

C1 June 2012 (MA)

$$\begin{aligned}
 \text{Q1)} \quad \int \left(6x^2 + \frac{2}{x^2} + 5 \right) dx &= \int (6x^2 + 2x^{-2} + 5) dx \\
 &= \frac{6x^3}{3} + \frac{2x^{-1}}{-1} + 5x + C \\
 &= 2x^3 - 2x^{-1} + 5x + C \\
 &= \boxed{2x^3 - \frac{2}{x} + 5x + C}
 \end{aligned}$$

$$\text{Q2a)} \quad (32)^{\frac{3}{5}} = (\sqrt[5]{32})^3 = 2^3 \quad \boxed{= 8}$$

$$\text{b)} \quad \left(\frac{25x^4}{4} \right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{25x^4}{4} \right)^{\frac{1}{2}}}$$

$$= \frac{1}{\left(\frac{5x^2}{2} \right)}$$

$$\boxed{= \frac{2}{5x^2}}$$

$$\begin{aligned}
 3) \quad \frac{2}{\sqrt{12}-\sqrt{8}} &= \frac{2 \times (\sqrt{12} + \sqrt{8})}{(\sqrt{12}-\sqrt{8})(\sqrt{12}+\sqrt{8})} \\
 &= \frac{2\sqrt{12} + 2\sqrt{8}}{12 + \sqrt{12}\sqrt{8} - \sqrt{12}\sqrt{8} - 8} \\
 &= \frac{2\sqrt{12} + 2\sqrt{8}}{4} \\
 &= \frac{2\sqrt{12} + 2\sqrt{8}}{\sqrt{16}} \\
 &= \frac{2\sqrt{4}\sqrt{3} + 2\sqrt{4}\sqrt{2}}{\sqrt{4}\sqrt{4}} \\
 &= \frac{4\sqrt{3} + 4\sqrt{2}}{4} \\
 &= \boxed{\sqrt{3} + \sqrt{2}}
 \end{aligned}$$

$$Q4) \quad y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

$$\begin{aligned}
 a) \quad \frac{dy}{dx} &= 15x^2 - 8x^{\frac{1}{3}} + 2 \\
 &= \boxed{15x^2 - 8\sqrt[3]{x} + 2}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \frac{d^2y}{dx^2} &= 30x - \frac{8x^{-\frac{2}{3}}}{3} \\
 &= \boxed{30x - \frac{8}{3x^{\frac{2}{3}}}}
 \end{aligned}$$

