$= \sqrt{\left(\frac{3}{2}\right)^2 + (-3)^2}$$

$$= \sqrt{\frac{9}{4} + 9}$$

$$= \sqrt{\frac{45}{4}} = \frac{\sqrt{45}}{2} = \frac{\sqrt{9}\sqrt{5}}{2}$$

$$= \boxed{\frac{3\sqrt{5}}{2} \text{ units}}$$

e)



If $CDE = 3 \times CD$, then $CDE = 3 \times \left(\frac{3}{2}\sqrt{5}\right)$

$= \frac{9}{2}\sqrt{5}$ units

$\overrightarrow{CD} = \begin{pmatrix} \frac{7}{2} - 2 \\ 1 - 4 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ -3 \end{pmatrix}$

$\therefore \overrightarrow{DE} = 3\begin{pmatrix} \frac{3}{2} \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ -9 \end{pmatrix}$

$\therefore$ E is at $\begin{pmatrix} \frac{7}{2} + \frac{9}{2} \\ 1 - 9 \end{pmatrix}$

$\therefore$ E is at $(8, -8)$

Length $AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{\left(\frac{19}{2}-\frac{7}{2}\right)^2+(4-1)^2}$

$= \sqrt{6^2+3^2}$

$= \sqrt{45} \quad = \sqrt{9\times 5} = \sqrt{9}\sqrt{5}$

$\underline{= 3\sqrt{5} \text{ units}}$

Length $AB = \sqrt{80} \quad = \sqrt{16\times 5} = \sqrt{16}\sqrt{5}$

$\underline{= 4\sqrt{5} \text{ units}}$

$\therefore$ $\underline{\text{Length } BD = \sqrt{5} \text{ units}}$

$\therefore AD = 3BD$

$\overrightarrow{AD} = \begin{pmatrix} \frac{7}{2}-\frac{19}{2} \\ 1-4 \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$

$\therefore \overrightarrow{DB} = \frac{1}{3}\begin{pmatrix} -6 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$\therefore$ B is at $\begin{pmatrix} \frac{7}{2}-2 \\ 1-1 \end{pmatrix}$

$\therefore$ B is at $\underline{\left(\frac{3}{2}, 0\right)}$

Area ACBE = Area Δ ABC + Area Δ ABE

$= \frac{1}{2}(AB)(CD) + \frac{1}{2}(AB)(DE)$

$= \frac{1}{2}(4\sqrt{5})\left(\frac{3}{2}\sqrt{5}\right) + \frac{1}{2}(4\sqrt{5})\left(\frac{9}{2}\sqrt{5} - \frac{3}{2}\sqrt{5}\right)$

$= \frac{1}{2}(6 \times 5) + \frac{1}{2}(4\sqrt{5})(3\sqrt{5})$

$= \frac{1}{2}(30) + \frac{1}{2}(60)$

$= 15 + 30$

$= \boxed{45 \text{ units}^2}$

Q10a) $f(x) = x^2(9-2x)$

When the curve cuts the x-axis, $f(x)=0$

$\therefore x^2(9-2x)=0$

Either $x^2=0$ or $9-2x=0$

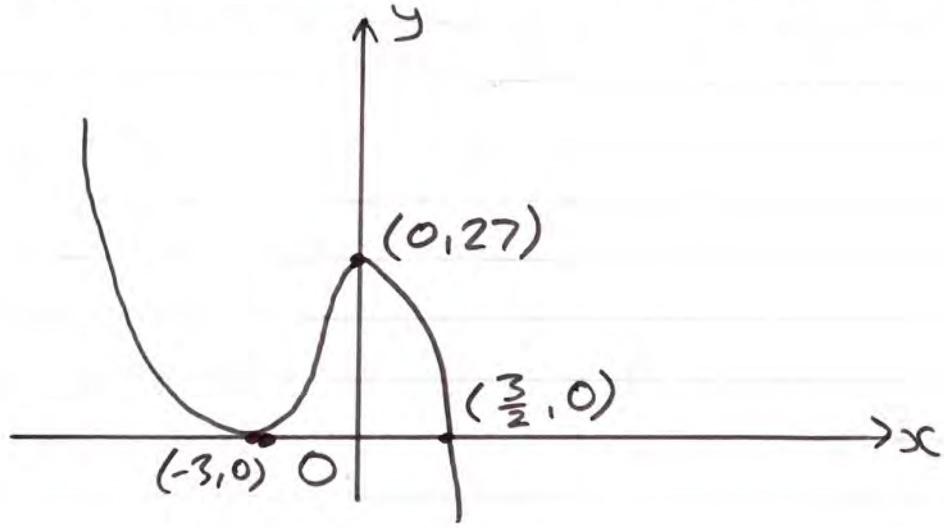
$x^2=0$ is the origin, O

$9-2x=0$,

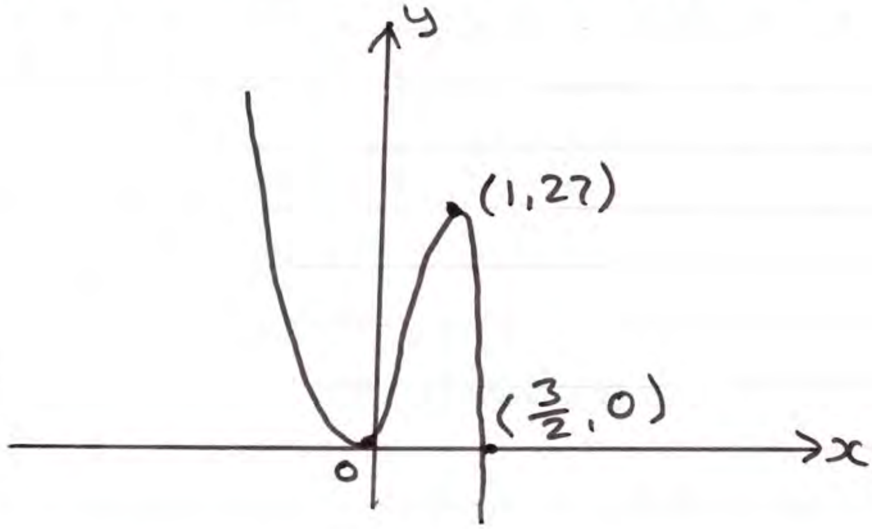
$2x=9 \Rightarrow x=\frac{9}{2}$

$\therefore \boxed{A \text{ is at } \left(\frac{9}{2}, 0\right)}$

b i) $y = f(x+3)$ - transformation vertically by '-3'



ii) $y = f(3x)$ - multiply x-coordinates by $\frac{1}{3}$:



c) $y = f(x) + k$ has a maximom point at (3, 10)

$f(x) + k$ - transformation ~~horiso~~ vertically of 'k'

If the original maximom point is at (3, 27) and the new maximom point is at (3, 10), compare the y-coordinates:

(3, 27) - (3, 10), transformation for y of -17.

$\therefore \boxed{k = -17}$