C1 January 2006 (MA)

Q1) $x^3 - 4x^2 + 3x = x(x^2 - 4x + 3)$

$= \boxed{x(x-3)(x-1)}$

Q2a) $U_2 = (1-3)^2 \quad = (-2)^2 \quad \underline{= 4}$

$U_3 = (4-3)^2 \quad = (1)^2 \quad \underline{= 1}$

$U_4 = (1-3)^2 \quad = (-2)^2 \quad \underline{= 4}$

b) We notice that for U_n, when n is even, $U_n = 4$, and when n is odd, U_n is 1.

$\therefore \boxed{U_{20} = 4}$

Q3a) $y = 5 - 2x$

Substitute in $x=3$

$y = 5 - 2(3) \ \underline{= -1}$

$\therefore P(3, -1)$ lies on the curve

b) $y = 5 - 2x \implies y = -2x + 5$

Gradient of line is -2.

$\therefore$ Gradient of perpendicular line is $\frac{1}{2}$

Equation: $y - (-1) = \frac{1}{2}(x-3) \implies y + 1 = \frac{1}{2}x - \frac{3}{2}$

$\boxed{x - 2y - 5 = 0}$

Q4a) $y = 2x^2 - \dfrac{6}{x^3}$, $x \neq 0$

$y = 2x^2 - 6x^{-3}$

$\dfrac{dy}{dx} = 4x + 18x^{-4}$

$= \boxed{4x + \dfrac{18}{x^4}}$

b) $\int (2x^2 - 6x^{-3})\,dx = \dfrac{2x^3}{3} - \dfrac{6x^{-2}}{-2} + C$

$= \dfrac{2x^3}{3} + 3x^{-2} + C$

$= \boxed{\dfrac{2x^3}{3} + \dfrac{3}{x^2} + C}$

Q5a) $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9}\,\sqrt{5} = \boxed{3\sqrt{5}}$

b) $\dfrac{2(3+\sqrt{5})}{3-\sqrt{5}} = \dfrac{6+2\sqrt{5}}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}}$

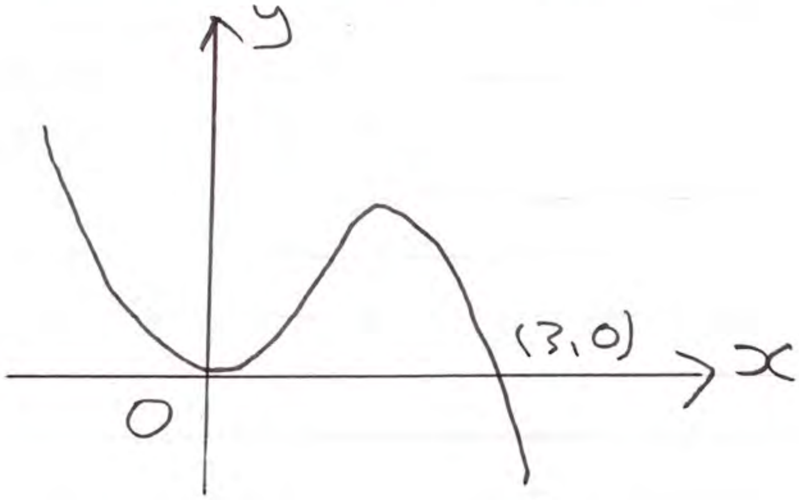
$= \dfrac{(6+2\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$

