C1 June 2009 (MA)

Q1a) $(3\sqrt{7})^2 = (3\sqrt{7})(3\sqrt{7}) = 9 \times 7 \boxed{= 63}$

b) $(8+\sqrt{5})(2-\sqrt{5}) = 16 - 8\sqrt{5} + 2\sqrt{5} - 5$

$= \boxed{11 - 6\sqrt{5}}$

Q2) $32\sqrt{2} = 2^5 \times 2^{\frac{1}{2}}$

$= 2^{11/2}$

$\therefore \boxed{a = \frac{11}{2}}$

Q3) $y = 2x^3 + \frac{3}{x^2}, \quad x \neq 0$

$y = 2x^3 + 3x^{-2}$

a) $\frac{dy}{dx} = 6x^2 - 6x^{-3}$

$= \boxed{6x^2 - \frac{6}{x^3}}$

b) $\int (2x^3 + 3x^{-2})\,dx = \frac{2x^4}{4} + \frac{3x^{-1}}{-1} + C$

$= \frac{x^4}{2} - 3x^{-1} + C$

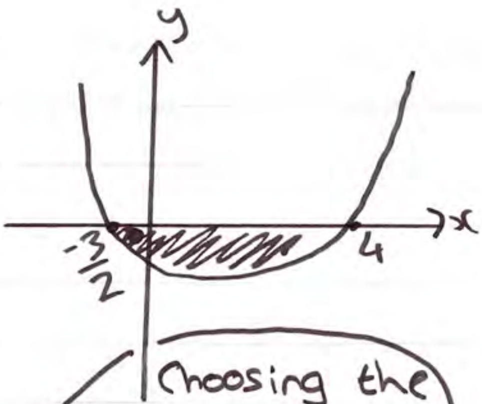
$= \boxed{\frac{x^4}{2} - \frac{3}{x} + C}$

Q4a) $4x - 3 > 7 - x$

$5x > 10$

$\boxed{x > 2}$

b) $2x^2 - 5x - 12 < 0$



For $2x^2 - 5x - 12 = 0$,

$(2x + 3)(x - 4) = 0$

Either $x = -\frac{3}{2}$ or $x = 4$

Choosing the values 'under' the x-axis, since $2x^2 - 5x - 12 < 0$

Set of possible values of x:

$\boxed{-\frac{3}{2} < x < 4}$

c) both $4x - 3 > 7 - x$ and $2x^2 - 5x - 12 < 0$

$x > 2$ and $-\frac{3}{2} < x < 4$

$2 < x$ and $-\frac{3}{2} < x < 4$

$\boxed{2 < x < 4}$

Q5) First term, $a = a$
Common difference, $d = d$

$U_n = a + (n-1)d$

$U_{10} = a + (10-1)d$

$U_{10} = a + 9d$

Also, $U_{40} = a + (40-1)d$

$U_{40} = a + 39d$

But, 2400 new houses were built in U_{10} and 600 new houses in U_{40}

$\therefore \quad a + 9d = 2400 \quad (1)$
$a + 39d = 600 \quad (2)$

a) $(2) - (1)$: $\quad 30d = -1800$

$$\boxed{d = -60}$$

b) Substitute into (1) for a:

$a + 9d = 2400$

$a + 9(-60) = 2400$

$a - 540 = 2400$

$$\boxed{a = 2940}$$

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{40} = \frac{40}{2}(2(2940) + (40-1) - 60)$

$S_{40} = 20(5880 - 2340)$

$S_{40} = 20 \times 3540$

$\boxed{S_{40} = 70800}$

Q6) $x^2 + 3px + p = 0$ has equal roots

If a quadratic has equal roots, then the discriminant is equal to zero.