$$Q5) a_1 = 3$$

$$a_{n+1} = 2a_n - c \quad (n \geq 1)$$

$$a) a_2 = 2a_1 - c$$

$$\boxed{a_2 = 6 - c}$$

$$b) a_3 = 2a_2 - c$$

$$a_3 = 2(6 - c) - c$$

$$a_3 = 12 - 2c - c$$

$$\boxed{a_3 = 12 - 3c}$$

$$c) \sum_{i=1}^4 a_i \geq 23$$

$$\sum_{i=1}^4 a_i = a_1 + a_2 + a_3 + a_4$$

$$a_4 = 2a_3 - c$$

$$a_4 = 2(12 - 3c) - c$$

$$a_4 = 24 - 6c - c$$

$$\underline{a_4 = 24 - 7c}$$

$$\therefore 3 + (6 - c) + (12 - 3c) + (24 - 7c) \geq 23$$

$$3 + 6 - c + 12 - 3c + 24 - 7c \geq 23$$

$$45 - 11c \geq 23$$

$$22 - 11c \geq 0$$

$$22 \geq 0 + 11c$$

$$22 \geq 11c$$

$$11c \leq 22$$

$$\therefore \boxed{c \leq 2}$$

Q6) First term, $a = 10p$
Common difference, $d = 5p$

a) $U_n = a + (n-1)d$

$$U_{15} = 10 + (15-1)5$$

$$U_{15} = 10 + 70$$

$$\boxed{U_{15} = 80p}$$

b) $S_n = \frac{n}{2} (2a + (n-1)d)$

$$S_{60} = \frac{60}{2} (2(10) + (60-1)5)$$

$$S_{60} = 30 (20 + 295)$$

$$S_{60} = 30 \times 315$$

$$\boxed{S_{60} = 9450p}$$

$\therefore S_{60}$ (total savings) are £94.50

- c) For the sister, First term, $a = 10p$
Common difference, $d = 10p$

$$S_m = \frac{m}{2} (2a + (m-1)d)$$

$$6300 = \frac{m}{2} (2(10) + (m-1)10)$$

$$6300 = \frac{m}{2} (20 + 10m - 10)$$

$$6300 = \frac{m}{2} (10 + 10m)$$

$$12600 = m(10 + 10m)$$

$$12600 = 10m + 10m^2$$

$$1260 = m + m^2$$

$$m(m+1) = 1260$$

$$\therefore m(m+1) = 35 \times 36$$

d) $m = 35$

$$Q7) \quad f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

$$\text{when } x=4, \quad f'(x) = \frac{1}{2}(4) - \frac{6}{\sqrt{4}} + 3$$

$$= 2 - 3 + 3$$

$$= \underline{2}$$

$\therefore$ the gradient of the tangent to $y=f(x)$ at point $P(4, -1)$ is 2

Equation of tangent: $y - y_1 = m(x - x_1)$

using $P(4, -1)$ and $m=2 \longrightarrow y - (-1) = 2(x - 4)$

$$y + 1 = 2x - 8$$

$$\boxed{y = 2x - 9}$$

$$b) \quad f'(x) = \frac{1}{2}x - 6x^{-\frac{1}{2}} + 3$$

$$f(x) = \int \left(\frac{1}{2}x - 6x^{-\frac{1}{2}} + 3 \right) dx$$

$$f(x) = \frac{1}{2} \cdot \frac{x^2}{2} - \frac{6x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c$$

$$\underline{f(x) = \frac{x^2}{4} - 12\sqrt{x} + 3x + c}$$

Since $y=f(x)$ passes through $(4, -1)$,
Substitute in $x=4$ and $y=-1$

$$-1 = \frac{4^2}{4} - 12\sqrt{4} + 3(4) + c$$

$$-1 = 4 - 24 + 12 + c$$

$$-1 = -8 + c$$

$$\therefore \underline{c = 7}$$

$$\boxed{f(x) = \frac{x^2}{4} - 12\sqrt{x} + 3x + 7}$$

$$\begin{aligned} \text{Q8a)} \quad 4x - 5 - x^2 &\equiv -(x^2 - 4x + 5) \\ &= -[x^2 - 4x + 4 - 4 + 5] \\ &= -[(x-2)^2 + 1] \\ &= \boxed{-1 - (x-2)^2} \end{aligned}$$

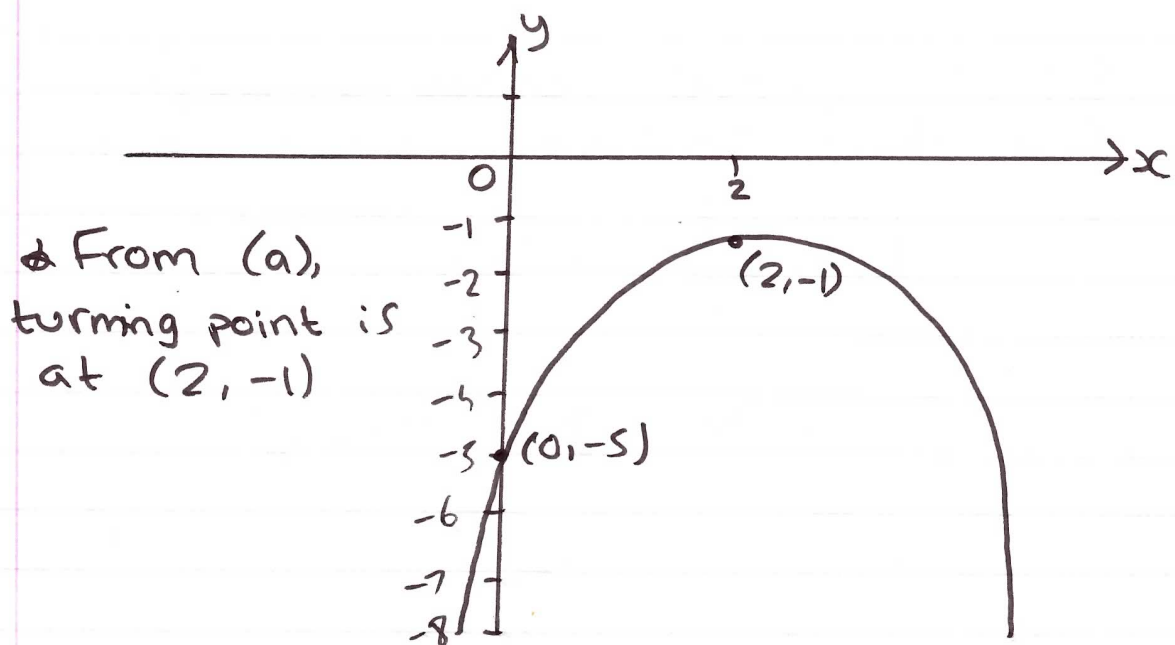
$$\text{b)} \quad b^2 - 4ac = (4)^2 - (4)(-1)(-5)$$

