

C1 June 2011 (MA)

$$Q1a) 25^{1/2} = \sqrt{25} \boxed{= 5}$$

$$b) 25^{-3/2} = \frac{1}{25^{3/2}} = \frac{1}{(\sqrt[2]{25})^3} = \boxed{\frac{1}{125}}$$

$$Q2) y = 2x^5 + 7 + \frac{1}{x^3}, x \neq 0$$

$$y = 2x^5 + 7 + x^{-3}$$

$$a) \frac{dy}{dx} = 10x^4 - 3x^{-4}$$

$$= \boxed{10x^4 - \frac{3}{x^4}}$$

$$b) \int (2x^5 + 7 + x^{-3}) dx = \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} + C$$

$$= \frac{x^6}{3} + 7x - \frac{x^{-2}}{2} + C$$

$$= \boxed{\frac{x^6}{3} + 7x - \frac{1}{2x^2} + C}$$

$$Q3) \text{ Gradient } PQ = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 6}{9 - (-1)}$$

$$= \underline{\underline{-\frac{3}{5}}}$$

$\therefore$ the gradient of the line perpendicular to PQ is $\frac{5}{3}$

$$\begin{aligned}\text{Midpoint PQ} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-1 + 9}{2}, \frac{6 + 0}{2} \right) \\ &= \underline{(4, 3)}\end{aligned}$$

Equation of perpendicular line:

$$y - y_1 = m(x - x_1)$$

Using Midpoint PQ (4, 3) and $m = \frac{5}{3}$:

$$y - 3 = \frac{5}{3}(x - 4)$$

$$3(y - 3) = 5(x - 4)$$

$$3y - 9 = 5x - 20$$

$$\boxed{5x - 3y - 11 = 0}$$

$$\begin{aligned} \text{Q4)} \quad x+y=2 &\Rightarrow x=2-y & \textcircled{1} \\ 4y^2-x^2=11 &\Rightarrow 4y^2-x^2=11 & \textcircled{2} \end{aligned}$$

Substitute $\textcircled{1}$ into $\textcircled{2}$:

$$4y^2 - (2-y)^2 = 11$$

$$4y^2 - (2-y)(2-y) = 11$$

$$4y^2 - (4 - 4y + y^2) = 11$$

$$4y^2 - 4 + 4y - y^2 - 11 = 0$$

$$3y^2 + 4y - 15 = 0$$

$$(3y - 5)(y + 3) = 0$$

$$\text{Either } y = \frac{5}{3} \text{ or } y = -3$$

Substitute into $\textcircled{1}$ for x :

$$\text{When } y = \frac{5}{3}, \quad x = 2 - \frac{5}{3} = \underline{\underline{\frac{1}{3}}}$$

$$\text{When } y = -3, \quad x = 2 - (-3) = \underline{\underline{5}}$$

$$\text{Solution set: } \boxed{x = \frac{1}{3}, y = \frac{5}{3} \text{ and } x = 5, y = -3}$$

Q5) $a_1 = k$
 $a_{n+1} = 5a_n + 3, n \geq 1$

a) $a_2 = 5a_1 + 3$

$$\boxed{a_2 = 5k + 3}$$

b) $a_3 = 5a_2 + 3$

$$a_3 = 5(5k + 3) + 3$$

$$a_3 = 25k + 15 + 3$$

$$\boxed{a_3 = 25k + 18}$$

c) $\sum_{r=1}^4 a_r = a_1 + a_2 + a_3 + a_4$

$$a_4 = 5a_3 + 3$$

$$a_4 = 5(25k + 18) + 3$$

$$a_4 = 125k + 90 + 3$$

$$\underline{a_4 = 125k + 93}$$

$$\therefore \sum_{r=1}^4 a_r = k + 5k + 3 + 25k + 18 + 125k + 93$$

$$= \boxed{156k + 114}$$

$$\therefore \sum_{r=1}^4 a_r = \boxed{6(26k + 19)}$$

$\therefore$ divisible by 6

$$\begin{aligned}
 \text{Q6a)} \quad \frac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}} &= \frac{6x + 3x^{\frac{5}{2}}}{x^{1/2}} \\
 &= \frac{6x}{x^{1/2}} + \frac{3x^{5/2}}{x^{1/2}} \\
 &= 6x^{\frac{1}{2}} + 3x^2
 \end{aligned}$$

$$\therefore p = \frac{1}{2} \quad \text{and} \quad q = 2$$

$$\text{b)} \quad \frac{dy}{dx} = \frac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$$

