C1 June 2010 (MA)

Q1) $\sqrt{75} - \sqrt{27} = \sqrt{25 \times 3} - \sqrt{9 \times 3}$

$= \sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{3}$

$= 5\sqrt{3} - 3\sqrt{3}$

$\boxed{= 2\sqrt{3}}$

Q2) $\int (8x^3 + 6x^{\frac{1}{2}} - 5)\,dx = \frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$

$= 2x^4 + 4x^{3/2} - 5x + c$

$= \boxed{2x^4 + 4x\sqrt{x} - 5x + c}$

Q 3a) $3(x-2) < 8 - 2x$

$3x - 6 < 8 - 2x$

$5x < 14$

$x < \frac{14}{5} \Rightarrow \boxed{x < 2.8}$

b) $(2x-7)(1+x) < 0$

$2x + 2x^2 - 7 - 7x < 0$

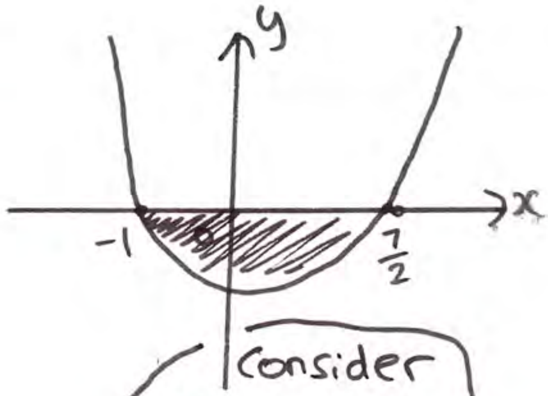
$2x^2 - 5x - 7 < 0$

$(2x-7)(x+1) \geq 0$

For $2x^2 - 5x - 7 = 0$,

$(2x-7)(x+1) = 0$

Either $x = \frac{7}{2}$ or $x = -1$



Set of possible values of x:

$$-1 < x < \frac{7}{2}$$

c) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$

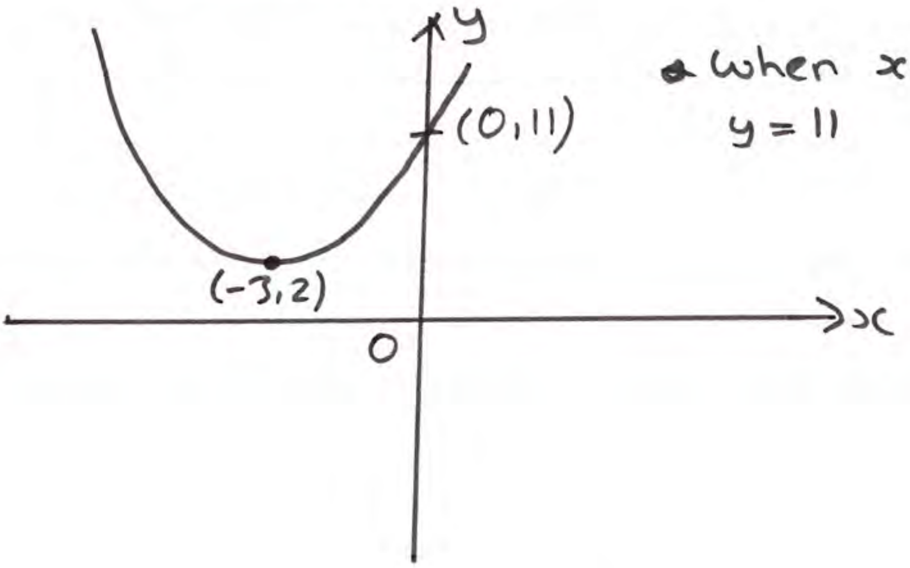
$x < \frac{14}{5}$ **and** $-1 < x < \frac{7}{2}$

$$-1 < x < \frac{14}{5}$$

Q4a) $x^2 + 6x + 11 = (x+3)^2 - 3^2 + 11$

$$= (x+3)^2 + 2$$

b)



