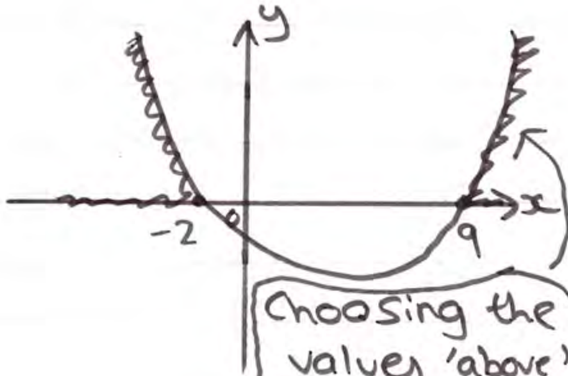
C1 June 2006 (MA)

Q1) $\int (6x^2 + 2 + x^{-\frac{1}{2}})\,dx = \dfrac{6x^3}{3} + 2x + \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$$= 2x^3 + 2x + 2x^{\frac{1}{2}} + c$$

$$= \boxed{2x^3 + 2x + 2\sqrt{x} + c}$$

Q2) $x^2 - 7x - 18 > 0$



Choosing the values 'above' the x-axis, since $x^2-7x-18>0$

For $x^2 - 7x - 18 = 0$,

$(x-9)(x+2) = 0$

Either $x = 9$ or $x = -2$

Set of possible values of x:

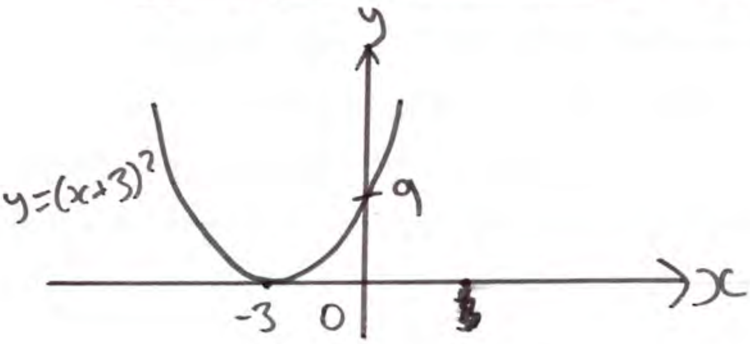
$\boxed{x < -2 \quad \text{or} \quad x > 9}$

Q3a) $y = (x+3)^2$

When $x = 0$, $y = (0+3)^2 \underline{= 9}$

When $y = 0$, $(x+3)^2 = 0$

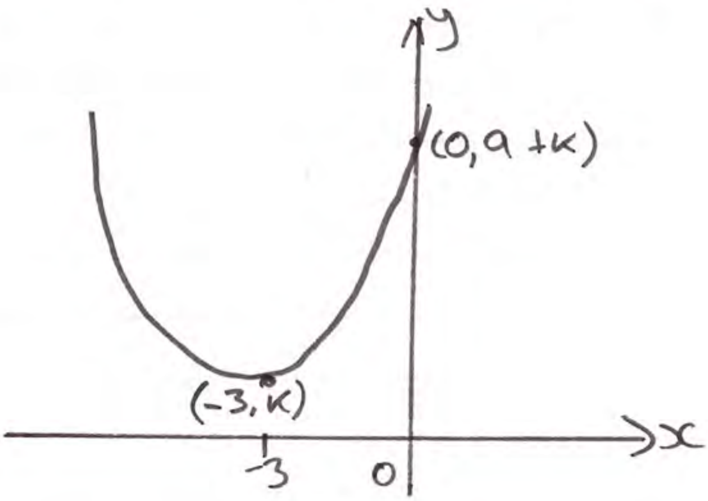
$\underline{\text{Either } x = -3 \text{ or } x = -3}$



b) $y = (x+3)^2 + k$

Let $f(x) = (x+3)^2$

Then $(x+3)^2 + k \equiv f(x) + k$, which is a transformation vertically of '$+k$'.