$$\frac{dy}{dx} = 6x^{\frac{1}{2}} + 3x^2$$

$$y = \int (6x^{\frac{1}{2}} + 3x^2) dx$$

$$y = \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3x^3}{3} + c$$

$$y = 4x^{3/2} + x^3 + c$$

$$\underline{y = 4x\sqrt{x} + x^3 + c}$$

Since $y = 90$ when $x = 4$, substitute in:

$$90 = 4(4)(\sqrt{4}) + 4^3 + c$$

$$90 = 4(4)(2) + 64 + c$$

$$90 = 96 + c$$

$$\therefore \underline{c = -6}$$

$$\boxed{y = 4x\sqrt{x} + x^3 - 6}$$

$$Q7a) \quad f(x) = x^2 + (k+3)x + k$$

$$b^2 - 4ac = (k+3)^2 - (4)(1)(k)$$

$$= (k+3)(k+3) - 4k$$

$$= k^2 + 6k + 9 - 4k$$

$$= \boxed{k^2 + 2k + 9}$$

$$b) \quad k^2 + 2k + 9 \equiv (k+1)^2 - 1^2 + 9$$

$$= (k+1)^2 - 1 + 9$$

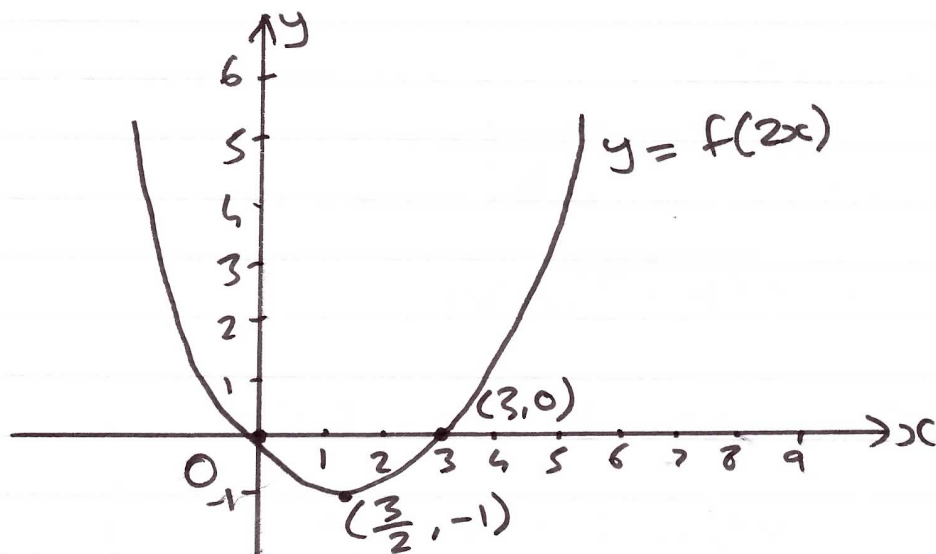
$$= \boxed{(k+1)^2 + 8}$$

- c) For a quadratic to have real roots, the discriminant must be greater than or equal to zero.

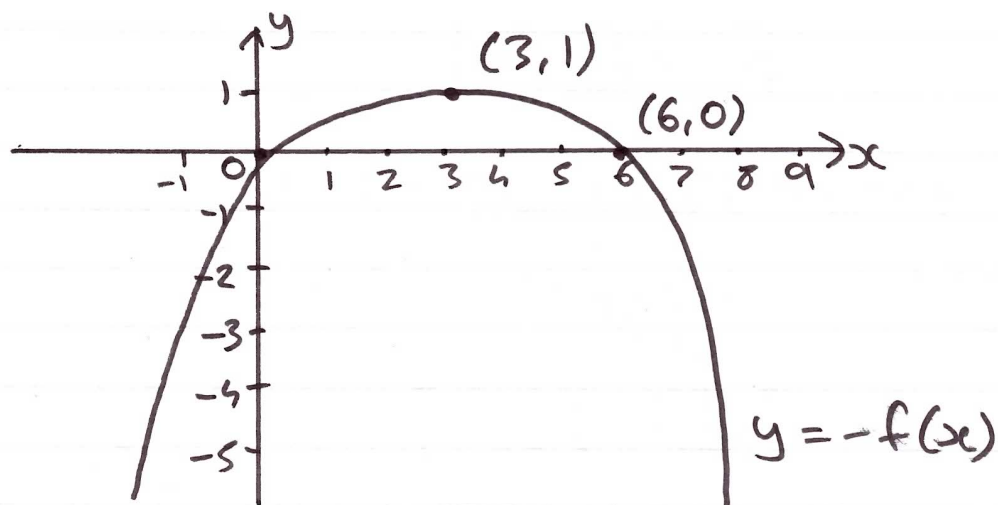
For this discriminant, $(k+1)^2 + 8$, the value inside the brackets is squared, so will always be positive. When you add 8, it will still be positive.