$\therefore b^2 - 4ac = 0$

$(3p)^2 - (4)(1)(p) = 0$

$9p^2 - 4p = 0$

$4p = 9p^2$

$9p = 4$

$\boxed{p = \frac{4}{9}}$

Q7) $a_1 = k$

$a_{n+1} = 2a_n - 7,\ n \geq 1$

a) $a_2 = 2a_1 - 7$

$\boxed{a_2 = 2k - 7}$

b) $a_3 = 2a_2 - 7$

$a_3 = 2(2k-7) - 7$

$a_3 = 4k - 14 - 7$

$\boxed{a_3 = 4k - 21}$

c) $\sum_{r=1}^{4} a_r = a_1 + a_2 + a_3 + a_4$

$a_4 = 2a_3 - 7$

$a_4 = 2(4k-21) - 7$

$a_4 = 8k - 42 - 7$

$\underline{a_4 = 8k - 49}$

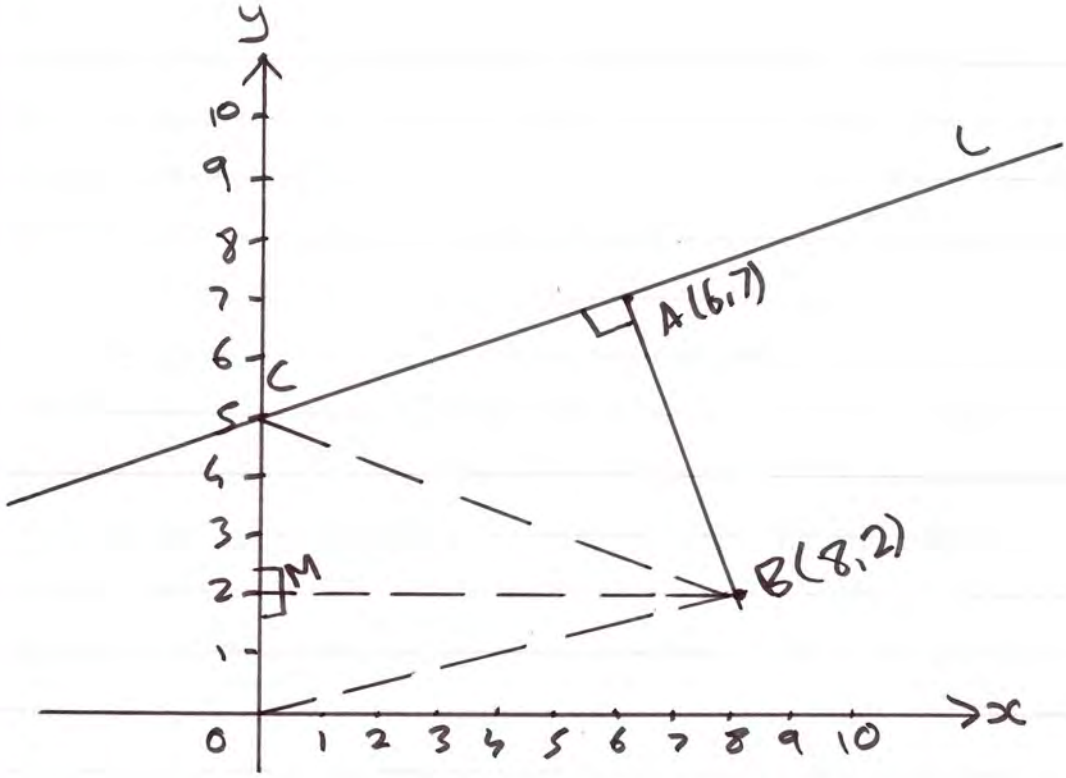
But $\sum_{r=1}^{4} a_r = 43$

$\therefore 43 = k + 2k - 7 + 4k - 21 + 8k - 49$

$15k = 120$

$\boxed{k = 8}$

Q8)



Gradient AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{7-2}{6-8}$

$= \dfrac{5}{-2}$

$= -\dfrac{5}{2}$

Since line L is perpendicular to AB, the gradient of line L is $-\left(-\dfrac{5}{2}\right)^{-1} = \dfrac{2}{5}$

