

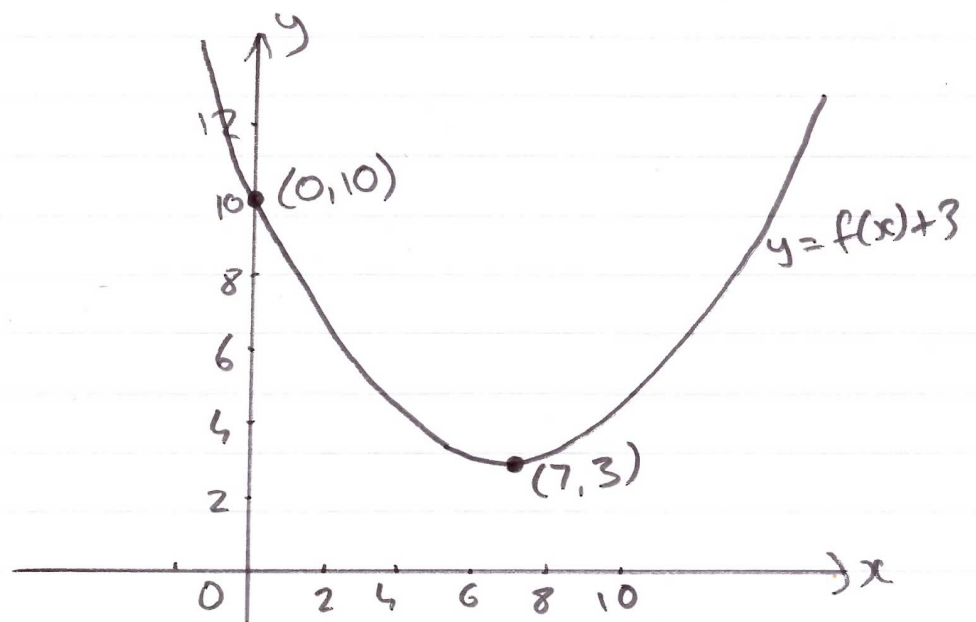
C1 June 2008 (MA)

$$Q1) \int (2 + 5x^2) dx = \boxed{2x + \frac{5x^3}{3} + C}$$

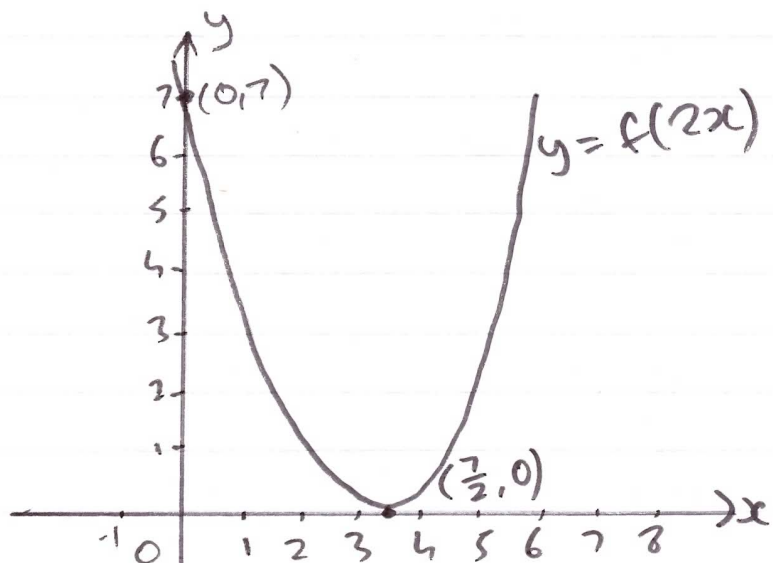
$$Q2) x^3 - 9x = x(x^2 - 9)$$

$$= \boxed{x(x-3)(x+3)}$$

Q3a) $y = f(x) + 3$ - transformation vertically of '+3'



b) $y = f(2x)$ - halve the x-coordinates:



