

(1 June 2010 (MA))

$$\begin{aligned}
 \text{Q1)} \quad \sqrt{75} - \sqrt{27} &= \sqrt{25 \times 3} - \sqrt{9 \times 3} \\
 &= \sqrt{25} \sqrt{3} - \sqrt{9} \sqrt{3} \\
 &= 5\sqrt{3} - 3\sqrt{3} \\
 &= \boxed{2\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q2)} \quad \int (8x^3 + 6x^{\frac{1}{2}} - 5) dx &= \frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + C \\
 &= 2x^4 + 4x^{3/2} - 5x + C \\
 &= \boxed{2x^4 + 4x\sqrt{x} - 5x + C}
 \end{aligned}$$

$$\text{Q3a)} \quad 3(x-2) < 8-2x$$

$$3x - 6 < 8 - 2x$$

$$5x < 14$$

$$x < \frac{14}{5} \Rightarrow \boxed{x < 2.8}$$

$$\text{b)} \quad (2x-7)(1+x) < 0$$

$$2x + 2x^2 - 7 - 7x < 0$$

$$2x^2 - 5x - 7 < 0$$

$$(2x-7)(x+1) < 0$$

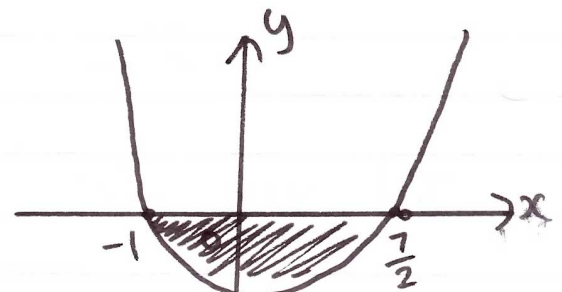
For $2x^2 - 5x - 7 = 0$,

$$(2x-7)(x+1) = 0$$

Either $x = \frac{7}{2}$ or $x = -1$

Set of possible values of x :

$$-1 < x < \frac{7}{2}$$



Consider the values 'under' the x -axis, since $2x^2 - 5x - 7 < 0$

c) both $3(x-2) < 8-2x$ and $(2x-7)(1+x) < 0$

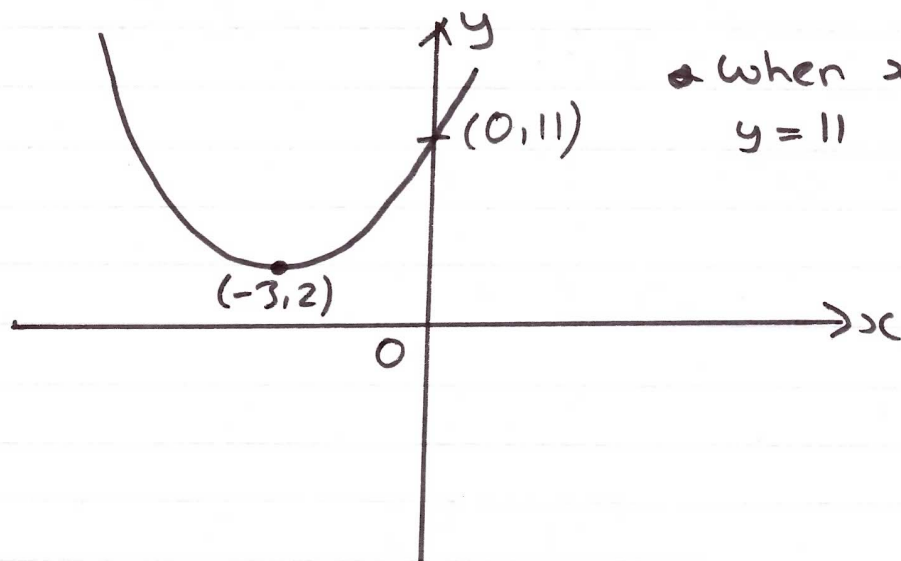
$$x < \frac{14}{5} \quad \underline{\text{and}} \quad -1 < x < \frac{7}{2}$$

$$-1 < x < \frac{14}{5}$$

Q4a) $x^2 + 6x + 11 = (x+3)^2 - 3^2 + 11$

$$= (x+3)^2 + 2$$

b)