Q4) $a_1 = 3$
$a_{n+1} = 3a_n - 5, \quad n \geq 1$

a) $a_2 = 3a_1 - 5$
$a_2 = 3(3) - 5$
$\boxed{a_2 = 4}$

$a_3 = 3a_2 - 5$
$a_3 = 3(4) - 5$
$\boxed{a_3 = 7}$

b) $\sum_{r=1}^{5} a_r = a_1 + a_2 + a_3 + a_4 + a_5$

$a_4 = 3a_3 - 5$
$= 3(7) - 5$
$= 16$

$$a_5 = 3a_4 - 5$$
$$= 3(16) - 5$$
$$= 43$$

$$\sum_{r=1}^{5} a_r = 3 + 4 + 7 + 16 + 43$$
$$\boxed{= 73}$$

Q5a) $y = x^4 + 6\sqrt{x} \Rightarrow y = x^4 + 6x^{1/2}$

$$\frac{dy}{dx} = 4x^3 + 3x^{-\frac{1}{2}}$$
$$= \boxed{4x^3 + \frac{3}{\sqrt{x}}}$$

b) $y = \frac{(x+4)^2}{x} \Rightarrow y = \frac{(x+4)(x+4)}{x}$

$$y = \frac{x^2 + 8x + 16}{x}$$

$$y = \frac{x^2}{x} + \frac{8x}{x} + \frac{16}{x}$$

$$y = x + 8 + 16x^{-1}$$

$$\frac{dy}{dx} = 1 - 16x^{-2}$$
$$= \boxed{1 - \frac{16}{x^2}}$$

Q6a) $(4+\sqrt{3})(4-\sqrt{3}) = 16 - 4\sqrt{3} + 4\sqrt{3} - 3$

$$\boxed{= 13}$$

b) $\dfrac{26}{4+\sqrt{3}} = \dfrac{26 \times (4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})} = \dfrac{104 - 26\sqrt{3}}{13}$

$$\boxed{= 8 - 2\sqrt{3}}$$

Q7) First term, $a = a$ km

Common difference, $d = d$ km

$U_n = a + (n-1)d$

$U_{11} = a + (11-1)d$

$U_{11} = a + 10d$

But $U_{11} = 9$ km, so $\underline{9 = a + 10d}$ (1)

Also, $S_n = \frac{n}{2}(2a + (n-1)d$

$S_{11} = \frac{n}{2}(2a + (11-1)d)$

$S_{11} = \frac{11}{2}(2a + 10d)$

$S_{11} = \frac{22a}{2} + \frac{110d}{2}$

$S_{11} = 11a + 55d$

But $S_{11} = 77$

$$77 = \frac{11}{2}(2a + 10d)$$

$$154 = 11(2a + 10d)$$

$$14 = 2a + 10d$$

$$7 = a + 5d \quad (2)$$

(1) − (2):

$$+2 = +5d$$

$$d = \frac{2}{5}$$

Substitute into (1) for a:

$$a + 10d = 9$$

$$a + 10\left(\frac{2}{5}\right) = 9$$

$$a + 4 = 9$$

$$a = 5$$

$$\therefore \boxed{a = 5 \text{ and } d = \frac{2}{5}}$$

8) $x^2+2px+(3p+4)=0$ has equal roots.

a) If a quadratic has equal roots, then the discriminant equals 0.

