

C1 June 2007 (MA)

$$Q1) (3+\sqrt{5})(3-\sqrt{5}) = 9 - 3\sqrt{5} + 3\sqrt{5} - 5$$

$$\boxed{= 4}$$

$$Q2a) 8^{\frac{4}{3}} = (\sqrt[3]{8})^4 = 2^4 \quad \boxed{= 16}$$

$$b) \frac{15x^{\frac{4}{3}}}{3x} = 5x^{\frac{1}{3}} \quad \boxed{= 5\sqrt[3]{x}}$$

$$Q3) y = 3x^2 + 4\sqrt{x}, x > 0$$

$$y = 3x^2 + 4x^{\frac{1}{2}}$$

$$a) \frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$$

$$\boxed{= 6x + \frac{2}{\sqrt{x}}}$$

$$b) \frac{d^2y}{dx^2} = 6 - x^{-\frac{3}{2}}$$

$$\boxed{= 6 - \frac{1}{x^{3/2}}}$$

$$c) \int (3x^2 + 4x^{\frac{1}{2}}) dx = \frac{3x^3}{3} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\boxed{= x^3 + \frac{8x^{3/2}}{3} + C}$$

Q4) First term, $a = 5p$.
Common difference, $d = 2p$

a) $U_n = a + (n-1)d$

$$U_{200} = 5 + (200-1)2$$

$$U_{200} = 5 + (199 \times 2)$$

$$U_{200} = 398 + 5$$

$$\boxed{U_{200} = 403p}$$

$\therefore$ She saves £4.03 in week 200.

b) $S_n = \frac{n}{2} (2a + (n-1)d)$

$$S_{200} = \frac{200}{2} (2(5) + (200-1)2)$$

$$S_{200} = 100 (10 + (199 \times 2))$$

$$S_{200} = 100 (10 + 398)$$

$$S_{200} = 100 \times 408$$

$$\boxed{S_{200} = 40800p}$$

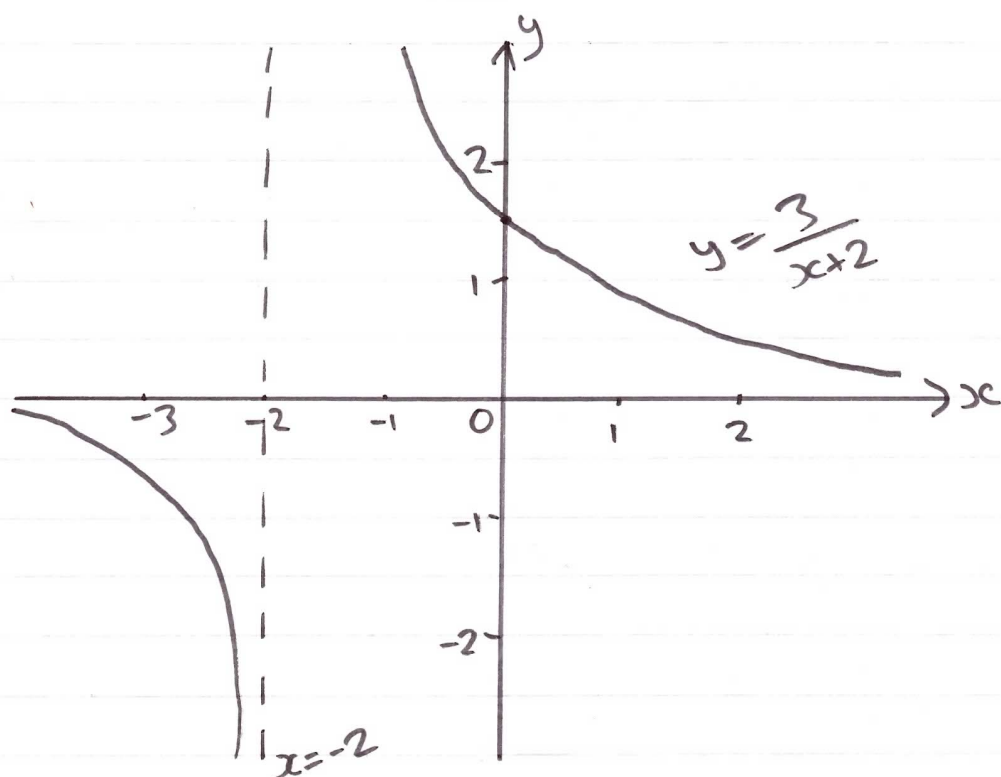
$\therefore$ She saves £408 over the complete 200 week period