when $x=0$, $y = 11$

c) $y = x^2 + 6x + 11$

$b^2 - 4ac = (6)^2 - (4)(1)(11)$

$= 36 - 44$

$= -8$

Q5) $a_{n+1} = \sqrt{(a_n^2 + 3)}$ $n \geq 1$ $a_1 = 2$

a) $a_2 = \sqrt{(a_1^2 + 3)}$

$a_2 = \sqrt{2^2 + 3}$

$a_2 = \sqrt{7}$

$a_3 = \sqrt{(a_2^2 + 3)}$

$a_3 = \sqrt{((\sqrt{7})^2 + 3)}$

$a_3 = \sqrt{10}$

b) $a_4 = \sqrt{(a_3^2 + 3)}$

$a_4 = \sqrt{(\sqrt{10})^2 + 3}$

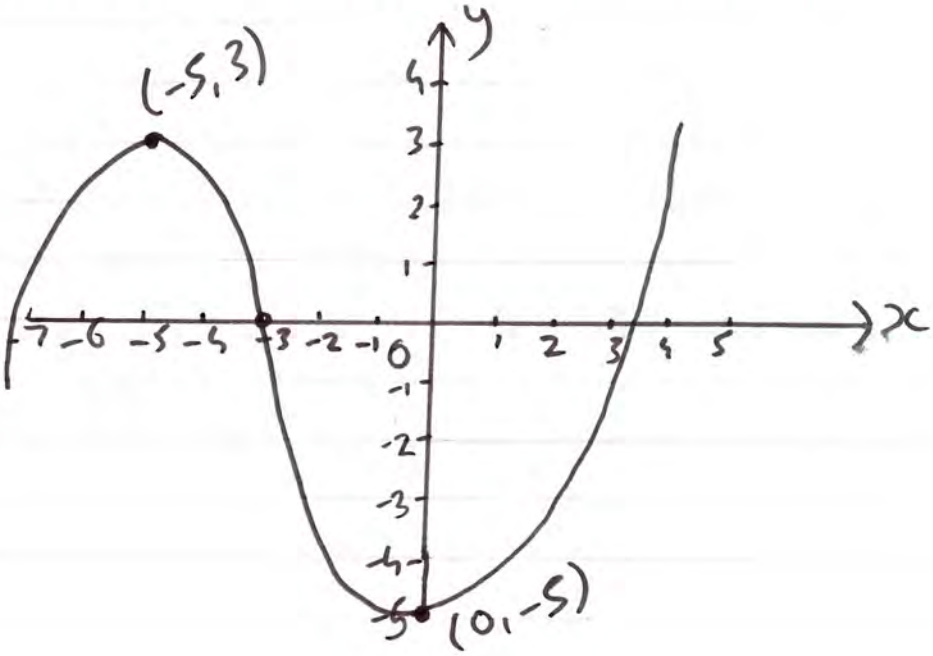
$a_4 = \sqrt{13}$

$a_5 = \sqrt{(a_4^2 + 3)}$

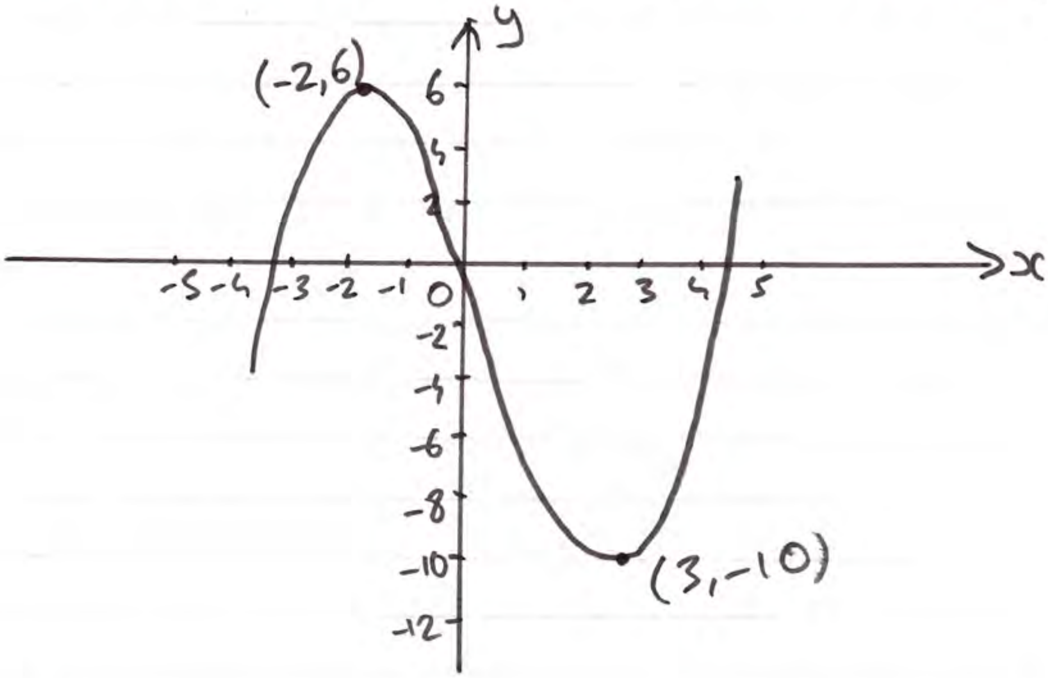
$a_5 = \sqrt{(\sqrt{13})^2 + 3}$

$a_5 = \sqrt{16} = 4$

Q6a) $y = f(x+3)$ - transformation horizontally (along the x-axis) of '-3':



b) $y = 2f(x)$ - multiply the y-coordinates by 2:



c) $y = f(x) + a$ - transformation vertically of '$+a$'.

If the minimum point of $y = f(x) + a$ is at $(3,0)$ then $\boxed{a = 5}$

