C1 January 2011 (MA)

Q1a) $16^{-\frac{1}{4}} = \dfrac{1}{16^{1/4}} = \dfrac{1}{\sqrt[4]{16}}$ $\boxed{= \dfrac{1}{2}}$

b) $x(2x^{-\frac{1}{4}})^4 = x(2^4 . x^{-\frac{1}{4}\times 4})$

$= x(16x^{-1})$

$= x\left(\dfrac{16}{x}\right)$

$= \dfrac{16\cancel{x}}{\cancel{x}}$

$\boxed{= 16}$

Q2) $\int(12x^5 - 3x^2 + 4x^{\frac{1}{3}})dx = \dfrac{12x^6}{6} - \dfrac{3x^3}{3} + \dfrac{4x^{\frac{4}{3}}}{\frac{4}{3}} + c$

$= \boxed{2x^6 - x^3 + 3x^{4/3} + c}$

Q3) $\dfrac{5-2\sqrt{3}}{\sqrt{3}-1} = \dfrac{5-2\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{(5-2\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{5\sqrt{3} + 5 - 6 - 2\sqrt{3}}{3 + \sqrt{3} - \sqrt{3} - 1}$

$= \dfrac{3\sqrt{3}-1}{2} = \boxed{-\dfrac{1}{2} + \dfrac{3}{2}\sqrt{3}}$

Q4) $a_1 = 2$

$a_{n+1} = 3a_n - c$

a) $a_2 = 3a_1 - c$

$$\boxed{a_2 = 6 - c}$$

b) $\sum_{i=1}^{3} a_i = 0$

$a_1 + a_2 + a_3 = 0$

$2 + (6-c) + [3(6-c) - c] = 0$

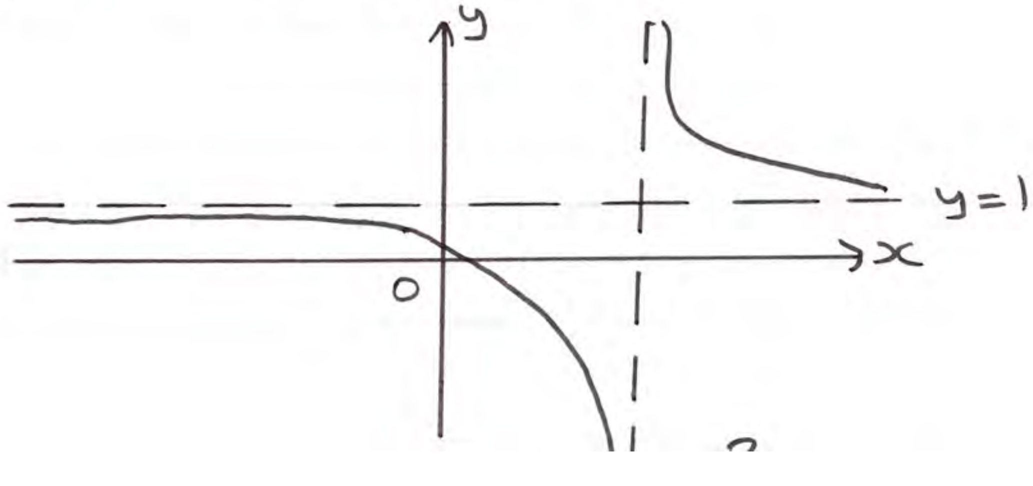
$2 + 6 - c + [18 - 3c - c] = 0$

$2 + 6 - c + 18 - 4c = 0$

$26 = 5c$

$$\boxed{c = \frac{26}{5}}$$

Q5a) $y = f(x-1)$ - transformation of $+1$ along the x-axis:



b) $f(x) = \frac{x}{x-2}, \quad x \neq 2$

$f(x-1) = \frac{x \pm 1}{(x \pm 1) - 2}$

When $x=0$, $f(x-1) = \frac{0 \pm 1}{(0 \pm 1) - 2}$

$$\boxed{= \frac{1}{3}}$$

Q6) First term $= a$. Common difference $= d$

$S_{10} = 162$

a) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}(2a + (10-1)d)$

$S_{10} = 5(2a + 9d)$

$162 = 10a + 45d$

$\therefore \boxed{10a + 45d = 162}$ ①

b) $U_6 = 17$

$U_n = a + (n-1)d$

$U_6 = a + (6-1)d$

$17 = a + 5d$

$\therefore \boxed{a + 5d = 17}$ ②

c) $10a + 45d = 162$ ① $\quad \times 1 \quad 10a + 45d = 162$

$a + 5d = 17$ ② $\quad \times 9 \quad 9a + 45d = 153$ $\}-$

$$\boxed{a = 9}$$

Substitute into ① for d:

$10a + 45d = 162$

$10(9) + 45d = 162$

$90 + 45d = 162$

$45d = 72$

$$\boxed{d = \frac{8}{5}}$$

Q7) $f'(x) = 12x^2 - 8x + 1$

$f(x) = \frac{12x^3}{3} - \frac{8x^2}{2} + x + C$

$\underline{f(x) = 4x^3 - 4x^2 + x + C}$

Since $f(x)$ passes through $(-1, 0)$,
substitute $x = -1$ and $f(x) = 0$

$0 = 4(-1)^3 - 4(-1)^2 + (-1) + c$

$0 = 4(-1) - 4(1) - 1 + c$

$0 = -4 - 4 - 1 + c$

$\therefore c = 9$

$\boxed{f(x) = 4x^3 - 4x^2 + x + 9}$

Q8) $x^2 + (k-3)x + (3-2k) = 0$ has two distinct real roots

a) If a quadratic has two distinct roots, then the discriminant is greater than

$\therefore b^2 - 4ac > 0$

$(k-3)^2 - (4)(1)(3-2k) > 0$

$(k-3)(k-3) - 4(3-2k) > 0$

$k^2 - 6k + 9 - 12 + 8k > 0$

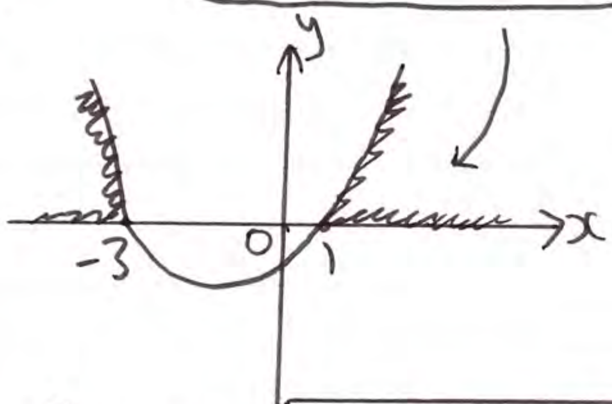
$\boxed{k^2 + 2k - 3 > 0}$

Choosing the valu 'above' the x-ax since $b^2 - 4ac > 0$

b) For $k^2 + 2k - 3 = 0$

$(k+3)(k-1) = 0$

Either $k = -3$ or $k = 1$



Set of possible values for k: $\boxed{k < -3 \text{ or } k > }$

Q9) $2y - 3x - k = 0$

a) Since A(1,4) lies on the curve, Substitute in $x=1$ and $y=4$:

$$2(4) - 3(1) - k = 0$$

$$8 - 3 - k = 0$$

$$5 - k = 0$$

$$\therefore \boxed{k = 5}$$

b) $2y - 3x - 5 = 0$

$$2y = 3x + 5$$

$$y = \frac{3}{2}x + \frac{5}{2}$$

So the $\boxed{\text{gradient is } \frac{3}{2}.}$

c) The gradient of a perpendicular line would have a gradient of $-\frac{2}{3}$.

Equation of perpendicular line:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{2}{3}(x - 1)$$

Using A(1,4) and $m = -\frac{2}{3}$

$$3(y-4) = -2(x-1)$$

$$3y - 12 = -2x + 2$$

$$3y - 12 + 2x - 2 = 0$$

$$\boxed{2x + 3y - 14 = 0}$$

d) When $2x + 3y - 14 = 0$ cuts the x-axis, $y = 0$

So, $2x + 3(0) - 14 = 0$

$$2x = 14$$

$$\underline{x = 7}$$

$\therefore$ $\boxed{B \text{ is at } (7,0)}$

e) Length $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(7-1)^2 + (0-4)^2}$$

$$= \sqrt{6^2 + (-4)^2}$$

$$= \sqrt{52} \quad = \sqrt{13 \times 4} \quad = \sqrt{4}\sqrt{13}$$

$$= \boxed{2\sqrt{13} \text{ units}}$$

Q10a)