Q5a) Let $f(x) = \frac{3}{x}$, $x \neq 0$

Then $\frac{3}{x+2} \equiv f(x+2)$, which is a transformation of '-2' along the x-axis

When $y=0$, $\frac{3}{x+2} = 0$
 $x \rightarrow \infty$

When $x=0$, $y = \frac{3}{2}$



b) Equations of the asymptotes for $y = \frac{3}{x+2}$ are:

$x = -2$ and $y = 0$

Q6a)

$$y = x - 4 \quad \textcircled{1}$$

$$2x^2 - xy = 8 \quad \textcircled{2}$$

Substitute $\textcircled{1}$ into $\textcircled{2}$:

$$2x^2 - x(x - 4) = 8$$

$$2x^2 - x^2 + 4x - 8 = 0$$

$$\boxed{x^2 + 4x - 8 = 0}$$

$$b) \quad x^2 + 4x - 8 = 0$$

$$(x+2)^2 - 2^2 - 8 = 0$$

$$(x+2)^2 - 4 - 8 = 0$$

$$(x+2)^2 = 12$$

$$x+2 = \pm\sqrt{12}$$

$$x = -2 \pm \sqrt{4 \times 3}$$

$$x = -2 \pm \sqrt{4}\sqrt{3}$$

$$\underline{x = -2 \pm 2\sqrt{3}}$$

Substitute into $\textcircled{1}$ for y :

$$\text{When } x = -2 \pm 2\sqrt{3}, \quad y = x - 4 \Rightarrow \underline{y = -6 \pm 2\sqrt{3}}$$

$$\text{Solution set: } \boxed{x = -2 + 2\sqrt{3}, y = -6 + 2\sqrt{3} \quad \text{and} \quad x = -2 - 2\sqrt{3}, y = -6 - 2\sqrt{3}}$$

Q7) $x^2 + kx + (k+3) = 0$ has different real roots.

a) If a quadratic has different real roots, then the discriminant is greater than 0.

$$\therefore b^2 - 4ac > 0$$

$$k^2 - (4)(1)(k+3) > 0$$

$$k^2 - 4(k+3) > 0$$

$$\therefore \boxed{k^2 - 4k - 12 > 0}$$

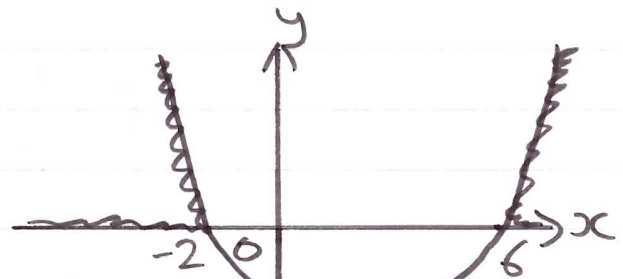
b) For $k^2 - 4k - 12 = 0$,

$$(k-6)(k+2) = 0$$

Either $k=6$ or $k=-2$

Set of possible values of k :