Q7) $y = 8x^3 - 4\sqrt{x} + \frac{3x^2+2}{x}$, $x > 0$

$$y = 8x^3 - 4x^{1/2} + \frac{3x^2}{x} + \frac{2}{x}$$

$$y = 8x^3 - 4x^{1/2} + 3x + 2x^{-1}$$

$$\frac{dy}{dx} = 24x^2 - 2x^{-1/2} + 3 - 2x^{-2}$$

$$= 24x^2 - \frac{2}{x^{1/2}} + 3 - \frac{2}{x^2}$$

$$= \boxed{24x^2 - \frac{2}{\sqrt{x}} + 3 - \frac{2}{x^2}}$$

Q8a) Gradient $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{4-0}{7-2}$$

$$= \frac{4}{5}$$

Equation of line : $y - y_1 = m(x - x_1)$

Using $B(2,0)$ and $m = \frac{4}{5}$ → $y - 0 = \frac{4}{5}(x-2)$

$$y = \frac{4}{5}x - \frac{8}{5}$$

$$\boxed{4x - 5y + 8 = 0}$$

b) Length $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(7-2)^2+(4-0)^2}$

$= \sqrt{5^2+4^2}$

$= \sqrt{25+16}$

$= \boxed{\sqrt{41} \text{ units}}$

c) Length $AC = AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{41} = \sqrt{(7-2)^2+(4-t)^2}$