$$b^2 - 4ac = 16 - 20$$

$$\boxed{b^2 - 4ac = -4}$$

$$\text{c)} \quad y = 4x - 5 - x^2$$

$$\text{When } \underline{x=0, y = -5}$$



Q9) $L_1: 4y + 3 = 2x$

a) When $y = 4$, $4(4) + 3 = 2x$

$$2(8) + 3 = 2x$$

$$19 = 2x$$

$$x = \frac{19}{2}$$

$$\therefore p = \frac{19}{2}$$

b) $4y + 3 = 2x$

$$4y = 2x - 3$$

$$y = \frac{1}{2}x - \frac{3}{4}$$

Gradient of L_1 is $\frac{1}{2}$

If the line L_2 is perpendicular to L_1 , then its gradient is -2.

Equation of L_2 : $y - y_1 = m(x - x_1)$

Using point $(2, 4)$ and $m = -2$ $\rightarrow y - 4 = -2(x - 2)$

$$y - 4 = -2x + 4$$

$$\boxed{2x + y - 8 = 0}$$

C) To find point of intersection, solve simultaneous equations:

$$y = \frac{1}{2}x - \frac{3}{4} \quad (1)$$

$$2x + y - 8 = 0 \quad (2)$$

Substitute (1) into (2):

$$2x + \left(\frac{1}{2}x - \frac{3}{4}\right) - 8 = 0$$

$$2x + \frac{1}{2}x - \frac{3}{4} - 8 = 0$$

$$\frac{5}{2}x = \frac{35}{4}$$

$$5x = \frac{35}{2}$$

$$\underline{x = \frac{7}{2}}$$

Substitute into ① for y :

$$y = \frac{1}{2}x - \frac{3}{4}$$

$$y = \frac{1}{2}\left(\frac{7}{2}\right) - \frac{3}{4}$$

$$y = \frac{7}{4} - \frac{3}{4}$$

$$\underline{y = 1}$$

$\therefore$ the coordinates of D are

$$\boxed{\left(\frac{7}{2}, 1\right)}$$

d) Length $CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{\left(\frac{7}{2} - 2\right)^2 + (1 - 4)^2}$$

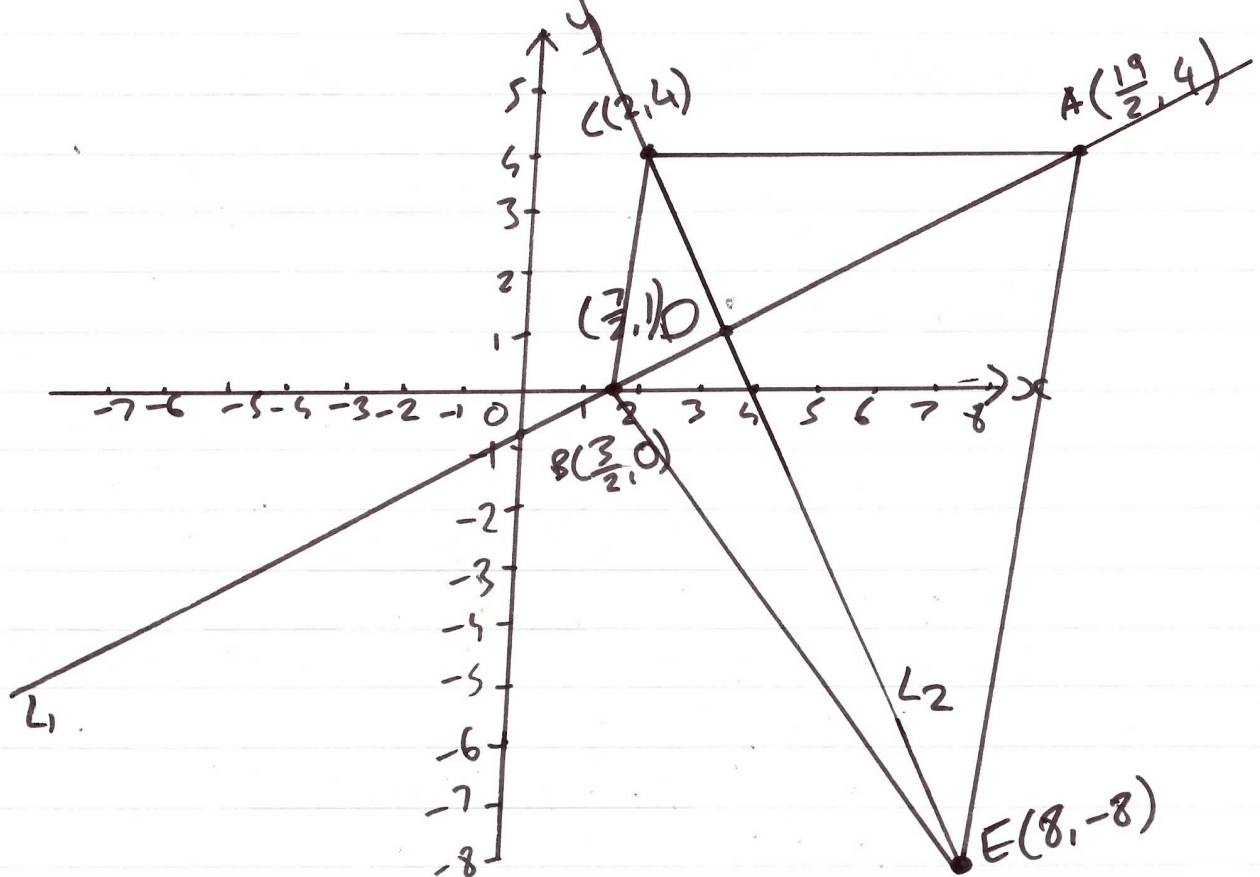
$$= \sqrt{\left(\frac{3}{2}\right)^2 + (-3)^2}$$