$$\therefore b^2-4ac=0$$

$$(2p)^2-(4)(1)(3p+4)=0$$

$$4p^2-4(3p+4)=0$$

$$4p^2-12p-16=0$$

$$p^2-3p-4=0$$

$$(p-4)(p+1)=0$$

Either $p=4$ or $p=-1$

Since $p>0$, $\boxed{p=4}$

b) $$x^2+2(4)x+(3(4)+4)=0$$

$$x^2+8x+16=0$$

$$(x+4)(x+4)=0$$

$$(x+4)^2=0$$

$$\boxed{x=-4}$$

Q9) $f(x) = (x^2 - 6x)(x-2) + 3x$

a) $f(x) = (x^3 - 2x^2 - 6x^2 + 12x) + 3x$

$f(x) = x^3 - 8x^2 + 15x$

$\boxed{f(x) = x(x^2 - 8x + 15)}$

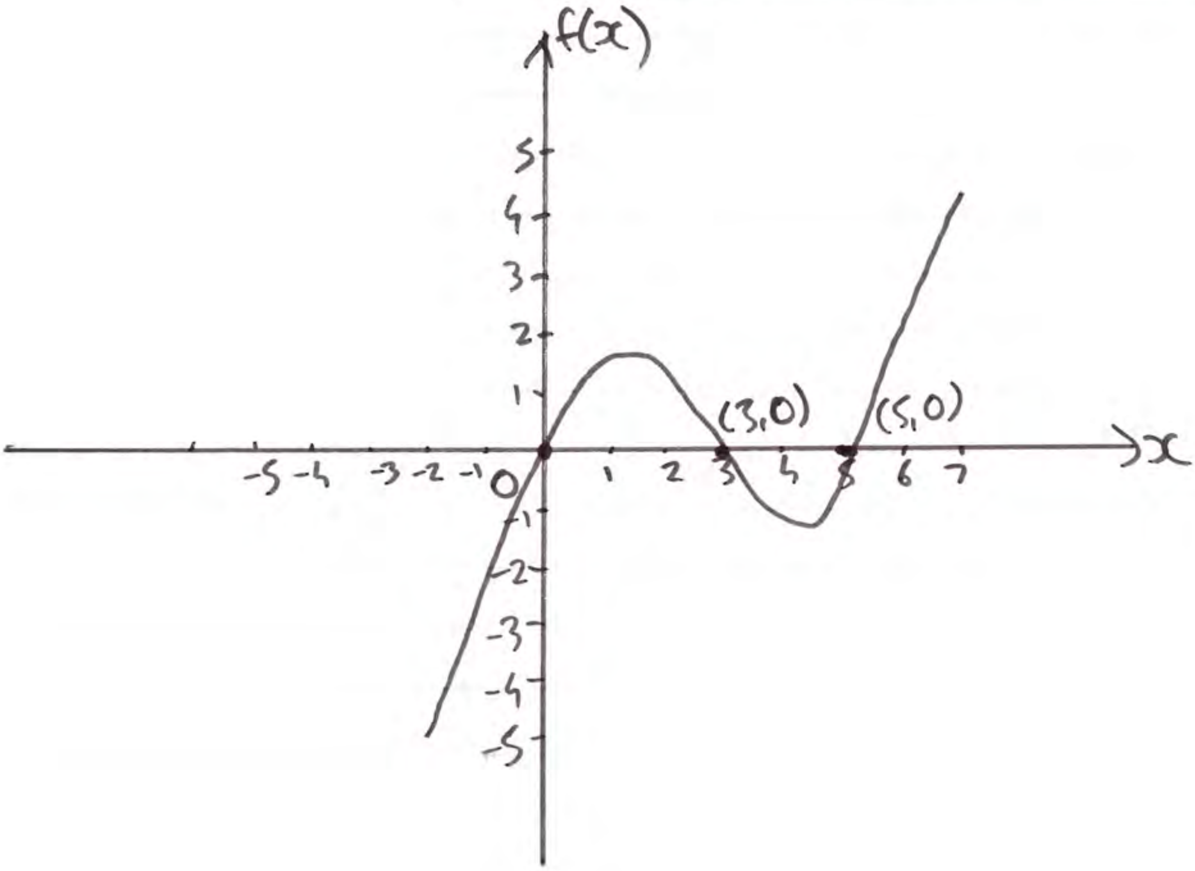
b) $f(x) = x(x^2 - 8x + 15)$

$\boxed{f(x) = x(x-5)(x-3)}$

c) When $x=0$, $f(x) = 0(0-5)(0-3) \underline{= 0}$

When $f(x)=0$, $x(x-5)(x-3)=0$

$\underline{\text{Either } x=0 \text{ or } x=3 \text{ or } x=5}$



Q10a) $f'(x) = 2x + \frac{3}{x^2}$

$f'(x) = 2x + 3x^{-2}$

$f(x) = \int (2x + 3x^{-2})\,dx$

$f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1} + c$

$f(x) = x^2 - 3x^{-1} + c$

$f(x) = x^2 - \frac{3}{x} + c$

Since $f(x)$ passes through $(3, 7\frac{1}{2})$,
Substitute in $x = 3$ and $f(x) = 7.5$:

$7.5 = 3^2 - \frac{3}{3} + c$

$7.5 = 9 - 1 + c$

$7.5 = 8 + c$

$\therefore c = -\frac{1}{2}$

$$\boxed{f(x) = x^2 - \frac{3}{x} - \frac{1}{2}}$$

b) $f(-2) = (-2)^2 - \frac{3}{-2} - \frac{1}{2}$

$= 4 + \frac{3}{2} - \frac{1}{2}$

$= 5$

$\therefore \boxed{f(-2) = 5}$

c) Using the point $(-2, 5)$,

when $x = -2$, $f'(x) = 2(-2) + \frac{3}{(-2)^2}$

$= -4 + \frac{3}{4}$

$= \frac{-13}{4}$

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $(-2, 5)$ and $m = \frac{-13}{4}$ $\rightarrow$ $y - 5 = \frac{-13}{4}(x - (-2))$

$4(y-5) = -13(x+2)$

$4y - 20 = -13x - 26$

$\boxed{13x + 4y + 6 = 0}$

Q11a) l_1 passes through $P(-1,2)$ and $Q(11,8)$

Gradient $PQ = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-2}{11-(-1)}$

$= \dfrac{6}{12}$

$= \dfrac{1}{2}$

Equation of l_1 : $y - y_1 = m(x - x_1)$

Using $P(-1,2)$ and $m = \frac{1}{2}$ $\longrightarrow$ $y - 2 = \frac{1}{2}(x-(-1))$

$2(y-2) = 1(x+1)$

$2y - 4 = x + 1$

$2y = x + 5$

$$\boxed{y = \frac{1}{2}x + \frac{5}{2}}$$

b) l_2 passes through $R(10,0)$ and is perpendicular to l_1.

If l_1 is perpendicular to l_2, then the gradient of l_2 is -2

Equation of l_2 : $y - y_1 = m(x - x_1)$

Using $R(10,0)$ and $m = -2$ → $y - 0 = -2(x - 10)$

$$\underline{y = -2x + 20}$$

To find point of intersection of l_1 and l_2, solve simultaneous equations

$$y = \frac{1}{2}x + \frac{5}{2} \quad (1)$$

$$y = -2x + 20 \quad (2)$$

Substitute (1) into (2):

$$\frac{1}{2}x + \frac{5}{2} = -2x + 20$$

$$\frac{5}{2}x - \frac{35}{2} = 0$$

$$\frac{5x - 35}{2} = 0$$

$$5x - 35 = 0$$

$$5x = 35$$

$$\underline{x = 7}$$

Substitute into ② for y:

$$y = -2x + 20$$

When $x = 7$, $y = -2(7) + 20 \underline{= 6}$

$\therefore$ the $\boxed{\text{coordinates of } S \text{ are } (7, 6)}$

c) Length $RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

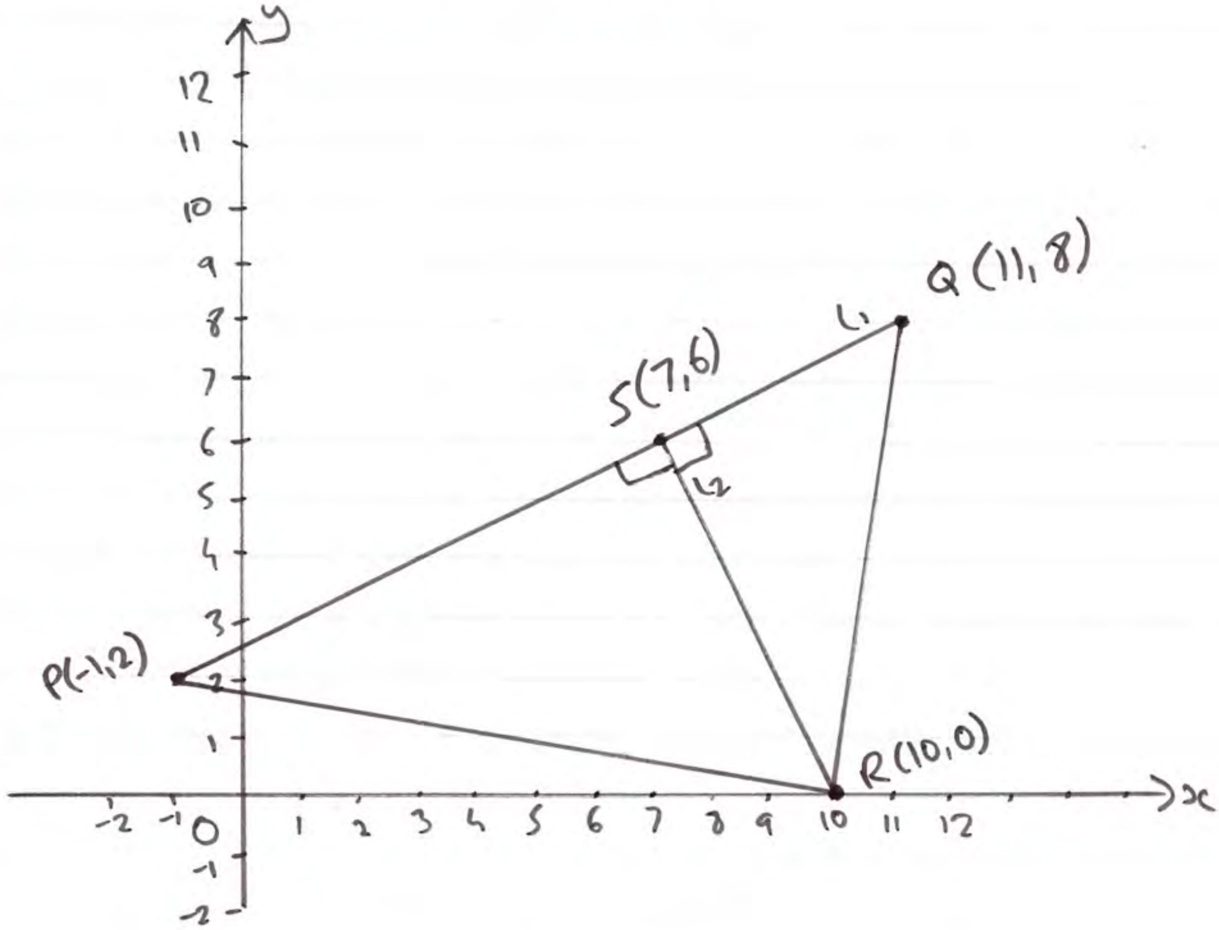
$$= \sqrt{(7-10)^2 + (6-0)^2}$$

$$= \sqrt{(-3)^2 + 6^2}$$

$$= \sqrt{45} = \sqrt{9 \times 5} = \sqrt{9}\sqrt{5}$$

$$= \boxed{3\sqrt{5} \text{ units}}$$

d)



The line through $P(-1,2)$ and $Q(11,8)$ is l_1, and the line l_2 is perpendicular to l_1 and passes through $R(10,0)$ and $S(7,6)$.

$\therefore P\hat{S}R$ and $Q\hat{S}R = 90°$

So we can use the formula Area $= \frac{1}{2}bh$

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(11-(-1))^2+(8-2)^2}$$

$$= \sqrt{12^2+6^2}$$

$$= \sqrt{180} = \sqrt{20\times 9} = \sqrt{5\times 4\times 9} = \sqrt{5}\sqrt{4}\sqrt{9}$$

$$= 6\sqrt{5} \text{ units}$$

Now, Area $= \frac{1}{2}bh = \frac{1}{2}(PQ)(RS)$

$$= \frac{1}{2}(6\sqrt{5})(3\sqrt{5})$$

$$= \frac{1}{2}(90)$$

$$= \boxed{45 \text{ units}^2}$$