$\sqrt{41} = \sqrt{5^2+(4-t)^2}$

$\sqrt{41} = \sqrt{25+(16-8t+t^2)}$

$\sqrt{41} = \sqrt{25+16-8t+t^2}$

$\sqrt{41} = \sqrt{41-8t+t^2}$

So, $-8t+t^2=0$

$t^2 = 8t$

$\boxed{t=8}$

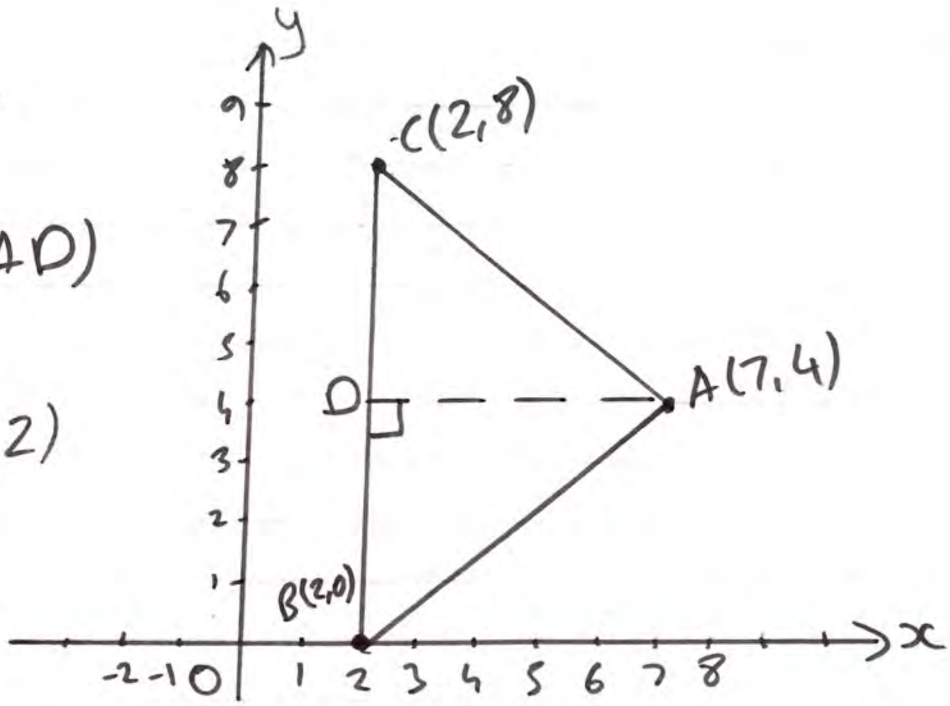
d) Area $= \frac{1}{2}bh$

$= \frac{1}{2}(BC)(AD)$

$= \frac{1}{2}(8)(7-2)$

$= \frac{1}{2}(8)(5)$

$= \boxed{20 \text{ units}^2}$



Q9) First term, $a =$ £a
common difference, $d =$ £d

After working for 30 days, this worker gets paid £40.75 on the last day.

a) $U_n = a + (n-1)d$

$U_{30} = a + (30-1)d$

$40.75 = a + 29d$

$\boxed{a + 29d = 40.75}$

b) A picker works for 30 days and earns a total of £1005.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{30} = \frac{30}{2}(2a + (30-1)d)$$

$$1005 = 15(2a + 29d)$$

$$1005 = 15(2a + 29d)$$

From (a), $29d = 40.75 - a$

$$\therefore 1005 = 15(2a + (40.75 - a))$$

$$1005 = 15(2a - a + 40.75)$$

$$\therefore \boxed{15(a + 40.75) = 1005}$$

c) $15a + 611.25 = 1005$

$$15a = 393.75$$

$$a = 26.25$$

so $\boxed{a = £26.25}$

$29d = 40.75 - a \Rightarrow 29d = 40.75 - 26.25 = 14.5$

$$\therefore \boxed{d = £0.50}$$

Q10a)