Q4) $f(x) = 3x + x^3, x > 0$

a) $f'(x) = 3 + 3x^2$

b) Given $f'(x) = 15,$

then $15 = 3 + 3x^2$

$$12 = 3x^2$$

$$4 = x^2$$

$$x = \pm 2$$

But $x > 0$, so $x = 2$

Q5) $x_1 = 1$

$$x_{n+1} = ax_n - 3, n \geq 1$$

a) $x_2 = a(x_1) - 3$

$$x_2 = a - 3$$

b) $x_3 = ax_2 - 3$

$$x_3 = a(a - 3) - 3$$

$$x_3 = a^2 - 3a - 3$$

c) Given $x_3 = 7,$

then $7 = a^2 - 3a - 3$

$$a^2 - 3a - 10 = 0$$

$$(a-5)(a+2) = 0$$

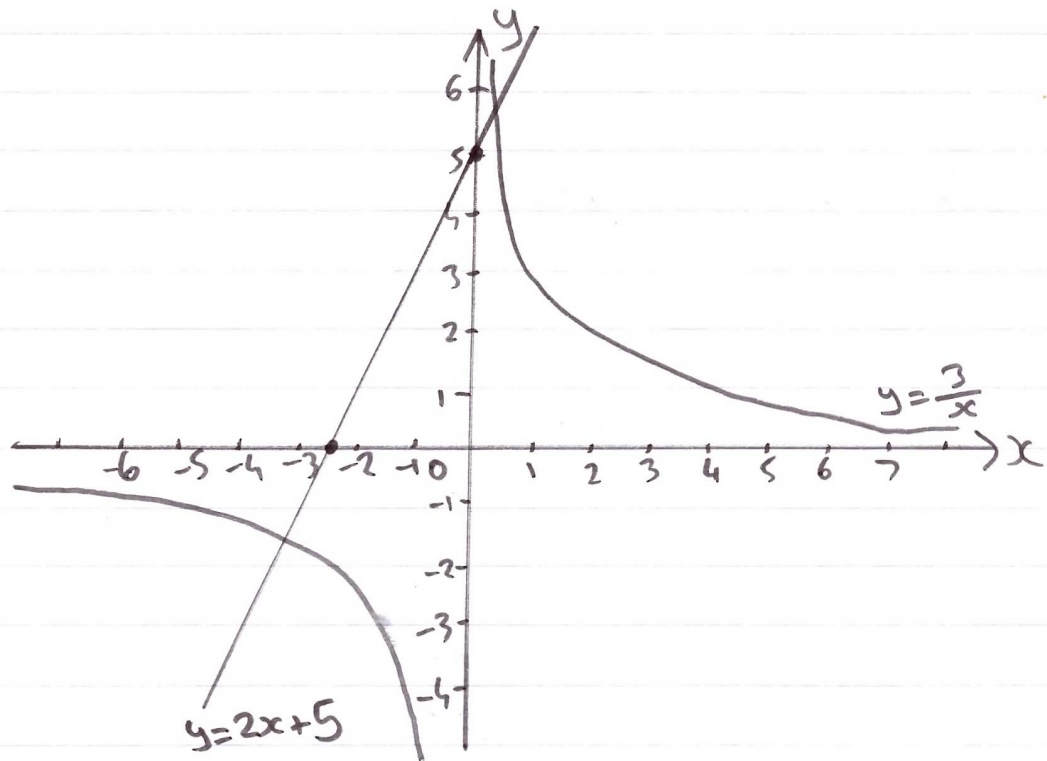
$$\boxed{\text{Either } a=5 \text{ or } a=-2}$$

Q6a) $y = \frac{3}{x}$: When $x \rightarrow \infty$, $y \rightarrow 0$
 When $x \rightarrow -\infty$, $y \rightarrow -0$

When $x \rightarrow 0$, $y \rightarrow \infty$
 When $x \rightarrow -0$, $y \rightarrow -\infty$

$y = 2x + 5$: When $x=0$, $y=5$

When $y=0$, $2x+5=0$
 $2x = -5$
 $x = -\frac{5}{2}$



b) To find points of intersection,
Solve simultaneous equations:

$$y = 2x + 5 \quad (1)$$

$$y = \frac{3}{x} \quad (2)$$

Substitute (1) into (2):

$$2x + 5 = \frac{3}{x}$$

$$2x + 5 - \frac{3}{x} = 0$$

$$\frac{2x^2 + 5x - 3}{x} = 0$$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$\text{Either } x = \frac{1}{2} \text{ or } x = -3$$

Substitute into (2) for y:

$$\text{When } x = \frac{1}{2}, y = \frac{3}{\frac{1}{2}} = \underline{6}$$

$$\text{When } x = -3, y = \frac{3}{-3} = \underline{-1}$$

The points of intersection are:

$$\boxed{\left(\frac{1}{2}, 6\right) \text{ and } (-3, -1)}$$

Q7) First term, $a = 5\text{km}$
Common difference, $d = 2\text{km}$

a) $U_n = a + (n-1)d$

$$U_4 = 5 + (4-1)2$$

$$U_4 = 5 + 6$$

$$\boxed{U_4 = 11\text{km}}$$

b) $U_n = a + (n-1)d$

$$U_n = 5 + (n-1)d$$

$$U_n = 5 + (n-1)2$$

$$U_n = 5 + 2n - 2$$

$$U_n = 3 + 2n$$

$$\boxed{U_n = 2n + 3}$$

c) $S_n = \frac{n}{2} (2a + (n-1)d)$

$$S_n = \frac{n}{2} (2(5) + (n-1)2)$$

$$S_n = \frac{n}{2} (10 + 2n - 2)$$

$$S_n = \frac{n}{2} (2n+8)$$

$$S_n = \frac{2n^2}{2} + \frac{8n}{2}$$

$$S_n = \frac{2n^2 + 8n}{2}$$

$$S_n = n^2 + 4n$$

$$\therefore \boxed{S_n = n(n+4) \text{ km}}$$

$$d) \quad 43 = 2n+3$$

$$43 - 3 = 2n$$

$$2n = 40$$

$$\boxed{n = 20}$$

$$e) \quad S_{20} = 20(20+4)$$

$$S_{20} = 20 \times 24$$

$$\boxed{S_{20} = 480 \text{ km}}$$

Q8) $2qx^2 + qx - 1 = 0$ has no real roots

a) If a quadratic has no real roots, then the discriminant is less than 0.

$$\therefore b^2 - 4ac < 0$$

$$q^2 - (4)(2q)(-1) < 0$$

$$q^2 - 8q(-1) < 0$$

$$\therefore \boxed{q^2 + 8q < 0}$$

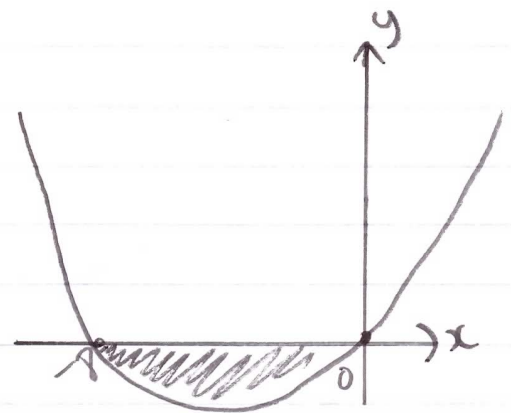
b) For $q^2 + 8q = 0$,

$$q(q + 8) = 0$$

Either $q = 0$ or $q = -8$

Set of possible values of q :

$$\boxed{-8 < q < 0}$$



Choosing the value 'under' the x-axis, since $q^2 + 8q < 0$

Q9a) $y = kx^3 - x^2 + x - 5$

$$\boxed{\frac{dy}{dx} = 3kx^2 - 2x + 1}$$

$$\begin{aligned}
 \text{b) When } x = -\frac{1}{2}, \quad \frac{dy}{dx} &= 3k\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1 \\
 &= 3k\left(\frac{1}{4}\right) + 1 + 1 \\
 &= \frac{3k}{4} + 2
 \end{aligned}$$

$\therefore$ the gradient of the tangent at $A = \frac{3k}{4} + 2$

For the line $2y - 7x + 1 = 0$,

$$2y = 7x - 1$$

$$y = \frac{7}{2}x - \frac{1}{2}$$

The gradient is $\frac{7}{2}$

Since the tangent at A is parallel to $2y - 7x + 1 = 0$, then both will have gradients of $\frac{7}{2}$.

$$\therefore \frac{3k}{4} + 2 = \frac{7}{2}$$

$$\frac{3k}{4} = \frac{3}{2}$$

$$3k = 6$$

$$\boxed{k = 2}$$

c) When $x = -\frac{1}{2}$, $y = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 5$

$$y = 2\left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) - \frac{1}{2} - 5$$