• When $x=0$,
 $y=11$

$$c) \quad y = x^2 + 6x + 11$$

$$b^2 - 4ac = (6)^2 - (4)(1)(11)$$

$$= 36 - 44$$

$$\boxed{= -8}$$

$$Q5) \quad a_{n+1} = \sqrt{(a_n^2 + 3)} \quad n \geq 1 \quad a_1 = 2$$

$$a) \quad a_2 = \sqrt{(a_1^2 + 3)}$$

$$a_2 = \sqrt{2^2 + 3}$$

$$\boxed{a_2 = \sqrt{7}}$$

$$a_3 = \sqrt{(a_2^2 + 3)}$$

$$a_3 = \sqrt{(\sqrt{7})^2 + 3}$$

$$\boxed{a_3 = \sqrt{10}}$$

$$b) \quad a_4 = \sqrt{(a_3^2 + 3)}$$

$$a_4 = \sqrt{(\sqrt{10})^2 + 3}$$

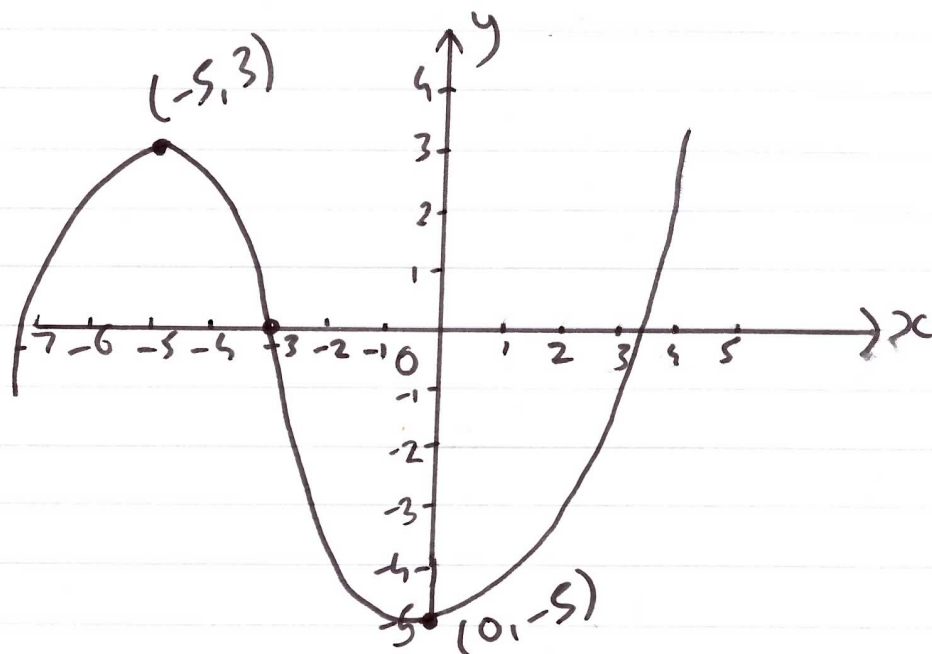
$$\boxed{a_4 = \sqrt{13}}$$

$$a_5 = \sqrt{(a_4^2 + 3)}$$

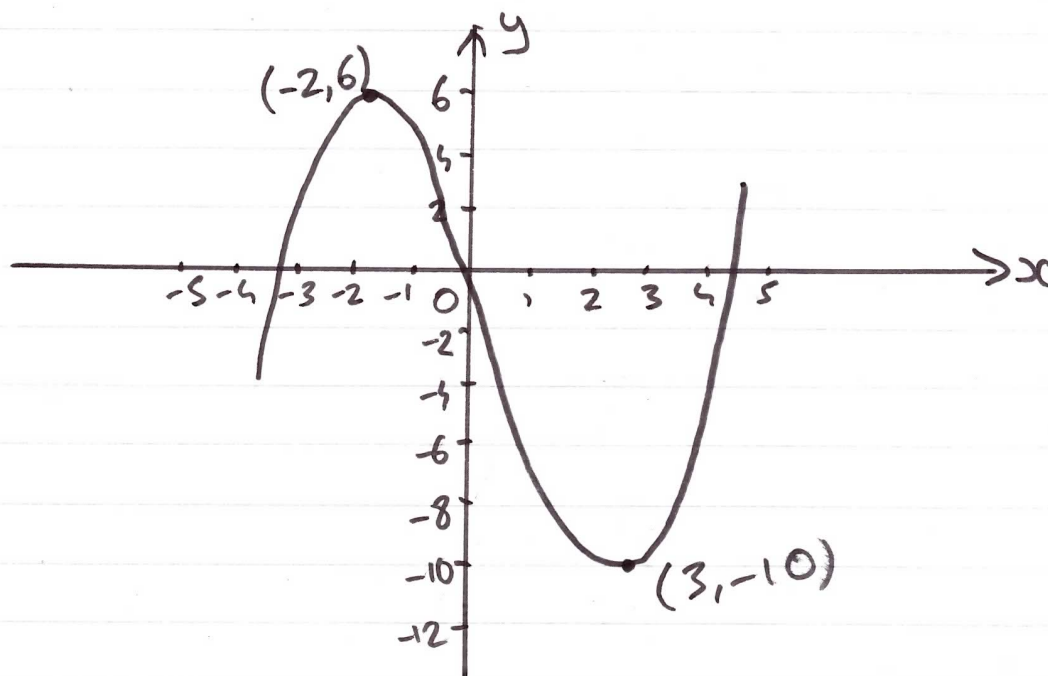
$$a_5 = \sqrt{(\sqrt{13})^2 + 3}$$

$$\boxed{a_5 = \sqrt{16} = 4}$$

Q6a) $y = f(x+3)$ - transformation horizontally (along the x -axis) of ' -3 ' :



b) $y = 2f(x)$ - multiply the y -coordinates by 2:



c) $y = f(x) + a$ - transformation vertically of ' $+a$ '.

If the minimum point of $y = f(x) + a$ is at $(3, 0)$ then $\boxed{a = 5}$

$$Q7) y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

$$y = 8x^3 - 4x^{1/2} + \frac{3x^2}{x} + \frac{2}{x}$$

