C1 June 2005 (MA)

Q1a) $8^{\frac{1}{3}} = \sqrt[3]{8}$ $\boxed{=2}$

b) $8^{-\frac{2}{3}} = \dfrac{1}{8^{2/3}} = \dfrac{1}{(\sqrt[3]{8})^2} = \dfrac{1}{(2)^2}$ $\boxed{= \dfrac{1}{4}}$

Q2a) $y = 6x - \dfrac{4}{x^2}$, $x \neq 0$

$y = 6x - 4x^{-2}$

$\dfrac{dy}{dx} = 6 + 8x^{-3}$

$= \boxed{6 + \dfrac{8}{x^3}}$

b) $\int (6x - 4x^{-2})\,dx = \dfrac{6x^2}{2} - \dfrac{4x^{-1}}{-1} + c$

$= 3x^2 + 4x^{-1} + c$

$= \boxed{3x^2 + \dfrac{4}{x} + c}$

Q3a) $x^2 - 8x - 29 \equiv (x-4)^2 - 4^2 - 29$

$= (x-4)^2 - 16 - 29$

$= \boxed{(x-4)^2 - 45}$

b) $(x-4)^2 - 45 = 0$

$(x-4)^2 = 45$

$$x-4=\pm\sqrt{45}$$

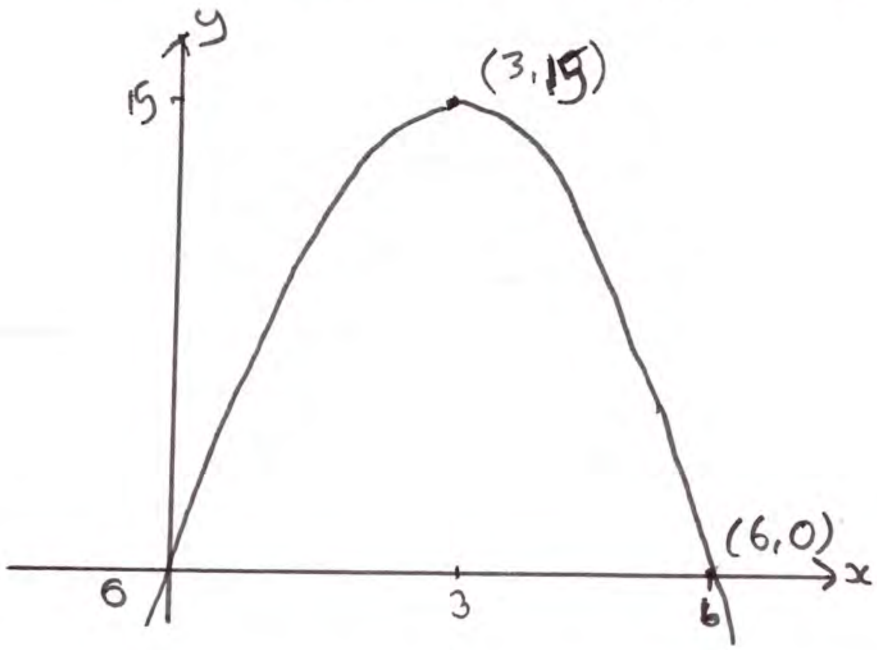
$$x=4\pm\sqrt{45}$$

$$x=4\pm\sqrt{9\times5}$$

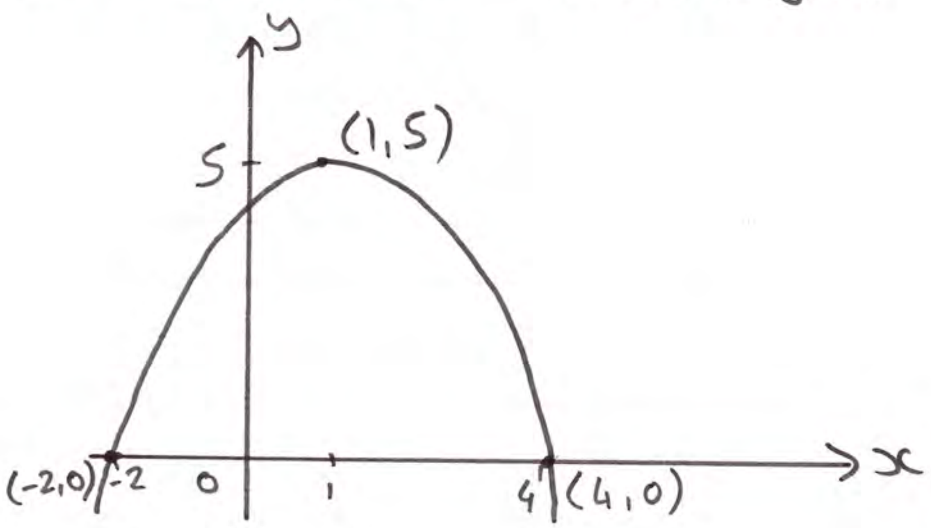
$$x=4\pm\sqrt{9}\sqrt{5}$$

$$\boxed{x=4\pm3\sqrt{5}}$$

Q4a) $y=3f(x)$ – transformation vertically of 'x3':



b) $y=f(x+2)$ – transformation horizontally of '–2':



Q5) $x-2y=1 \Rightarrow x=2y+1$ ①

$x^2+y^2=29 \Rightarrow x^2+y^2=29$ ②

Substitute ① into ②:

$(2y+1)^2+y^2=29$

$(2y+1)(2y+1)+y^2=29$

$4y^2+4y+1+y^2-29=0$

$5y^2+4y-28=0$

$(5y+14)(y-2)=0$

Either $y=-\frac{14}{5}$ or $y=2$

Substitute into ① for x:

When $y=\frac{-14}{5}$, $x=2\left(-\frac{14}{5}\right)+1$

$x=-\frac{28}{5}+1$

$x=-\frac{23}{5}$

When $y=2$, $x=2(2)+1=5$