$$\therefore f(x) = 0 \text{ has real roots.}$$

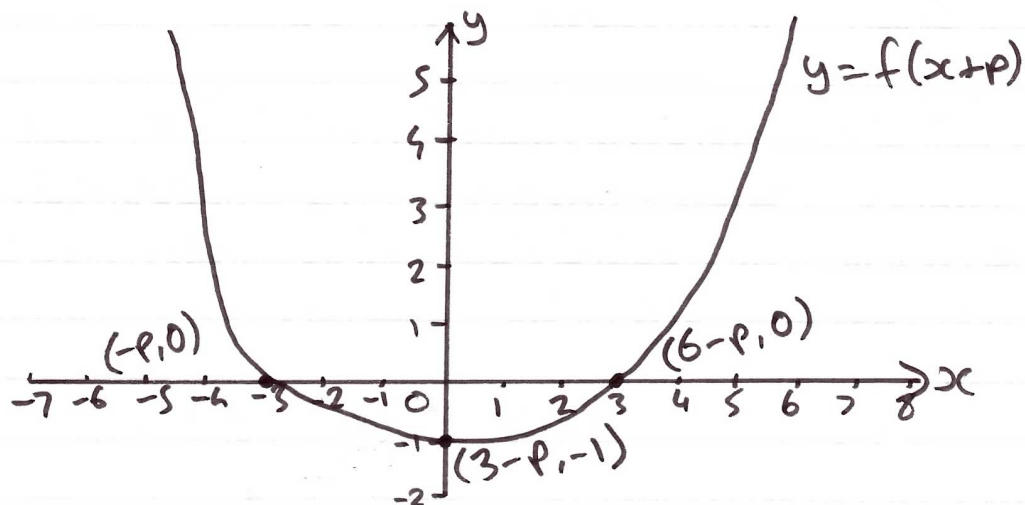
Q8a) $y = f(2x)$ - halve the x -coordinates:



b) $y = -f(x)$ - reflection in the x -axis:



c) $y = f(x+p)$ - transformation horizontally of ' $-p$ '



Q9a) $2 + 4 + 6 + \dots + 100$

First term, $a = 2$

Common difference, $d = 2$

There are 50 even numbers between 1 and 100, so use S_{50} .

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{50} = \frac{50}{2} (2(2) + (50-1)2)$$

$$S_{50} = 25(4 + (49 \times 2))$$

$$S_{50} = 25(4 + 98)$$

$$S_{50} = 25 \times 102$$

$$\boxed{S_{50} = 2550}$$

$$\therefore 2 + 4 + 6 + \dots + 100 = 2550$$

b) $k + 2k + 3k + \dots + 100$

i) $U_n = a + (n-1)d$

First term, $a = k$

Common difference, $d = k$

$$100 = k + (n-1)k$$

$$100 = k + kn - k$$

$$100 = kn$$

$$\therefore \boxed{n = \frac{100}{k}}$$

$$ii) S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{\frac{100}{k}} = \frac{\frac{100}{k}}{2} (2k + (\frac{100}{k} - 1)k)$$

