

(1 June 2005 (MA))

$$Q1a) 8^{\frac{1}{3}} = \sqrt[3]{8} \boxed{= 2}$$

$$b) 8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{(2)^2} \boxed{= \frac{1}{4}}$$

$$Q2a) y = 6x - \frac{4}{x^2}, x \neq 0$$

$$y = 6x - 4x^{-2}$$

$$\frac{dy}{dx} = 6 + 8x^{-3}$$

$$\boxed{= 6 + \frac{8}{x^3}}$$

$$b) \int (6x - 4x^{-2}) dx = \frac{6x^2}{2} - \frac{4x^{-1}}{-1} + C$$

$$= 3x^2 + 4x^{-1} + C$$

$$\boxed{= 3x^2 + \frac{4}{x} + C}$$

$$Q3a) x^2 - 8x - 29 \equiv (x-4)^2 - 4^2 - 29$$

$$= (x-4)^2 - 16 - 29$$

$$\boxed{= (x-4)^2 - 45}$$

$$b) (x-4)^2 - 45 = 0$$

$$(x-4)^2 = 45$$

$$x - 4 = \pm \sqrt{45}$$

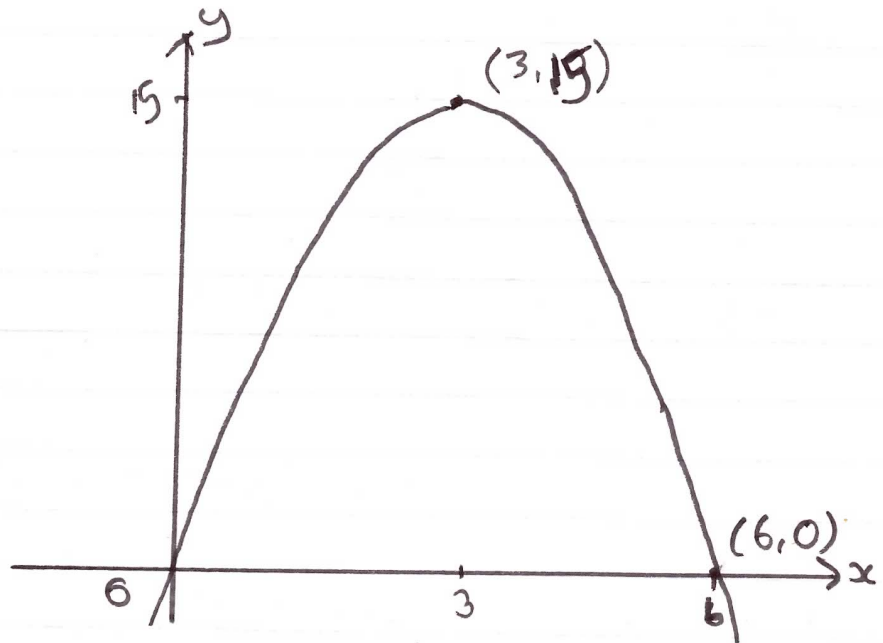
$$x = 4 \pm \sqrt{45}$$

$$x = 4 \pm \sqrt{9 \times 5}$$

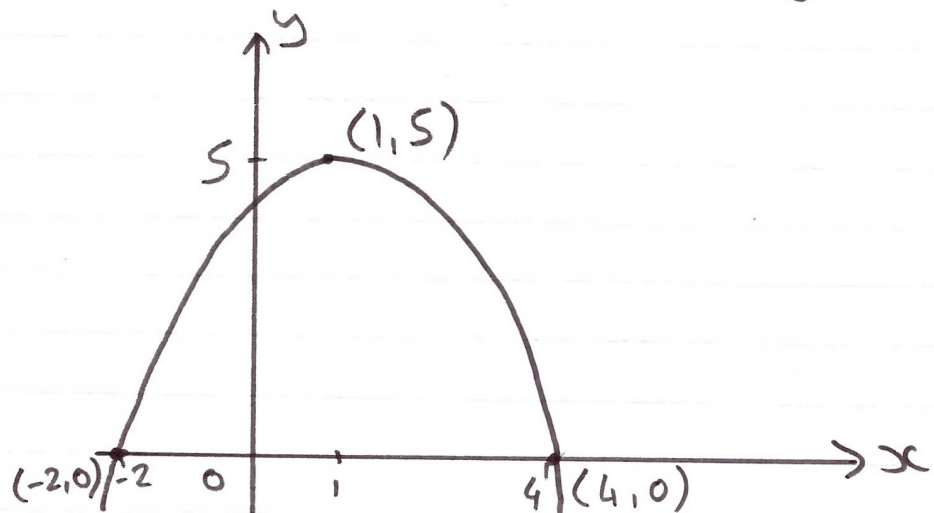
$$x = 4 \pm \sqrt{9} \sqrt{5}$$

$$\boxed{x = 4 \pm 3\sqrt{5}}$$

Q4a) $y = 3f(x)$ - transformation vertically of 'x3':