Solution set: $\boxed{x=-\frac{23}{5},\ y=-\frac{14}{5} \text{ and } x=5, y=2}$

Q6a) $3(2x+1) > 5-2x$

$6x+3 > 5-2x$

$8x > 2$

$\boxed{x > \frac{1}{4}}$

b) $2x^2-7x+3>0$

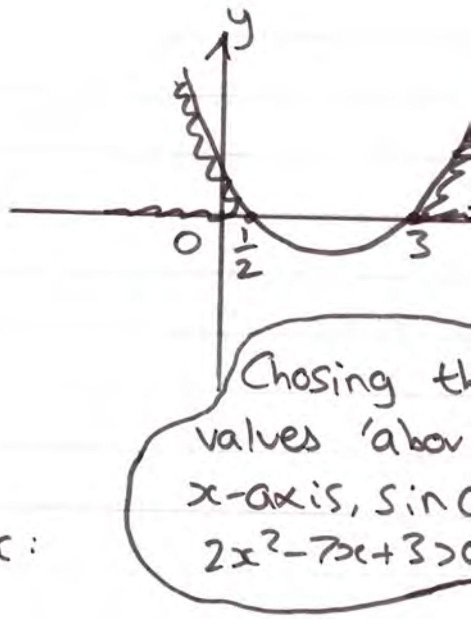
For $2x^2-7x+3=0$

$(2x-1)(x-3)=0$

Either $x=\frac{1}{2}$ or $x=3$

Set of possible values of x:

$\boxed{x<\frac{1}{2} \quad \text{or} \quad x>3}$



c) both $3(2x+1)>5-2x$ and $2x^2-7x+3>0$

$x>4^{-1}$ and $\left(x<\frac{1}{2} \text{ or } x\right.$

$x>\frac{1}{4}$ and $\left(x<\frac{1}{2} \text{ or } x\right.$

$\boxed{\frac{1}{4}<x<\frac{1}{2} \quad \text{or} \quad x>3}$

Q7a) $\dfrac{(3-\sqrt{x})^2}{\sqrt{x}} = \dfrac{(3-\sqrt{x})(3-\sqrt{x})}{\sqrt{x}}$

$$= \frac{9-3\sqrt{x}-3\sqrt{x}+x}{\sqrt{x}}$$

$$= \frac{9-6\sqrt{x}+x}{\sqrt{x}}$$

$$= \frac{9}{\sqrt{x}} - \frac{6\sqrt{x}}{\sqrt{x}} + \frac{x}{\sqrt{x}}$$

$$= \frac{9}{x^{1/2}} - \frac{6x^{1/2}}{x^{1/2}} + \frac{x}{x^{1/2}}$$

$$= \boxed{9x^{-\frac{1}{2}} - 6 + x^{1/2}}$$

b) $\dfrac{dy}{dx} = \dfrac{(3-\sqrt{x})^2}{\sqrt{x}}, \; x>0$

$$\frac{dy}{dx} = 9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$$

$$y = \int (9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}})\,dx$$

$$y = \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} - 6x + \frac{x^{3/2}}{\frac{3}{2}} + C$$