$$y = 8x^3 - 4x^{1/2} + 3x + 2x^{-1}$$

$$\frac{dy}{dx} = 24x^2 - 2x^{-1/2} + 3 - 2x^{-2}$$

$$= 24x^2 - \frac{2}{x^{1/2}} + 3 - \frac{2}{x^2}$$

$$= \boxed{24x^2 - \frac{2}{\sqrt{x}} + 3 - \frac{2}{x^2}}$$

$$Q8a) \text{ Gradient } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 0}{7 - 2}$$

$$= \underline{\underline{\frac{4}{5}}}$$

$$\text{Equation of line: } y - y_1 = m(x - x_1)$$

Using B(2,0)
and $m = \frac{4}{5}$

$$\rightarrow y - 0 = \frac{4}{5}(x - 2)$$

$$y = \frac{4}{5}x - \frac{8}{5}$$

$$\boxed{4x - 5y + 8 = 0}$$

$$\begin{aligned}
 \text{b) Length } AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(7 - 2)^2 + (4 - 0)^2} \\
 &= \sqrt{5^2 + 4^2} \\
 &= \sqrt{25 + 16} \\
 &= \boxed{\sqrt{41} \text{ units}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) Length } AC &= AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 \sqrt{41} &= \sqrt{(7 - 2)^2 + (4 - t)^2} \\
 \sqrt{41} &= \sqrt{5^2 + (4 - t)^2} \\
 \sqrt{41} &= \sqrt{25 + (16 - 8t + t^2)} \\
 \sqrt{41} &= \sqrt{25 + 16 - 8t + t^2} \\
 \sqrt{41} &= \sqrt{41 - 8t + t^2}
 \end{aligned}$$