b) $y = f(x+2)$ - transformation horizontally of '-2':



$$\begin{aligned} \text{Q5)} \quad x-2y=1 &\Rightarrow x=2y+1 & \textcircled{1} \\ x^2+y^2=29 &\Rightarrow x^2+y^2=29 & \textcircled{2} \end{aligned}$$

Substitute $\textcircled{1}$ into $\textcircled{2}$:

$$(2y+1)^2 + y^2 = 29$$

$$(2y+1)(2y+1) + y^2 = 29$$

$$4y^2 + 4y + 1 + y^2 - 29 = 0$$

$$5y^2 + 4y - 28 = 0$$

$$(5y + 14)(y - 2) = 0$$

$$\text{Either } y = -\frac{14}{5} \text{ or } y = 2$$

Substitute into $\textcircled{1}$ for x :

$$\text{When } y = -\frac{14}{5}, \quad x = 2\left(-\frac{14}{5}\right) + 1$$

$$x = -\frac{28}{5} + 1$$

$$x = -\frac{23}{5}$$

$$\text{When } y = 2, \quad x = 2(2) + 1 = 5$$

$$\text{Solution set: } \boxed{x = -\frac{23}{5}, y = -\frac{14}{5} \text{ and } x = 5, y = 2}$$

Q6a) $3(2x+1) > 5-2x$

$$6x+3 > 5-2x$$

$$8x > 2$$

$$x > \frac{1}{4}$$

b) $2x^2 - 7x + 3 > 0$

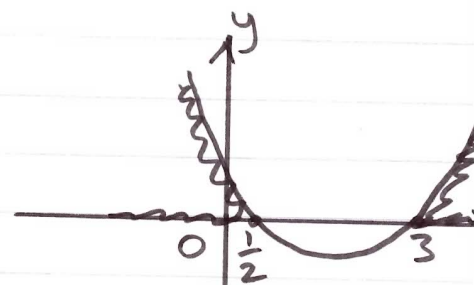
For $2x^2 - 7x + 3 = 0$

$$(2x-1)(x-3) = 0$$

Either $x = \frac{1}{2}$ or $x = 3$

Set of possible values of x :

$$x < \frac{1}{2} \text{ or } x > 3$$



Choosing the values 'above' x -axis, since $2x^2 - 7x + 3 > 0$

c) both $3(2x+1) > 5-2x$ and $2x^2 - 7x + 3 > 0$

$$x > \frac{1}{4} \quad \underline{\text{and}} \quad \left(x < \frac{1}{2} \text{ or } x > 3 \right)$$

$$x > \frac{1}{4} \quad \underline{\text{and}} \quad \left(x < \frac{1}{2} \text{ or } x > 3 \right)$$

$$\frac{1}{4} < x < \frac{1}{2} \text{ or } x > 3$$

$$\begin{aligned}
 \text{Q7a)} \quad \frac{(3-\sqrt{x})^2}{\sqrt{x}} &= \frac{(3-\sqrt{x})(3-\sqrt{x})}{\sqrt{x}} \\
 &= \frac{9 - 3\sqrt{x} - 3\sqrt{x} + x}{\sqrt{x}} \\
 &= \frac{9 - 6\sqrt{x} + x}{\sqrt{x}} \\
 &= \frac{9}{\sqrt{x}} - \frac{6\sqrt{x}}{\sqrt{x}} + \frac{x}{\sqrt{x}} \\
 &= \frac{9}{x^{1/2}} - \frac{6x^{1/2}}{x^{1/2}} + \frac{x}{x^{1/2}} \\
 &= \boxed{9x^{-1/2} - 6 + x^{1/2}}
 \end{aligned}$$