Equation of line l: $y - y_1 = m(x - x_1)$

Using $A(6,7)$ and $m = \frac{2}{5}$ $\rightarrow$ $y - 7 = \frac{2}{5}(x - 6)$

$5(y-7) = 2(x-6)$

$5y - 35 = 2x - 12$

$\boxed{2x - 5y + 23 = 0}$

b) When l intersects the y-axis, $x = 0$

So, $2(0) - 5y + 23 = 0$

$23 = 5y$

$y = \frac{23}{5}$

$\therefore$ the coordinates of C are $\boxed{\left(0, \frac{23}{5}\right)}$

c) Area $\Delta OCB = \frac{1}{2}bh = \frac{1}{2}(OC)(MB)$

$= \frac{1}{2}\left(\frac{23}{5}\right)(8)$

$= \frac{184}{10} = \boxed{\frac{92}{5} \text{ units}^2}$

Q9) $f(x) = \dfrac{(3-4\sqrt{x})^2}{\sqrt{x}}$, $x>0$

a) $f(x) = \dfrac{(3-4x^{1/2})(3-4x^{1/2})}{x^{1/2}}$

$f(x) = (9 - 12x^{1/2} - 12x^{1/2} + 16x)x^{-1/2}$

$f(x) = \dfrac{9 - 24x^{1/2} + 16x}{x^{1/2}}$

$f(x) = \dfrac{9}{x^{1/2}} - \dfrac{24x^{1/2}}{x^{1/2}} + \dfrac{16x}{x^{1/2}}$

$f(x) = 9x^{-1/2} - 24 + 16x^{1/2}$

$\boxed{f(x) = 9x^{-1/2} + 16x^{1/2} - 24}$

b) $f'(x) = -\dfrac{9}{2}x^{-3/2} + 8x^{-1/2}$

$f'(x) = -\dfrac{9}{2x^{3/2}} + \dfrac{8}{x^{1/2}}$

$\boxed{f'(x) = \dfrac{8}{\sqrt{x}} - \dfrac{9}{2x\sqrt{x}}}$

c) $f'(9) = \dfrac{8}{\sqrt{9}} - \dfrac{9}{2(9)(\sqrt{9})}$

$= \dfrac{8}{3} - \dfrac{9}{54}$ $\boxed{= \dfrac{5}{2}}$

Q10a) $x^3 - 6x^2 + 9x = x(x^2 - 6x + 9)$

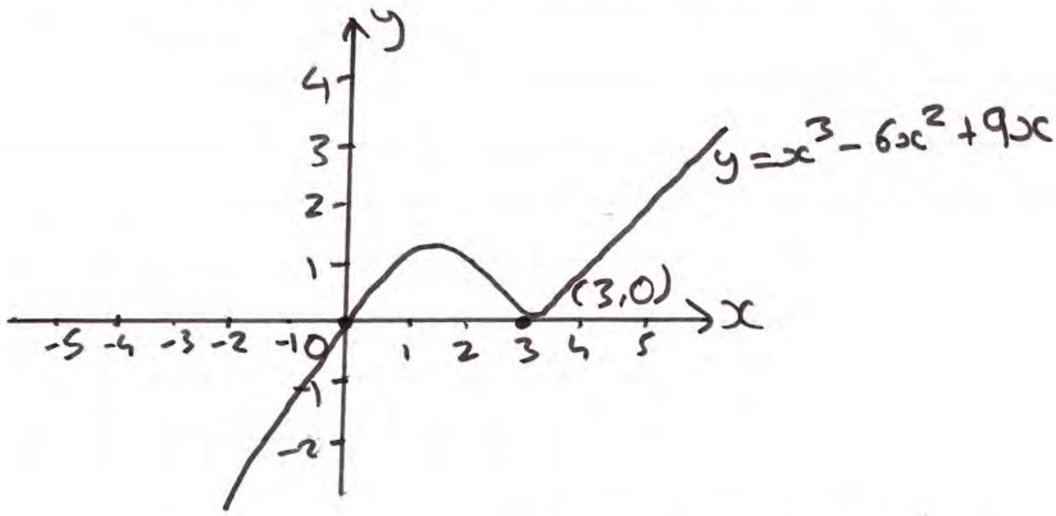
$= x(x-3)(x-3)$

$= \boxed{x(x-3)^2}$

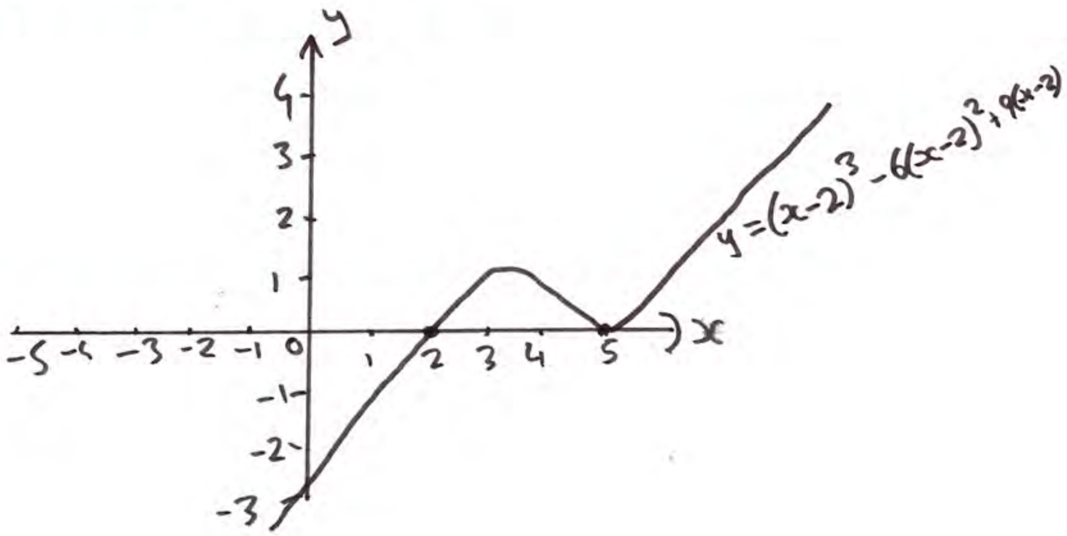
b) When $x=0,\ y=0$

When $y=0$, $x(x-3)^2 = 0$

Either $x=0$ or $x=3$ or $x=3$