i) $y = x(4-x)$

When $x = 0,\ y = 0$

When $y = 0,\ x(4-x) = 0$

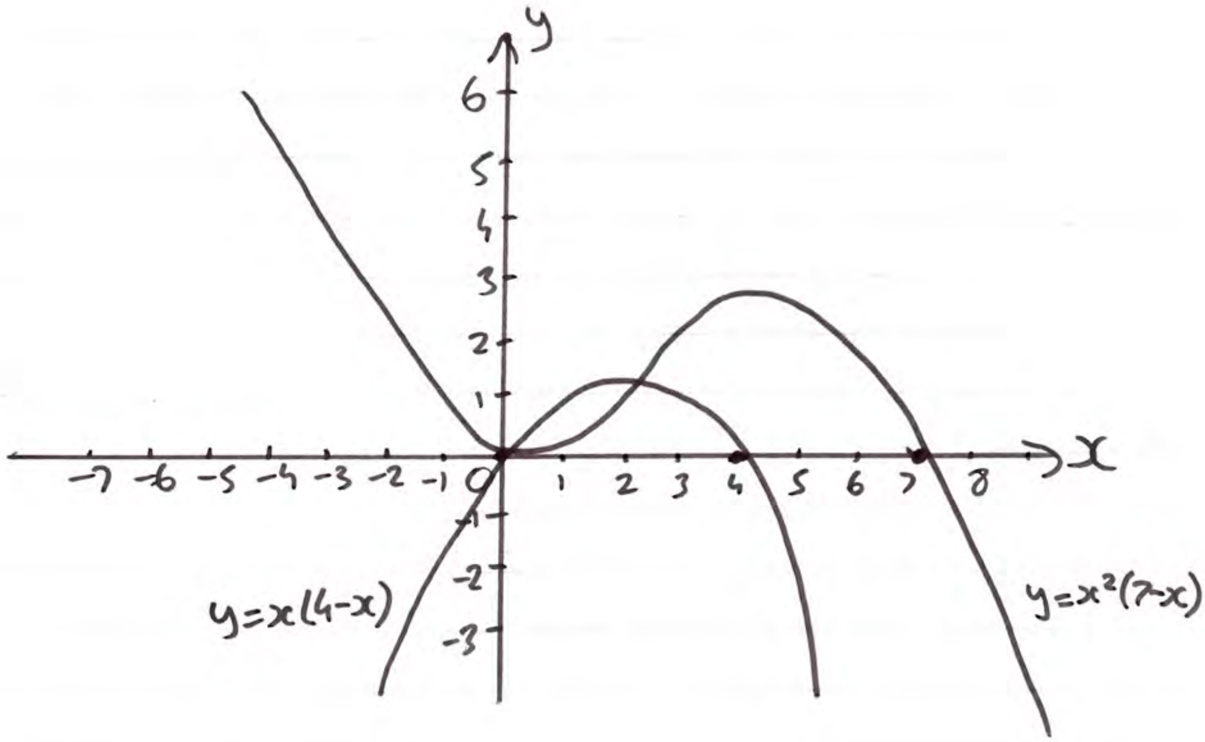
Either $x = 0$ or $x = 4$

ii) $y = x^2(7-x)$

When $x = 0,\ y = 0$

When $y = 0,\ x^2(7-x) = 0$

Either $x = 0$ or $x = 0$ or $x = 7$



b) To find points of intersection, solve simultaneous equations:

$y = x(4-x)$ ①
$y = x^2(7-x)$ ②

Substitute ① into ②:

$x(4-x) = x^2(7-x)$

$4x - x^2 = 7x^2 - x^3$

$x^3 - 8x^2 + 4x = 0$

$\therefore \boxed{x(x^2 - 8x + 4) = 0}$

c) $x(x^2 - 8x + 4) = 0 \Rightarrow x^2 - 8x + 4 = 0$

$(x-4)^2 - 4^2 + 4 = 0$

$(x-4)^2 = 12$

$x = 4 \pm \sqrt{12} \Rightarrow x = 4 \pm 2\sqrt{3}$

From the sketch, $\underline{x = 4 - 2\sqrt{3}}$, since $4+2\sqrt{3} > 7$, which is further than the x-coordinate for the x-intercept of $y = x^2(7-x)$.

Substitute into ① for y: $(4-2\sqrt{3})(4-(4-2\sqrt{3}))$

$y = (4-2\sqrt{3})(2\sqrt{3})$

$\underline{y = 8\sqrt{3} - 12}$

$\therefore$ the $\boxed{\text{coordinates of A are } (4-2\sqrt{3}, 8\sqrt{3} - 12)}$

Q11) $\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$

Given P(4, 5) lies on y,

a) $y = f(x) = \int \left(3x - \frac{5}{\sqrt{x}} - 2 \right) dx$

$= \int \left(3x - \frac{5}{x^{1/2}} - 2 \right) dx$

$= \int \left(3x - 5x^{-1/2} - 2 \right) dx$

$= \frac{3x^2}{2} - \frac{5x^{1/2}}{1/2} - 2x + C$

$= \frac{3x^2}{2} - 10\sqrt{x} - 2x + C$

Since this curve passes through P(4,5), substitute in $x = 4$ and $y = 5$

$5 = \frac{3(4)^2}{2} - 10(\sqrt{4}) - 2(4) + C$

$5 = 24 - 20 - 8 + C$

$5 - 24 + 20 + 8 = C$

$\therefore C = 9$

$y = f(x) = \frac{3x^2}{2} - 10\sqrt{x} - 2x + 9$

b) When $x = 4$, $\frac{dy}{dx} = 3(4) - \frac{5}{\sqrt{4}} - 2$

$= 12 - \frac{5}{2} - 2$

$= \frac{15}{2}$

Equation of tangent at P: $y - y_1 = m(x - x_1)$

Using $P(4, 5)$ and $m = \frac{15}{2}$ → $y - 5 = \frac{15}{2}(x - 4)$

$2(y - 5) = 15(x - 4)$

$2y - 10 = 15x - 60$

$15x - 2y - 50 = 0$