$$\text{So, } -8t + t^2 = 0$$

$$t^2 = 8t$$

$$\boxed{t = 8}$$

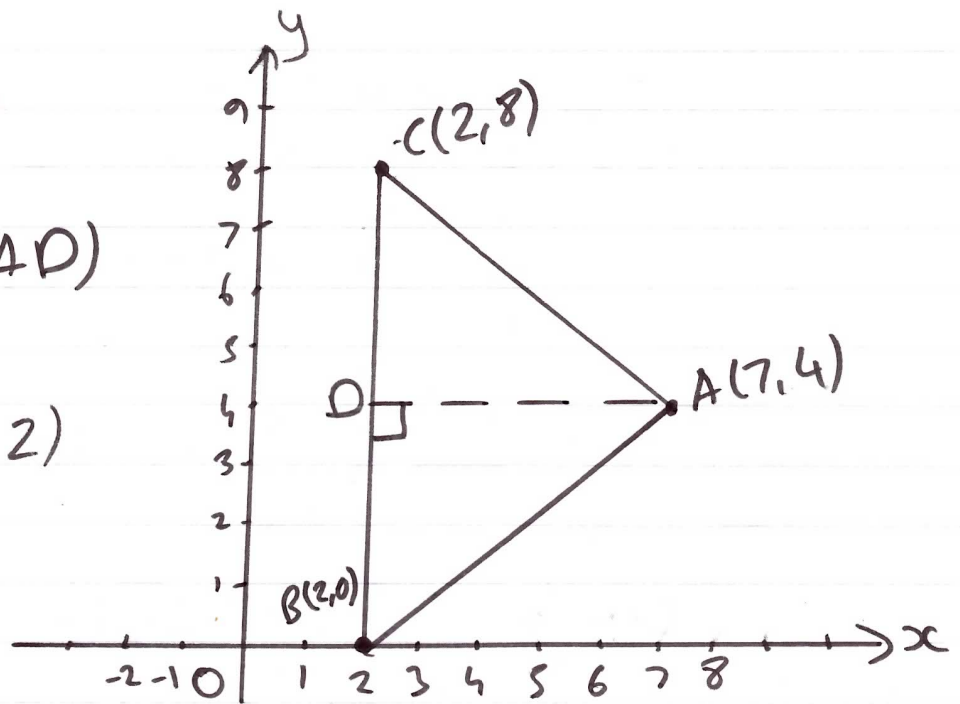
$$d) \text{ Area} = \frac{1}{2}bh$$

$$= \frac{1}{2}(BC)(AD)$$

$$= \frac{1}{2}(8)(7-2)$$

$$= \frac{1}{2}(8)(5)$$

$$= \boxed{20 \text{ units}^2}$$



Q9) First term, $a = \pounds a$
Common difference, $d = \pounds d$

After working for 30 days, this worker gets paid $\pounds 40.75$ on the last day.

$$a) \quad U_n = a + (n-1)d$$

$$U_{30} = a + (30-1)d$$

$$40.75 = a + 29d$$

$$\boxed{a + 29d = 40.75}$$

- b) A picker works for 30 days and earns a total of £1005.

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{30} = \frac{30}{2} (2a + (30-1)d)$$

$$1005 = 15(2a + 29d)$$

$$1005 = 15(2a + 29d)$$

From (a), $29d = 40.75 - a$

$$\therefore 1005 = 15(2a + (40.75 - a))$$

$$1005 = 15(2a - a + 40.75)$$

$$\therefore \boxed{15(a + 40.75) = 1005}$$

c) $15a + 611.25 = 1005$

$$15a = 393.75$$

$$a = 26.25$$

$$\text{So } \boxed{a = £26.25}$$

$$29d = 40.75 - a \Rightarrow 29d = 40.75 - 26.25 = 14.5$$

$$\therefore \boxed{d = £0.50}$$

Q10a)