$$\boxed{k < -2 \text{ or } k > 6}$$



Choosing the values 'above' the x -axis, since $k^2 - 4k - 12 > 0$

Q8) $a_1 = k$

$$a_{n+1} = 3a_n + 5, \quad n \geq 1$$

a) $a_2 = 3a_1 + 5$

$$\boxed{a_2 = 3k + 5}$$

b) $a_3 = 3a_2 + 5$

$$a_3 = 3(3k+5) + 5 \Rightarrow \boxed{a_3 = 9k + 20}$$

$$\text{ci)} \quad \sum_{r=1}^4 a_r = a_1 + a_2 + a_3 + a_4$$

$$a_4 = 3a_3 + 5$$

$$a_4 = 3(9k+20) + 5$$

$$\underline{a_4 = 27k + 65}$$

$$\therefore \sum_{r=1}^4 a_r = k + (6k+5) + (9k+20) + (27k+65)$$

$$= k + 3k + 5 + 9k + 20 + 27k + 65$$

$$= 40k + 90$$

$$\boxed{= 10(4k+9)}$$

$$\text{ii)} \quad \frac{40k+90}{10} = 4k+9, \text{ which means } 40k+90 \text{ is divisible by } 10.$$

$$\text{Q9a)} \quad f'(x) = 6x^2 - 10x - 12$$

$$f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x + C$$

$$\underline{f(x) = 2x^3 - 5x^2 - 12x + C}$$

Since $f(x)$ passes through $(5, 65)$,
substitute in $x=5$ and $y=65$:

$$65 = 2(5)^3 - 5(5)^2 - 12(5) + C$$

$$65 = 2(125) - 5(25) - 60 + C$$

$$65 = 250 - 125 - 60 + C$$

$$65 = 65 + C$$

$$\therefore \underline{C = 0}$$

$$\boxed{f(x) = 2x^3 - 5x^2 - 12x}$$

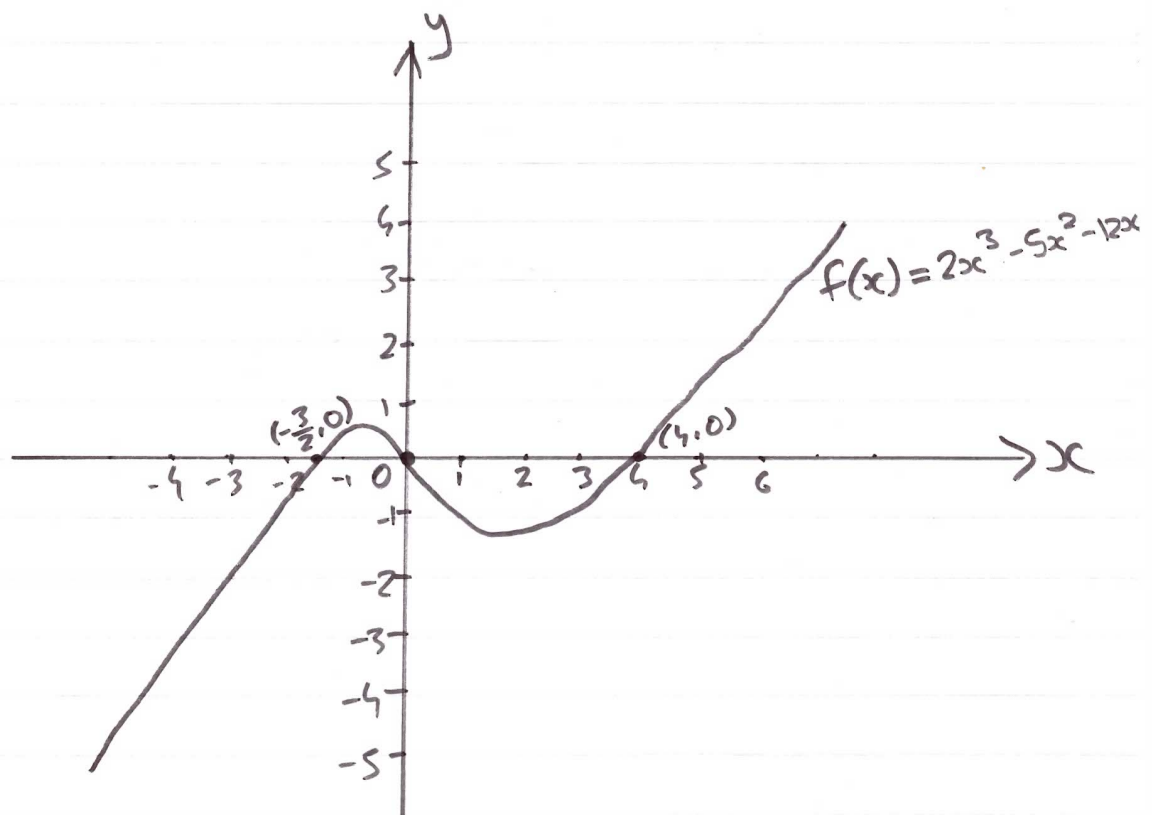
$$b) f(x) = x(2x^2 - 5x - 12)$$

$$\boxed{f(x) = x(2x + 3)(x - 4)}$$

$$c) \text{ When } x=0, f(x) = 0(2(0)+3)(0-4) = \underline{0}$$

$$\text{When } f(x)=0, x(2x+3)(x-4)=0$$

$$\text{Either } x=0 \text{ or } x=-\frac{3}{2} \text{ or } x=4$$



$$Q10) \quad y = x^2(x-6) + \frac{4}{x}, \quad x > 0$$

$$a) \quad \text{When } x=1, y = 1^2(1-6) + \frac{4}{1}$$

$$y = 1(-5) + 4$$

$$y = -1$$

$\therefore P$ is at $(1, -1)$

$$\text{When } x=2, y = 2^2(2-6) + \frac{4}{2}$$

$$y = 4(-4) + 2$$

$$y = -14$$

$\therefore Q$ is at $(2, -14)$

$$\text{Length } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2-1)^2 + (-14-(-1))^2}$$

$$= \sqrt{1 + (-13)^2}$$

$$= \boxed{\sqrt{170} \text{ units}}$$

$$b) \quad y = x^2(x-6) + \frac{4}{x}, \quad x > 0$$

$$y = x^3 - 6x^2 + 4x^{-1}$$

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 - 12x - 4x^{-2} \\ &= 3x^2 - 12x - \frac{4}{x^2} \end{aligned}$$

$$\begin{aligned} \text{At } P(1, -1), \frac{dy}{dx} &= 3(1)^2 - 12(1) - \frac{4}{1^2} \\ &= 3 - 12 - 4 \\ &= \underline{-13} \end{aligned}$$

$\therefore$ the gradient of the tangent at P is -13.

$$\begin{aligned} \text{At } Q(2, -14), \frac{dy}{dx} &= 3(2)^2 - 12(2) - \frac{4}{2^2} \\ &= 12 - 24 - 1 \\ &= \underline{-13} \end{aligned}$$

$\therefore$ the gradient of the tangent at Q is -13.

Since the tangents at P and Q are both gradients of -13, they are parallel.

- c) If the gradient of the tangent at P is -13 , then the gradient of the normal at P is $\frac{1}{13}$.

Equation of normal at P: $y - y_1 = m(x - x_1)$

Using $P(1, -1)$ and $m = \frac{1}{13}$

$$\rightarrow y - (-1) = \frac{1}{13}(x - 1)$$

$$13(y + 1) = 1(x - 1)$$

$$13y + 13 = x - 1$$

$$\boxed{x - 13y - 14 = 0}$$

Q11) $l_1: y = 3x + 2$ $l_2: 3x + 2y - 8 = 0$

a) Gradient of l_2 : $2y = -3x + 8$

$$y = -\frac{3}{2}x + 4$$

$$\therefore \text{Gradient of } l_2 = -\frac{3}{2}$$

- b) To find coordinates of the point of intersection, solve simultaneous equations:

$$y = 3x + 2 \quad \textcircled{1}$$

$$3x + 2y - 8 = 0 \quad \textcircled{2}$$

Substitute $\textcircled{1}$ into $\textcircled{2}$

$$3x + 2(3x + 2) - 8 = 0$$

$$3x + 6x + 4 - 8 = 0$$

$$9x - 4 = 0$$

$$9x = 4$$

$$\underline{x = \frac{4}{9}}$$

Substitute into ① for y:

$$y = 3x + 2$$

$$\text{When } x = \frac{4}{9}, y = 3\left(\frac{4}{9}\right) + 2$$

$$y = \frac{12}{9} + 2$$

$$y = \frac{4}{3} + 2$$

$$\underline{y = \frac{10}{3}}$$

∴ the coordinates of the point of intersection, P are $\boxed{\left(\frac{4}{9}, \frac{10}{3}\right)}$

c) For l_1 , when $y=1$, $1=3x+2$

$$-1=3x$$

$$x = -\frac{1}{3}$$

$\therefore A$ is at $(-\frac{1}{3}, 1)$

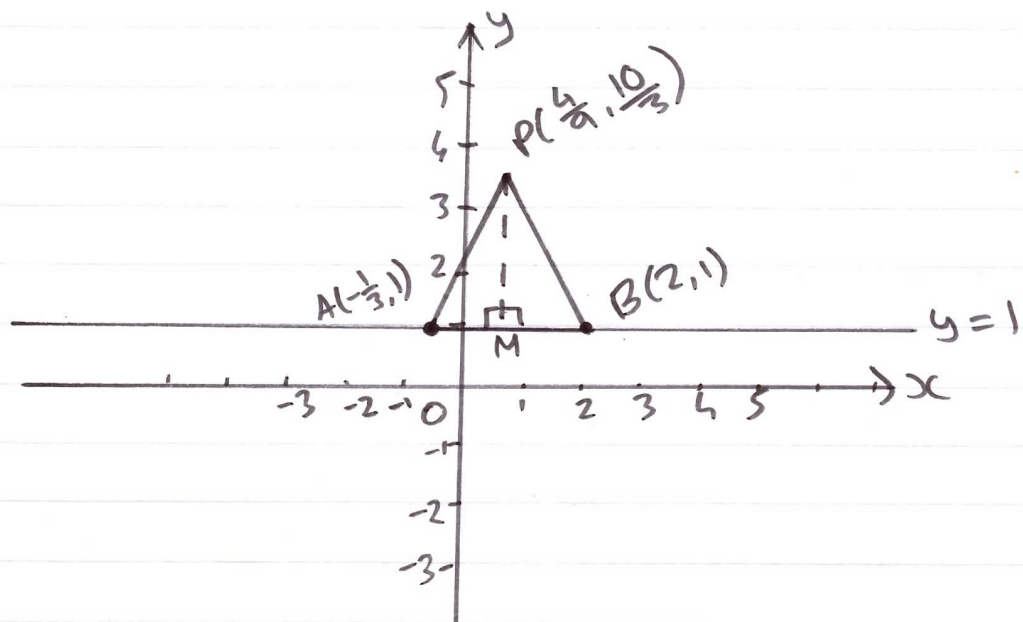
For l_2 , when $y=1$, $3x+2(1)-8=0$

$$3x+2-8=0$$

$$3x=6$$

$$x=2$$

$\therefore B$ is at $(2, 1)$



$$\begin{aligned}\text{Length } AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(2 - (-\frac{1}{3}))^2 + (1 - 1)^2} \\&= \sqrt{(\frac{7}{3})^2 + 0^2} \\&= \sqrt{\frac{49}{9}} \\&= \frac{7}{3} \text{ units}\end{aligned}$$

$$\text{Length } PM = \frac{10}{3} - 1 = \frac{7}{3} \text{ units}$$

$$\begin{aligned}\text{Area } \triangle ABP &= \frac{1}{2}bh = \frac{1}{2}(AB)(PM) \\&= \frac{1}{2}\left(\frac{7}{3}\right)\left(\frac{7}{3}\right) \\&= \frac{1}{2}\left(\frac{49}{9}\right) \\&= \boxed{\frac{49}{18} \text{ units}^2}\end{aligned}$$