$= \dfrac{18 + 6\sqrt{5} + 6\sqrt{5} + 10}{9 + 3\sqrt{5} - 3\sqrt{5} - 5}$

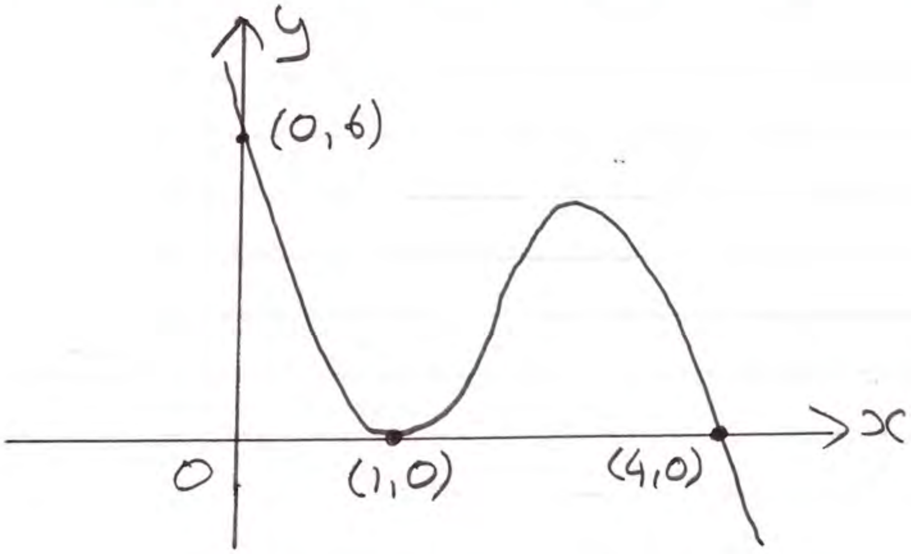
$= \dfrac{28 + 12\sqrt{5}}{4}$

$= \boxed{7 + 3\sqrt{5}}$

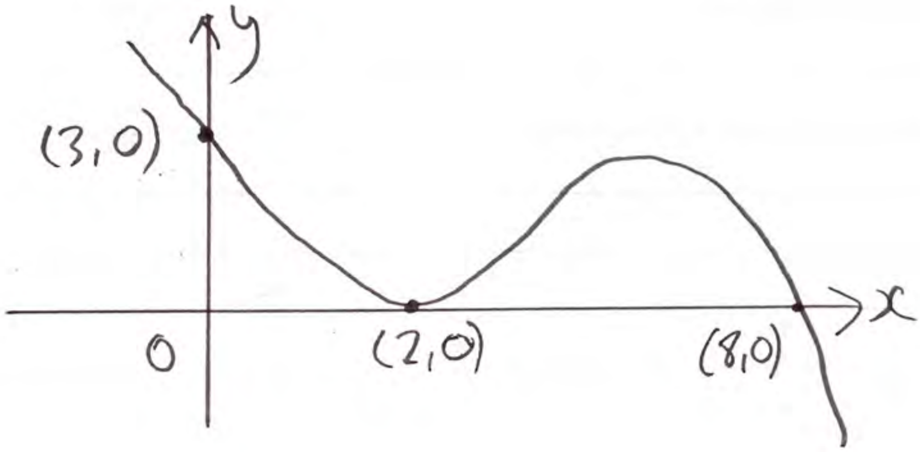
Q6a) $y=f(x+1)$: transformation of x-coordinates to the left by 1:



b) $y=2f(x)$: multiply y-coordinates by 2:



c) $y=f(\frac{1}{2}x)$ - multiply x-coordinates by 2:



Q7a) $a = 500,\ d = 200$

$S_n = \frac{n}{2}(2a + (n-1)d$

$S_2 = \frac{2}{2}(2(500) + (2-1)200)$

$= 1(1000 + 200)$

$= \boxed{£1200}$

b) $a = 500,\ d = 200$

$U_n = a + (n-1)d$

$U_8 = 500 + (7)200$

$= \boxed{£1900}$

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_8 = \frac{8}{2}(2(500) + (8-1)d)$

$= 4(1000 + 1400)$

$= 4 \times 2400$

$= \boxed{£9600}$

d) $32000 = \frac{1}{2}n(2(500) + (n-1)200)$

$64000 = n(1000 + 200n - 200)$

$64000 = n(800 + 200n)$

$64000 = 800n + 200n^2$

$200n^2 + 800n - 64000 = 0$

$n^2 + 4n - 320 = 0$

$(n+20)(n-16) = 0$

Since n has to be positive, $\underline{n=16}$

$\therefore$ Alice is 26 when she receives her last allowance

Q8) $f'(x) = 3 + \frac{5x^2+2}{x^{1/2}}, \quad x > 0$

$f'(x) = 3 + \frac{5x^2}{x^{1/2}} + \frac{2}{x^{1/2}}$

$f'(x) = 3 + 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$

$f(x) = 3x + \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$f(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C$