$$= \sqrt{\frac{9}{4} + 9}$$

$$= \sqrt{\frac{45}{4}} = \frac{\sqrt{45}}{2} = \frac{\sqrt{9 \times 5}}{2}$$

$$= \boxed{\frac{3\sqrt{5}}{2} \text{ units}}$$

e)



If $CE = 3 \times CD$, then $CE = 3 \times (\frac{3}{2}\sqrt{5})$
 $= \frac{9}{2}\sqrt{5}$ units

$$\vec{CD} = \begin{pmatrix} \frac{7}{2} - 2 \\ 1 - 4 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ -3 \end{pmatrix}$$

$$\therefore \vec{DE} = 3 \begin{pmatrix} \frac{3}{2} \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ -9 \end{pmatrix}$$

$$\therefore E \text{ is at } \begin{pmatrix} \frac{7}{2} + \frac{9}{2} \\ 1 - 9 \end{pmatrix}$$

$$\therefore \underline{E \text{ is at } (8, -8)}$$

$$\begin{aligned}
 \text{Length } AD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(\frac{19}{2} - \frac{7}{2}\right)^2 + (4 - 1)^2} \\
 &= \sqrt{6^2 + 3^2} \\
 &= \sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \sqrt{5} \\
 &= \underline{3\sqrt{5} \text{ units}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Length } AB &= \sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \sqrt{5} \\
 &= \underline{4\sqrt{5} \text{ units}}
 \end{aligned}$$

$$\therefore \underline{\text{Length } BD = \sqrt{5} \text{ units}}$$

$$\therefore AD = 3BD$$

$$\vec{AD} = \begin{pmatrix} \frac{7}{2} - \frac{19}{2} \\ 1 - 4 \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$$

$$\begin{aligned}
 \therefore \vec{DB} &= \frac{1}{3} \begin{pmatrix} -6 \\ -3 \end{pmatrix} \\
 &= \begin{pmatrix} -2 \\ -1 \end{pmatrix}
 \end{aligned}$$

$$\therefore B \text{ is at } \begin{pmatrix} \frac{7}{2} - 2 \\ 1 - 1 \end{pmatrix}$$

$$\therefore B \text{ is at } \underline{\underline{\begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}}}$$

$$\begin{aligned}
\text{Area } ACBE &= \text{Area } \triangle ABC + \text{Area } \triangle ABE \\
&= \frac{1}{2} (AB)(CD) + \frac{1}{2} (AB)(DE) \\
&= \frac{1}{2} (4\sqrt{5})\left(\frac{3}{2}\sqrt{5}\right) + \frac{1}{2} (4\sqrt{5})\left(\frac{9}{2}\sqrt{5} - \frac{3}{2}\sqrt{5}\right) \\
&= \frac{1}{2} (6 \times 5) + \frac{1}{2} (4\sqrt{5})(3\sqrt{5}) \\
&= \frac{1}{2} (30) + \frac{1}{2} (60) \\
&= 15 + 30 \\
&= \boxed{45 \text{ units}^2}
\end{aligned}$$