$$\text{b)} \quad \frac{dy}{dx} = \frac{(3-\sqrt{x})^2}{\sqrt{x}}, x > 0$$

$$\frac{dy}{dx} = 9x^{-1/2} - 6 + x^{1/2}$$

$$y = \int (9x^{-1/2} - 6 + x^{1/2}) dx$$

$$y = \frac{9x^{1/2}}{\frac{1}{2}} - 6x + \frac{x^{3/2}}{\frac{3}{2}} + C$$

$$y = 18x^{1/2} - 6x + \frac{2x^{3/2}}{3} + C$$

$$\underline{y = 18\sqrt{x} - 6x + \frac{2x\sqrt{x}}{3} + C}$$

Since $y = \frac{2}{3}$ at $x = 1$, substitute into equation:

$$y = 18\sqrt{x} - 6x + \frac{2x\sqrt{x}}{3} + c$$

$$\frac{2}{3} = 18(\sqrt{1}) - 6(1) + \frac{2(1)(\sqrt{1})}{3} + c$$

$$\frac{2}{3} = 18 - 6 + \frac{2}{3} + c$$

$$0 = 12 + c$$

$$\therefore \underline{c = -12}$$

$$\boxed{y = 18\sqrt{x} - 6x + \frac{2x\sqrt{x}}{3} - 12}$$

Q8a) Equation of l : $y - y_1 = m(x - x_1)$

Using $(9, -4)$ and $m = \frac{1}{3}$:

$$y - (-4) = \frac{1}{3}(x - 9)$$

$$3(y + 4) = 1(x - 9)$$

$$3y + 12 = x - 9$$

$$\boxed{x - 3y - 21 = 0}$$

b) Equation of l_2 : $y - y_1 = m(x - x_1)$

Since l_2 passes through the origin, $(0,0)$ can be used as (x_1, y_1) , and we are given the gradient, $m = -2$

$$y - 0 = -2(x - 0)$$

$$\underline{y = -2x}$$

To find point of intersection between l_1 and l_2 , solve simultaneous equations:

$$x - 3y - 21 = 0 \quad \textcircled{1}$$

$$y = -2x \quad \textcircled{2}$$

Substitute $\textcircled{2}$ into $\textcircled{1}$:

$$x - 3(-2x) - 21 = 0$$

$$x + 6x - 21 = 0$$

$$7x = 21$$

$$\underline{x = 3}$$

Substitute into $\textcircled{2}$ for y :