$$S_{\frac{100}{k}} = \frac{100}{2k} (2k + \frac{100k}{k} - k)$$

$$S_{\frac{100}{k}} = \frac{100}{2k} (k + 100)$$

$$S_{\frac{100}{k}} = \frac{100k}{2k} + \frac{10000}{2k}$$

$$\boxed{S_{\frac{100}{k}} = 50 + \frac{5000}{k}}$$

$$c) (2k+1), (4k+4), (6k+7), \dots,$$

$$\text{First term, } a = (2k+1)$$

$$\text{Common difference, } d = (2k+3)$$

$$U_n = a + (n-1)d$$

$$U_{50} = (2k+1) + (50-1)(2k+3)$$

$$U_{50} = (2k+1) + (49)(2k+3)$$

$$U_{50} = 2k + 1 + 98k + 147$$

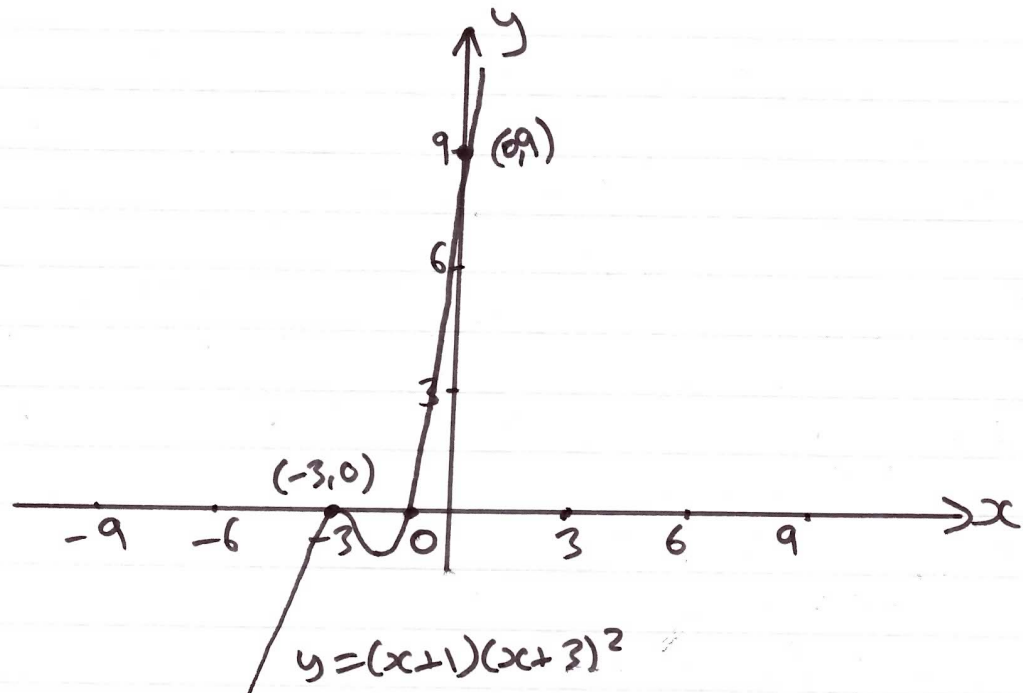
$$\boxed{U_{50} = 100k + 148}$$

Q10a) $y = (x+1)(x+3)^2$

When $x=0$, $y = (0+1)(0+3)^2 = \underline{9}$

When $y=0$, $(x+1)(x+3)^2 = 0$

Either $x=-1$ or $x=-3$ or $x=-3$



b) $\frac{dy}{dx} = \frac{dy}{dx} ((x+1)(x+3)^2)$

$$y = (x+1)(x+3)^2$$

$$y = (x+1)(x+3)(x+3)$$

$$y = (x+1)(x^2 + 6x + 9)$$

$$y = x^3 + 7x^2 + 15x + 9$$

$$\frac{dy}{dx} = 3x^2 + 14x + 15$$

$$\begin{aligned} \text{c) When } x = -5, \frac{dy}{dx} &= 3(-5)^2 + 14(-5) + 15 \\ &= 3(25) - 70 + 15 \\ &= 75 - 70 + 15 \\ &= \underline{20} \end{aligned}$$

$\therefore$ the gradient of the tangent at A is 20

$$\text{When } x = -5, y = (-5)^3 + 7(-5)^2 + 15(-5) + 9$$

$$y = -125 + 7(25) - 75 + 9$$

$$y = -125 + 175 - 75 + 9$$

$$\underline{y = -16}$$

Equation of tangent at A: $y - y_1 = m(x - x_1)$

Using A (-5, -16)
and $m = 20$

$$\rightarrow y - (-16) = 20(x - (-5))$$

$$y + 16 = 20(x + 5)$$

$$y + 16 = 20x + 100$$

$$\boxed{y = 20x + 84}$$

- d) If the tangents at A and B are parallel, then their gradients are both equal.

$$\therefore \frac{dy}{dx} = 20 = 3x^2 + 14x + 15$$

$$\therefore 3x^2 + 14x + 15 = 20$$

$$3x^2 + 14x - 5 = 0$$

$$(3x - 1)(x + 5) = 0$$

$$\text{Either } x = \frac{1}{3} \text{ or } x = -5$$

We already know the coordinates when $x = -5$, which is A(-5, -16).

So, the point B has x-coordinate $\frac{1}{3}$.