i) $y = x(4-x)$

When $x=0, y=0$

When $y=0, x(4-x)=0$

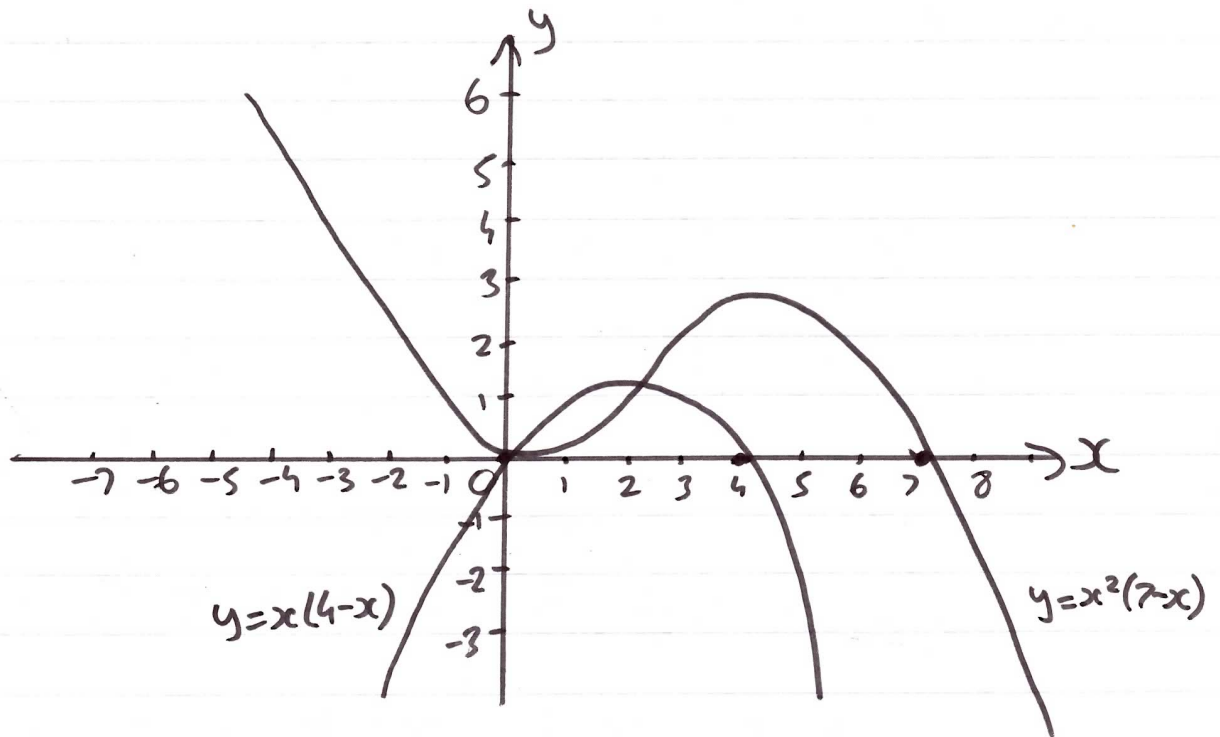
Either $x=0$ or $x=4$

ii) $y = x^2(7-x)$

When $x=0, y=0$

When $y=0, x^2(7-x)=0$

Either $x=0$ or $x=0$ or $x=7$



b) To find points of intersection,
Solve simultaneous equations:

$$y = x(4-x) \quad (1)$$

$$y = x^2(7-x) \quad (2)$$

Substitute (1) into (2):

$$x(4-x) = x^2(7-x)$$

$$4x - x^2 = 7x^2 - x^3$$

$$x^3 - 8x^2 + 4x = 0$$

$$\therefore \boxed{x(x^2 - 8x + 4) = 0}$$

$$c) \quad x(x^2 - 8x + 4) = 0 \Rightarrow x^2 - 8x + 4 = 0$$

$$(x-4)^2 - 4^2 + 4 = 0$$

$$(x-4)^2 = 12$$

$$x = 4 \pm \sqrt{12} \Rightarrow x = 4 \pm 2\sqrt{3}$$

From the sketch, $x = 4 - 2\sqrt{3}$, since $4 + 2\sqrt{3} > 7$,
which is further than the x-coordinate
for the x-intercept of $y = x^2(7-x)$.

Substitute into (1) for y: $(4 - 2\sqrt{3})(4 - (4 - 2\sqrt{3}))$

$$y = (4 - 2\sqrt{3})(2\sqrt{3})$$

$$y = 8\sqrt{3} - 12$$

$\therefore$ the coordinates of A are $(4 - 2\sqrt{3}, 8\sqrt{3} - 12)$

$$Q11) \frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given $P(4, 5)$ lies on y ,

$$\begin{aligned} a) \quad y = f(x) &= \int \left(3x - \frac{5}{\sqrt{x}} - 2 \right) dx \\ &= \int \left(3x - \frac{5}{x^{1/2}} - 2 \right) dx \\ &= \int (3x - 5x^{-1/2} - 2) dx \\ &= \frac{3x^2}{2} - \frac{5x^{1/2}}{1/2} - 2x + C \\ &= \frac{3x^2}{2} - 10\sqrt{x} - 2x + C \end{aligned}$$

Since this curve passes through $P(4, 5)$,
Substitute in $x=4$ and $y=5$

$$5 = \frac{3(4)^2}{2} - 10(\sqrt{4}) - 2(4) + C$$

$$5 = 24 - 20 - 8 + C$$

$$5 - 24 + 20 + 8 = C$$

$$\therefore \underline{C = 9}$$

$$\boxed{y = f(x) = \frac{3x^2}{2} - 10\sqrt{x} - 2x + 9}$$

$$b) \text{ When } x=4, \frac{dy}{dx} = 3(4) - \frac{5}{\sqrt{4}} - 2$$

$$= 12 - \frac{5}{2} - 2$$

$$= \underline{\underline{\frac{15}{2}}}$$

Equation of tangent at P: $y - y_1 = m(x - x_1)$

Using $P(4, 5)$
and $m = \frac{15}{2}$ $\rightarrow y - 5 = \frac{15}{2}(x - 4)$

$$2(y - 5) = 15(x - 4)$$

$$2y - 10 = 15x - 60$$

$$\boxed{15x - 2y - 50 = 0}$$