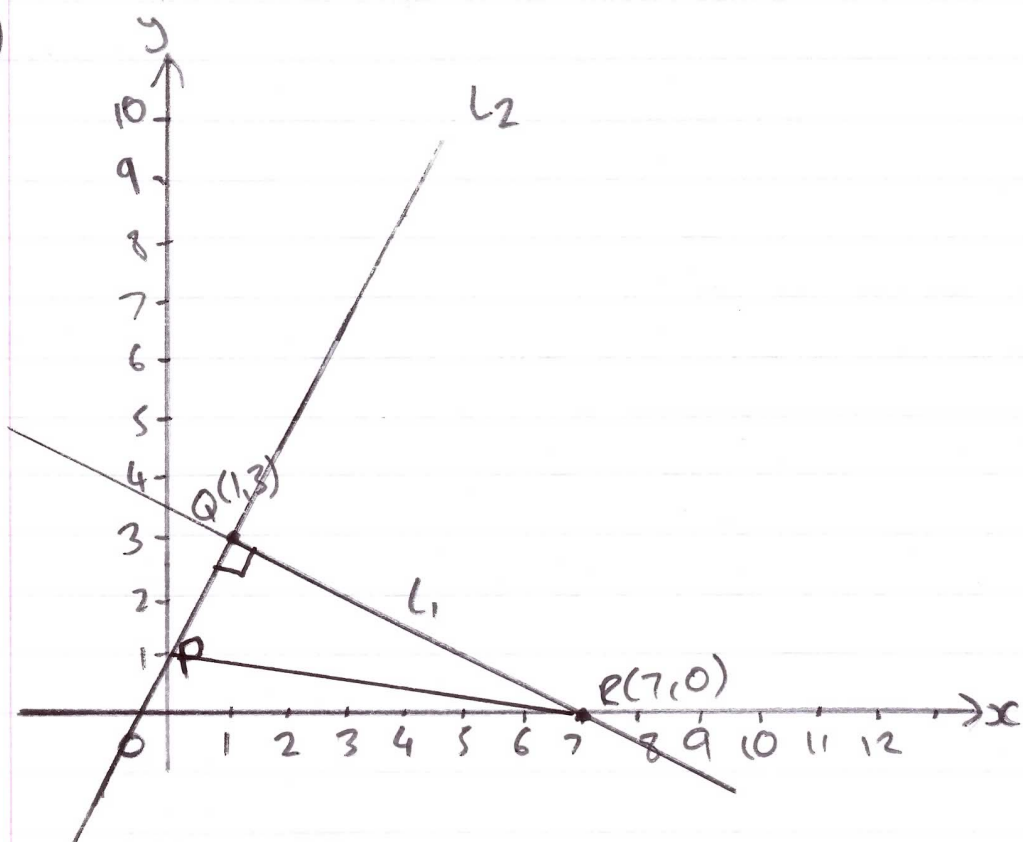
$$y = -\frac{2}{8} - \frac{1}{4} - \frac{1}{2} - 5$$

$$y = -\frac{1}{4} - \frac{1}{4} - \frac{1}{2} - 5$$

$$\underline{y = -6}$$

$\therefore$ the coordinates of A are $\boxed{\left(-\frac{1}{2}, -6\right)}$

Q10)



$$\begin{aligned}
 \text{a) Length } QR &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(7 - 1)^2 + (0 - 3)^2} \\
 &= \sqrt{6^2 + (-3)^2} \\
 &= \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5} \\
 &= \underline{3\sqrt{5} \text{ units}}
 \end{aligned}$$

$$\therefore \boxed{a = 3}$$

$$\begin{aligned}
 \text{b) Gradient } QR &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{0 - 3}{7 - 1} \\
 &= -\frac{3}{6} \\
 &= \underline{-\frac{1}{2}}
 \end{aligned}$$

$\therefore$ the Gradient of the perpendicular line, l_2 is 2

Equation of l_2 : $y - y_1 = m(x - x_1)$

Using $Q(1, 3)$ and $m = 2$ $\rightarrow y - 3 = 2(x - 1)$

$$y - 3 = 2x - 2$$

$$\boxed{y = 2x + 1}$$

c) When l_2 crosses the y -axis, $x=0$

$$\therefore y = 2(0) + 1 \Rightarrow y = 1$$

$$\therefore \boxed{P \text{ is at } (0, 1)}$$

d) Length $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(1 - 0)^2 + (3 - 1)^2}$$

$$= \sqrt{1^2 + 2^2}$$

$$\boxed{= \sqrt{5} \text{ units}}$$

$$\text{Area } \triangle PQR = \frac{1}{2}bh = \frac{1}{2}(QR)(PQ)$$

$$= \frac{1}{2}(3\sqrt{5})(\sqrt{5})$$

$$= \frac{1}{2}(15)$$

$$\boxed{= \frac{15}{2} \text{ units}^2}$$

$$Q11a) \frac{dy}{dx} = \frac{(x^2 + 3)^2}{x^2}, x \neq 0$$

$$\frac{dy}{dx} = \frac{(x^2 + 3)(x^2 + 3)}{x^2}$$

$$\frac{dy}{dx} = \frac{x^4 + 6x^2 + 9}{x^2}$$

$$\frac{dy}{dx} = \frac{x^4}{x^2} + \frac{6x^2}{x^2} + \frac{9}{x^2}$$

$$\boxed{\frac{dy}{dx} = x^2 + 6 + 9x^{-2}}$$

$$b) y = \int (x^2 + 6 + 9x^{-2}) dx$$

$$y = \frac{x^3}{3} + 6x + \frac{9x^{-1}}{-1} + C$$

$$y = \frac{x^3}{3} + 6x - \frac{9}{x} + C$$

Since the curve passes through (3, 20),
 substitute in $x=3$ and $y=20$:

$$20 = \frac{3^3}{3} + 6(3) - \frac{9}{3} + C$$

$$20 = \frac{27}{3} + 18 - 3 + C$$

$$20 = 9 + 18 - 3 + C$$

$$\therefore \underline{C = -4} \Rightarrow \boxed{y = \frac{x^3}{3} + 6x - \frac{9}{x} - 4}$$