c) Let $f(x) = x^3 - 6x^2 + 9x$, then $(x-2)^3 - 6(x-2)^2 + 9(x-2) \equiv f(x-2)$, which is a horizontal tranformation of '+2'.



Q11a) $y = x^3 - 2x^2 - x + 9, \ x > 0$

Sobstitute in $x=2$ and $y=7$:

$7 = 2^3 - 2(2)^2 - 2 + 9$

$7 = 8 - 8 - 2 + 9$

$\underline{7 = 7}$

$\therefore$ P (2,7) lies on the curve

b) $\frac{dy}{dx} = 3x^2 - 4x - 1$

At P, $\frac{dy}{dx} = 3(2)^2 - 4(2) - 1$

$= 12 - 8 - 1$

$\underline{= 3}$

$\therefore$ the gradient of the tangent at P is 3.

Equation of tangent at P: $y - y_1 = m(x - x_1)$

Using P(2,7) and m=3 $\longrightarrow$ $y - 7 = 3(x-2)$

$y - 7 = 3x - 6$

$\boxed{y = 3x + 1}$

c) If the tangent at Q is perpendicular to the tangent at P, then the gradient of the tangent at Q is $-\frac{1}{3}$.

$$\therefore \frac{dy}{dx} = 3x^2 - 4x - 1 = -\frac{1}{3}$$

$$3x^2 - 4x - \frac{2}{3} = 0$$

$$9x^2 - 12x - 2 = 0$$

$$x^2 - \frac{4}{3}x - \frac{2}{9} = 0$$

$$\left(x - \frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 - \frac{2}{9} = 0$$

$$\left(x - \frac{2}{3}\right)^2 - \frac{4}{9} - \frac{2}{9} = 0$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{2}{3}$$

$$x - \frac{2}{3} = \pm\sqrt{\frac{2}{3}}$$

$$x = \frac{2}{3} \pm \frac{\sqrt{2}}{\sqrt{3}}$$

$$x = \frac{2}{3} \pm \frac{\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$x = \frac{2}{3} \pm \frac{\sqrt{6}}{3}$$

$$x = \frac{1}{3}(2 \pm \sqrt{6})$$

We already know that the x-coordinate of Q is positive,

$\therefore$ the x-coordinate of Q is $\boxed{\frac{1}{3}(2+\sqrt{6})}$