$f(x) = 3x + 2x^2\sqrt{x} + 4\sqrt{x} + C$

Using the point (1,6):

$6 = 3(1) + 2(1)^2(\sqrt{1}) + 4\sqrt{1} + c$

$6 = 3 + 2 + 4 + c$

$\underline{c = -3}$

$\therefore \boxed{f(x) = 3x + 2x^2\sqrt{x} + 4\sqrt{x} - 3}$

Q9a) When curve cuts x-axis, y=0

$\therefore (x-1)(x^2-4) = 0$

$(x-1)(x-2)(x+2) = 0$

$\therefore$ <u>x-coordinate of P is -2</u>

<u>x-coordinate of Q is 2</u>

b) $y = (x-1)(x^2-4)$

$y = x^3 - 4x - x^2 + 4$

$y = x^3 - x^2 - 4x + 4$

$\boxed{\frac{dy}{dx} = 3x^2 - 2x - 4}$

c) At $x = -1$, $\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4$

$= 3 + 2 - 4$

$\underline{= 1}$

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $(-1, 6)$: $y - 6 = 1(x - (-1))$

$$y - 6 = x + 1$$

$$\boxed{y = x + 7}$$

d) Since the gradient of the tangent at R is equal, $3x^2 - 2x - 4 = 1$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0$$

$$x = \frac{5}{3} \quad \text{or} \quad x = -1$$

We already know that $(-1, 6)$ lies on a tangent to the curve, so find the coordinates when $x = \frac{5}{3}$:

$$y = (x - 1)(x^2 - 4)$$

$$y = \left(\frac{5}{3} - 1\right)\left(\left(\frac{5}{3}\right)^2 - 4\right)$$

$$y = \left(\frac{2}{3}\right)\left(\frac{25}{9} - 4\right)$$

$$y = \left(\frac{2}{3}\right)\left(-\frac{11}{9}\right)$$

$$y = -\frac{22}{27}$$

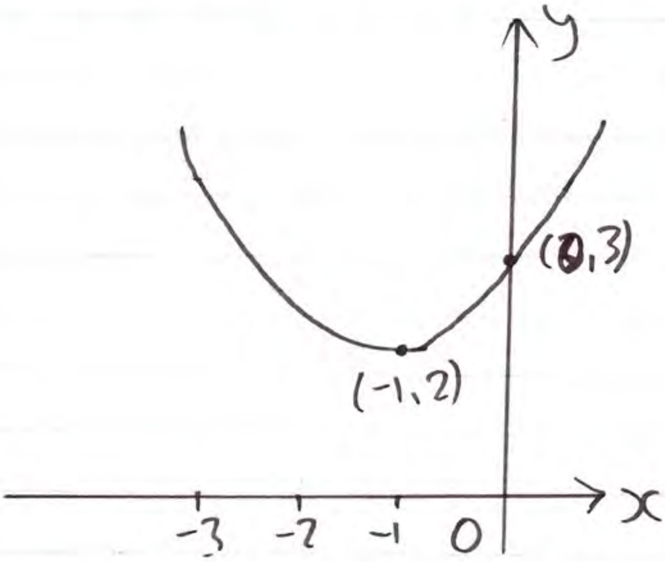
$\therefore$ coordinates of R are $\boxed{\left(\frac{5}{3}, -\frac{22}{27}\right)}$

Q10a) $x^2+2x+3 = (x+1)^2 - 1 + 3$

$$= \underline{(x+1)^2+2}$$

$\therefore$ $\boxed{a=1 \quad \text{and} \quad b=2}$

b) sketch of $y=x^2+2x+3$:



- $x=0$, $y=3$
- Turning point at $(-1,2)$ [from (a)]

c) $b^2-4ac = 2^2 - 4(1)(3) \underline{=-8}$

$\therefore$ there are no real roots at $y=0$, and so the curve doesn't cross the x-axis

d) $x^2+kx+3=0$ has no real roots, so $b^2-4ac<0$

$k^2-4(1)(3)<0$

$k^2-12<0$

$k^2<12$

$$\boxed{-2\sqrt{3} < k < 2\sqrt{3}}$$