Q10a) $f(x) = x^2(9-2x)$

When the curve cuts the x -axis, $f(x)=0$

$$\therefore x^2(9-2x)=0$$

Either $x^2=0$ or $9-2x=0$

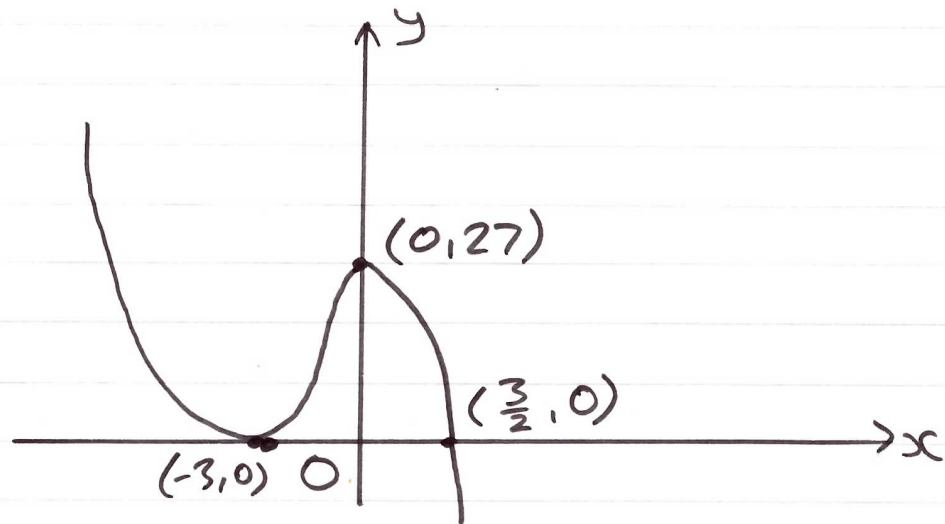
$x^2=0$ is the origin, 0

$$9-2x=0,$$

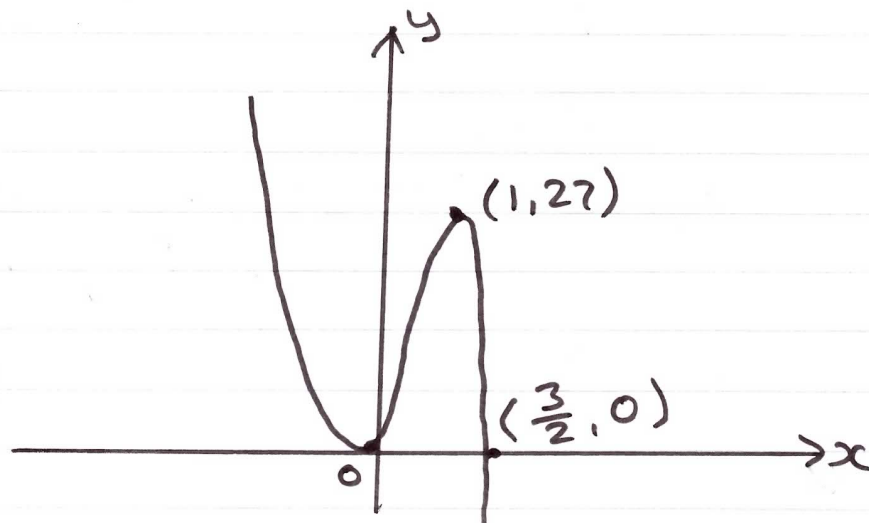
$$2x=9 \Rightarrow x=\frac{9}{2}$$

$$\therefore \boxed{A \text{ is at } \left(\frac{9}{2}, 0\right)}$$

b i) $y = f(x+3)$ - transformation vertically by '-3'.



ii) $y = f(3x)$ - multiply x-coordinates by $\frac{1}{3}$:



c) $y = f(x) + k$ has a maximum point at $(3, 10)$
 $f(x) + k$ - transformation ~~horizo~~ vertically of 'k'

If the original maximum point is at $(3, 27)$ and the new maximum point is at $(3, 10)$, compare the y-coordinates:

$(3, 27) - (3, 10)$, transformation for y of -17.

$$\therefore \boxed{k = -17}$$