i) $y = x(x+2)(3-x)$

When $x = 0$, $y = 0(0+2)(3-0) \underline{= 0}$

When $y = 0$, $x(x+2)(3-x) = 0$

$\underline{\text{Either } x = 0 \text{ or } x = -2 \text{ or } x = 3}$

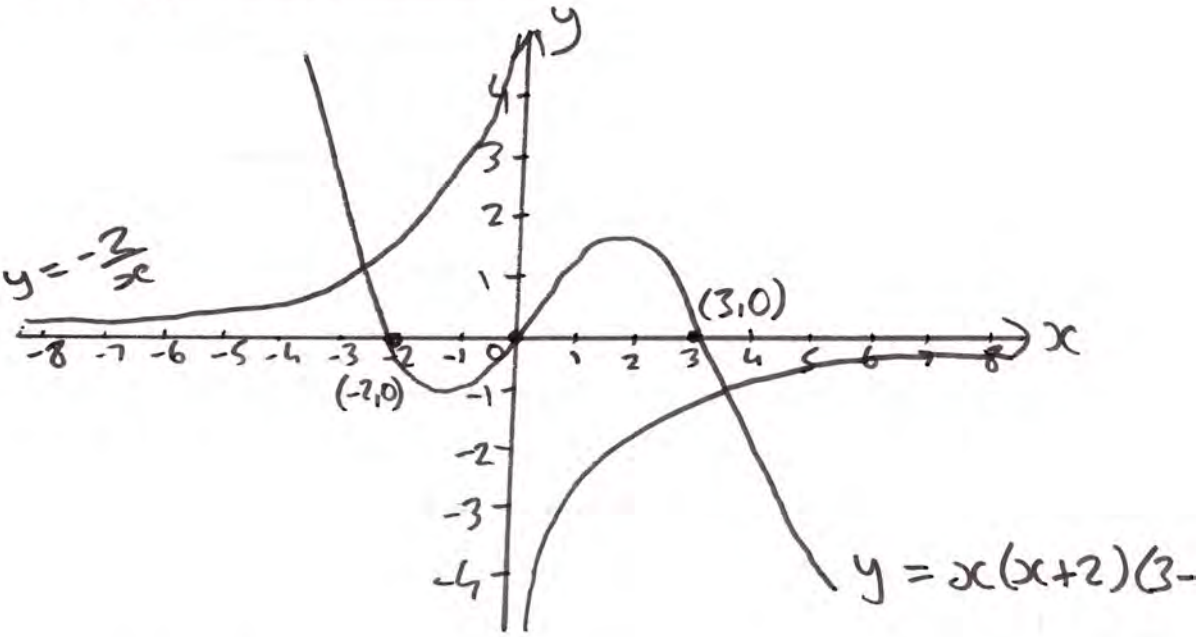
ii) $y = -\dfrac{2}{x}$

When $\underline{x \to 0,\ y \to -\infty}$

When $\underline{x \to -0,\ y \to \infty}$

When $\underline{y \to 0,\ x \to -\infty}$

When $\underline{y \to -0,\ x \to \infty}$



b) For the equation $x(x+2)(3-x) + \dfrac{2}{x} = 0$,

there would be $\underline{\text{2 distinct real solutions,}}$ since there are 2 points of intersection between the two curves.

Q11a) $y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \ x > 0$

$y = \frac{1}{2}x^3 - 9x^{3/2} + 8x^{-1} + 30$

$\frac{dy}{dx} = \frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$

$= \boxed{\frac{3x^2}{2} - \frac{27\sqrt{x}}{2} - \frac{8}{x^2}}$

b) Substitute in $x=4$ and $y=-8$:

$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30$

$-8 = \frac{1}{2}(4)^3 - 9(4)^{3/2} + \frac{8}{4} + 30$

$-8 = \frac{1}{2}(64) - 9(4)(\sqrt{4}) + 2 + 30$

$-8 = 32 - 72 + 2 + 30$

$\underline{-8 = -8}$

$\therefore$ the point $P(4,-8)$ lies on the curve

c) When $x=4$, $\frac{dy}{dx} = \frac{3(4)^2}{2} - \frac{27(\sqrt{4})}{2} - \frac{8}{4^2}$

$$= \frac{3(16)}{2} - \frac{27(2)}{2} - \frac{8}{16}$$

$$= \frac{48}{2} - \frac{54}{2} - \frac{1}{2}$$

$$= -\frac{7}{2}$$

$\therefore$ the gradient of the normal at P is $\frac{2}{7}$.

Equation of normal: $y - y_1 = m(x - x_1)$

Using $P(4, -8)$ and $m = \frac{2}{7}$

$$y - (-8) = \frac{2}{7}(x-4)$$

$$7(y+8) = 2(x-4)$$

$$7y + 56 = 2x - 8$$

$$7y + 56 + 8 - 2x = 0$$

$$\boxed{-2x + 7y + 64 = 0}$$