$$y = 18x^{\frac{1}{2}} - 6x + \frac{2x^{3/2}}{3} + C$$

$$y = 18\sqrt{x} - 6x + \frac{2x\sqrt{x}}{3} + C$$

Since $y=\frac{2}{3}$ at $x=1$, Substitute into equation:

$$y = 18\sqrt{x} - 6x + \frac{2x\sqrt{x}}{3} + c$$

$$\frac{2}{3} = 18(\sqrt{1}) - 6(1) + \frac{2(1)(\sqrt{1})}{3} + c$$

$$\frac{2}{3} = 18 - 6 + \frac{2}{3} + c$$

$$0 = 12 + c$$

$$\therefore \underline{c = -12}$$

$$\boxed{y = 18\sqrt{x} - 6x + \frac{2x\sqrt{x}}{3} - 12}$$

Q8a) Equation of l_1: $y - y_1 = m(x - x_1)$

Using $(9, -4)$ and $m = \frac{1}{3}$:

$$y - (-4) = \frac{1}{3}(x - 9)$$

$$3(y + 4) = 1(x - 9)$$

$$3y + 12 = x - 9$$

$$\boxed{x - 3y - 21 = 0}$$

b) Equation of l_2 : $y - y_1 = m(x - x_1)$

Since l_2 passes through the origin, $(0,0)$ can be used as (x_1, y_1), and we are given the gradient, $m = -2$

$$y - 0 = -2(x - 0)$$

$$\underline{y = -2x}$$

To find point of intersection between l_1 and l_2, solve simultaneous equations:

$$x - 3y - 21 = 0 \quad (1)$$
$$y = -2x \quad (2)$$

Substitute (2) into (1):

$$x - 3(-2x) - 21 = 0$$

$$x + 6x - 21 = 0$$

$$7x = 21$$

$$\underline{x = 3}$$

Substitute into (2) for y:

$$y = -2(3) \Rightarrow \underline{y = -6}$$

$\therefore$ $\boxed{P \text{ is at } (3, -6)}$

c) When l_1 crosses the y-axis, $x=0$

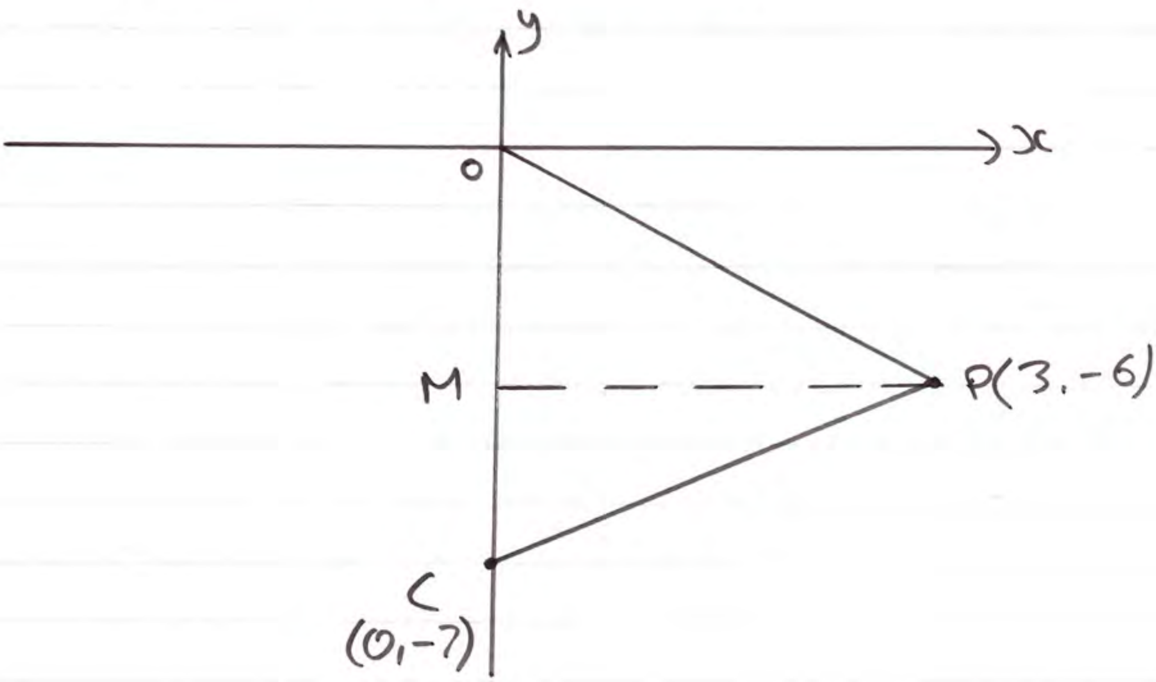
So, $x - 3y - 21 = 0$

$0 - 3y - 21 = 0$

$3y = -21$

$\underline{y = -7}$

$\therefore$ C is at $(0, -7)$



Area $\Delta OCP = \frac{1}{2}bh = \frac{1}{2}(OC)(MP)$

$= \frac{1}{2}(7)(3)$

$= \boxed{\frac{21}{2} \text{ units}^2}$

Q9a) First term, a = a.
Commondifference, d = d.

$$S_n = a + (a+d) + \dots + (a+(n-1)d)$$

$$S_n = (a+(n-1)d) + \dots + a$$

flip over first and last term

$$2S_n = (2a+(n-1)d) + \dots + (2a+(n-1)d)$$

$$2S_n = n(2a+(n-1)d)$$

$$\boxed{S_n = \frac{n}{2}(2a+(n-1)d)}$$

b) First term, a = £149.
Common difference, d = −2

$$U_n = a+(n-1)d$$

$$U_{21} = 149 + (21-1) - 2$$

$$U_{21} = 149 + (20) - 2$$

$$U_{21} = 149 - 40$$

$$\boxed{U_{21} = £109}$$

c) $S_n = \frac{n}{2}(2a+(n-1)d)$

$$S_n = \frac{n}{2}(2(149) + (n-1) - 2)$$

$$S_n = \frac{n}{2}(298 - 2n + 2)$$

$$S_n = \frac{n}{2}(300 - 2n)$$

But $S_n = £5000$

So, $5000 = \frac{n}{2}(300 - 2n)$

$10\,000 = n(300 - 2n)$

$10\,000 = 300n - 2n^2$

$2n^2 - 300n + 10\,000 = 0$

$\therefore \boxed{n^2 - 150n + 5000 = 0}$

d) $(n - 50)(n - 100) = 0$

$\boxed{\text{Either } n=50 \text{ or } n=100}$

e) $U_{50} = 149 + (50-1)-2$
$= 149 - 98$
$= \underline{51}$

$u_{100} = 149 + (100-1)-2$
$= 149 - 198$
$= \underline{-47}$

$\therefore n=100$ is not a sensible solution to the repayment problem since $U_{100} < 0$

Q10) $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$

a) Substitute in $x = 3$ and $y = 0$:

$0 = \frac{1}{3}(3)^3 - 4(3)^2 + 8(3) + 3$

$0 = \frac{1}{3}(27) - 4(9) + 24 + 3$

$0 = 9 - 36 + 24 + 3$

$\underline{0 = 0}$

$\therefore$ P(3, 0) lies on the curve

b) $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$

$\frac{dy}{dx} = \underline{x^2 - 8x + 8}$

When $x = 3$, $\frac{dy}{dx} = 3^2 - 8(3) + 8 = 9 - 24 + 8 = $

Equation of tangent: $y - y_1 = m(x - x_1)$

Using P(3,0) and $m = -7$ $\rightarrow$ $y - 0 = -7(x - 3)$

$\boxed{y = -7x + 21}$

c) If the tangent is parallel to another tangent at a different point, then the gradients are both equal.

$$\therefore \frac{dy}{dx} = -7 = x^2 - 8x + 8$$

$$x^2 - 8x + 8 = -7$$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0$$

$$\underline{\text{Either } x = 5 \text{ or } x = 3}$$

We already know that when $x=3$, $y=0$, which is point P.

So, at $x=5$, this is point Q.

When $x=5$, $y = \frac{1}{3}(5)^3 - 4(5)^2 + 8(5) + 3$

$$y = \frac{1}{3}(125) - 4(25) + 40 + 3$$

$$y = \frac{125}{3} - 100 + 43$$

$$y = \frac{125}{3} - 57$$

$$y = -\frac{46}{3}$$

$\therefore$ Q has coordinates $\boxed{\left(5, -\frac{46}{3}\right)}$