$$y = -2(3) \Rightarrow \underline{y = -6}$$

$$\therefore \boxed{P \text{ is at } (3, -6)}$$

c) When l_1 crosses the y -axis, $x=0$

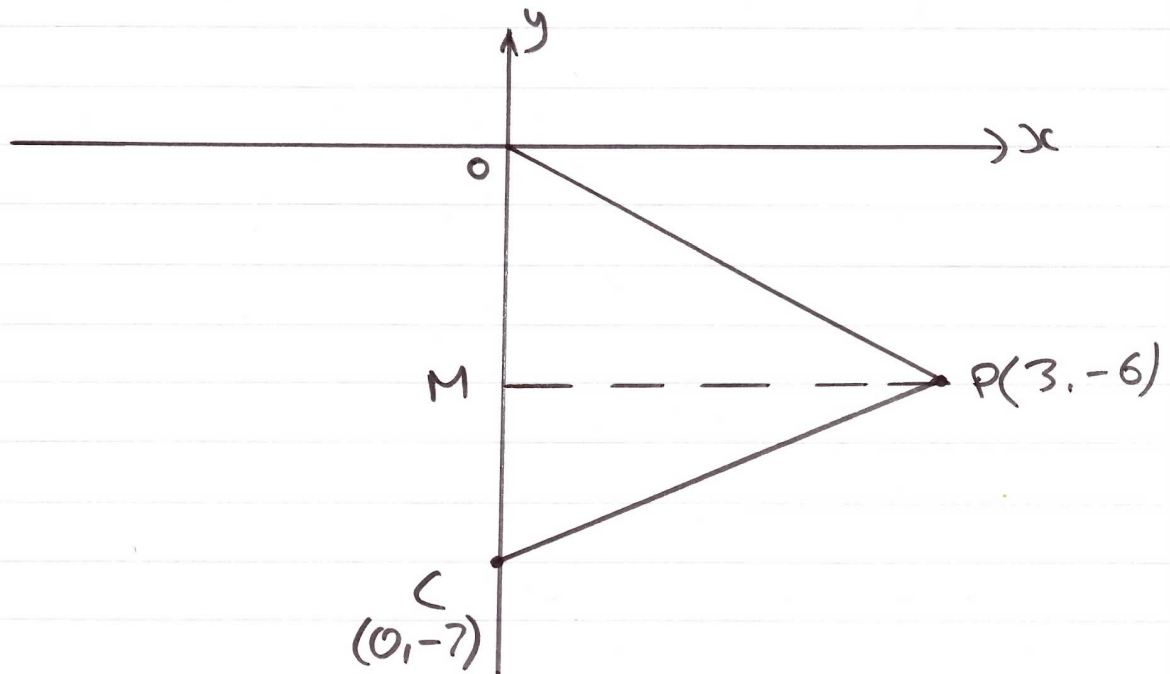
$$\text{So, } x - 3y - 21 = 0$$

$$0 - 3y - 21 = 0$$

$$3y = -21$$

$$\underline{y = -7}$$

$\therefore C$ is at $(0, -7)$



$$\text{Area } \triangle OCP = \frac{1}{2} bh = \frac{1}{2} (OC)(MP)$$

$$= \frac{1}{2} (7)(3)$$

$$= \boxed{\frac{21}{2} \text{ units}^2}$$

Q9a) First term, $a = a$.
Common difference, $d = d$.

$$S_n = a + (a+d) + \dots + (a+(n-1)d)$$

$$S_n = (a+(n-1)d) + \dots + a$$

flip over
first and
last term

$$2S_n = (2a+(n-1)d) + \dots + (2a+(n-1)d)$$

$$2S_n = n(2a+(n-1)d)$$

$$S_n = \frac{n}{2} (2a+(n-1)d)$$

b) First term, $a = £149$.
Common difference, $d = -2$

$$U_n = a + (n-1)d$$

$$U_{21} = 149 + (21-1)-2$$

$$U_{21} = 149 + (20) - 2$$

$$U_{21} = 149 - 40$$

$$U_{21} = £109$$

c) $S_n = \frac{n}{2} (2a+(n-1)d)$

$$S_n = \frac{n}{2} (2(149) + (n-1) - 2)$$

$$S_n = \frac{n}{2} (298 - 2n + 2)$$

$$S_n = \frac{n}{2} (300 - 2n)$$

But $S_n = £5000$

$$\text{So, } 5000 = \frac{n}{2} (300 - 2n)$$

$$10000 = n (300 - 2n)$$

$$10000 = 300n - 2n^2$$

$$2n^2 - 300n + 10000 = 0$$

$$\therefore \boxed{n^2 - 150n + 5000 = 0}$$

d) $(n - 50)(n - 100) = 0$

$$\boxed{\text{Either } n = 50 \text{ or } n = 100}$$

e)
$$\begin{aligned} U_{50} &= 149 + (50 - 1) - 2 \\ &= 149 - 98 \\ &= \underline{51} \end{aligned}$$

$$\begin{aligned} U_{100} &= 149 + (100 - 1) - 2 \\ &= 149 - 198 \\ &= \underline{-47} \end{aligned}$$

$\therefore n = 100$ is not a sensible solution to the repayment problem since $U_{100} < 0$

Q10) $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$

a) Substitute in $x=3$ and $y=0$:

$$0 = \frac{1}{3}(3)^3 - 4(3)^2 + 8(3) + 3$$

$$0 = \frac{1}{3}(27) - 4(9) + 24 + 3$$

$$0 = 9 - 36 + 24 + 3$$

$$\underline{0 = 0}$$

$\therefore P(3,0)$ lies on the curve

b) $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$

$$\frac{dy}{dx} = \underline{x^2 - 8x + 8}$$

When $x=3$, $\frac{dy}{dx} = 3^2 - 8(3) + 8 = 9 - 24 + 8 = \underline{-7}$

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $P(3,0)$ and $m = -7$ $\rightarrow y - 0 = -7(x - 3)$

$$\boxed{y = -7x + 21}$$

- c) If the tangent is parallel to another tangent at a different point, then the gradients are both equal.

$$\therefore \frac{dy}{dx} = -7 = x^2 - 8x + 8$$

$$x^2 - 8x + 8 = -7$$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0$$

$$\underline{\text{Either } x=5 \text{ or } x=3}$$

We already know that when $x=3$, $y=0$, which is point P.

So, at $x=5$, this is point Q.

$$\text{When } x=5, y = \frac{1}{3}(5)^3 - 4(5)^2 + 8(5) + 3$$

$$y = \frac{1}{3}(125) - 4(25) + 40 + 3$$

$$y = \frac{125}{3} - 100 + 43$$

$$y = \frac{125}{3} - 57$$

$$\underline{y = -\frac{46}{3}}$$

$$\therefore Q \text{ has coordinates } \boxed{\left(5, -\frac{46}{3}\right)}$$