Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Tuesday 8 January 2019

Morning (Time: 2 hours 30 minutes)	Paper Reference **WMA01/01**

Core Mathematics C12
Advanced Subsidiary

You must have: Mathematical Formulae and Statistical Tables (Blue)	Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Jan 2019
C12

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1. A line l passes through the points $A(5, -2)$ and $B(1, 10)$.

Find the equation of l, writing your answer in the form $y = mx + c$ where m and c are constants.

(3)

① $A(5, -2)$ $B(1, 10)$
x_1 y_1 x_2 y_2.

gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{10 - -2}{1 - 5} = \dfrac{12}{-4} = -3$

$y = mx + c$.

$y = -3x + c$ $(1, 10)$

$10 = -3 + c$.

$c = 13$

$\therefore$ $y = -3x + 13$.

Leave blank

2. Given $y = 2^x$, express each of the following in terms of y. Write each expression in its simplest form.

(a) 2^{2x} **(1)**

(b) 2^{x+3} **(1)**

(c) $\frac{1}{4^{2x-3}}$ **(2)**

(a) $(2^x)^2$

$y = 2^x$ $\quad = (y)^2$

$= \underline{y^2}$

(b) $2^x \cdot 2^3$

$= 8 \cdot y \quad = \underline{8y}$

(c) $\frac{1}{4^{2x} \cdot 4^{-3}}$

$4 = 2^2$

$(2^{2x})^2$

$= (2^x)^4$

$= y^4$ -- (1)

$4^{-3} = \frac{1}{64}$

$\therefore 1 \div \frac{1}{64} = 64$

$= \frac{64}{y^3}$

Leave blank

3. A curve has equation

$$y = \sqrt{2}x^2 - 6\sqrt{x} + 4\sqrt{2}, \qquad x > 0$$

Find the gradient of the curve at the point $P(2, 2\sqrt{2})$.

Write your answer in the form $a\sqrt{2}$, where a is a constant.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

3. $\frac{dy}{dx} = 2\sqrt{2}\,x - 6x^{-1/2} \cdot \frac{1}{2}$

$= 2\sqrt{2}\,x - 3x^{-1/2}$

at $x = 2$.

$= 4\sqrt{2} - 3(2)^{-1/2}$

$= 4\sqrt{2} - \frac{3}{\sqrt{2}}$

$\frac{8-3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

$= \frac{5\sqrt{2}}{2}$

$a = \frac{5}{2}$

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4. A sequence is defined by

$$u_1 = k, \text{ where } k \text{ is a constant}$$

$$u_{n+1} = 4u_n - 3, \quad n \geqslant 1$$

(a) Find u_2 and u_3 in terms of k, simplifying your answers as appropriate.

(3)

Given $\sum_{n=1}^{3} u_n = 18$

(b) find k.

(3)

(4). $u_1 = k$.

$u_{1+1} = 4u_1 - 3$

$u_2 = \underline{\underline{4k-3}}$

$u_3 = 4u_2 - 3$

$= 4(4k-3) - 3$

$= 16k - 12 - 3$

$u_3 = 16k - 15$

(b) $u_1 + u_2 + u_3$

$k + 4k - 3 + 16k - 15$

$21k - 18 = 18$

$21k = 36$

$k = \frac{12}{7}$

Leave blank

5. (a) Use the binomial theorem to find the first 4 terms, in ascending powers of x, of the expansion of

$$\left(1-\frac{x}{2}\right)^8$$

Give each term in its simplest form.

(4)

(b) Use the answer to part (a) to find an approximate value to 0.9^8

Write your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

(a) $\binom{8}{0}(1)^8\left(\frac{-x}{2}\right)^0 + \binom{8}{1}\left(\frac{-x}{2}\right) + \binom{8}{2}\left(\frac{-x}{2}\right)^2 + \binom{8}{3}\left(\frac{-x}{2}\right)^3$

$= 1 - 4x + 7x^2 - 7x^3 \ldots$

(b) $1 - \frac{x}{2} = 0.9$

$\underline{x = 0.2}$

$1 - 4(0.2) + 7(0.2)^2 - 7(0.2)^3$

$= \frac{53}{125} = \underline{\underline{0.424}}$

Leave blank

6. (a) Sketch the graph of $y = 1 + \cos x, \quad 0 \leqslant x \leqslant 2\pi$

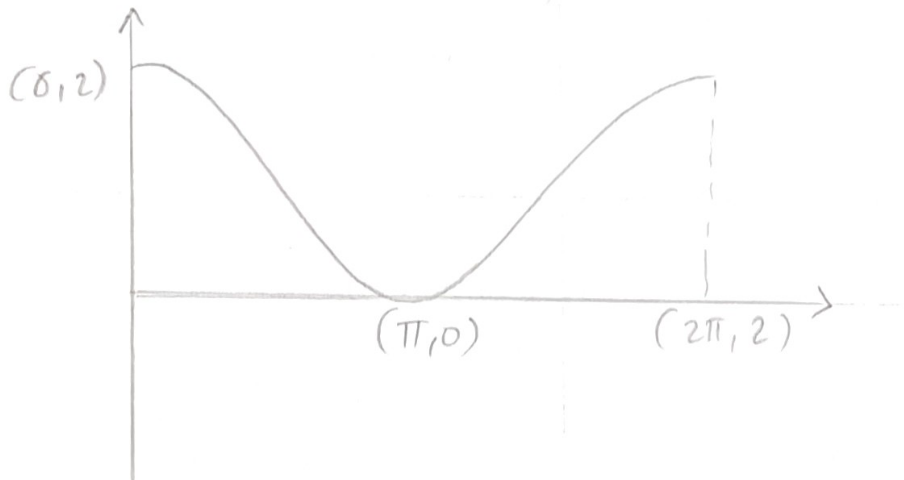
Show on your sketch the coordinates of the points where your graph meets the coordinate axes.

(3)

(b) Use the trapezium rule, with 6 strips of equal width, to find an approximate value for

$$\int_0^{2\pi} (1 + \cos x)\,dx$$

(4)



translation of y co-ordinates 1 unit up.

(b) $\frac{2\pi}{6} = \frac{1}{3}\pi \rightarrow h.$

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	2	$\frac{3}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$	2

$$\text{Area} = \frac{1}{2} \times \frac{1}{3}\pi \times \left[2 + 2 + 2\left(\frac{3}{2} + \frac{1}{2} + \frac{1}{2} + \frac{3}{2}\right)\right]$$

$$= \underline{\underline{2\pi}}$$

Leave blank

7. The equation $2x^2 + 5px + p = 0$, where p is a constant, has no real roots.

Find the set of possible values for p.

(5)

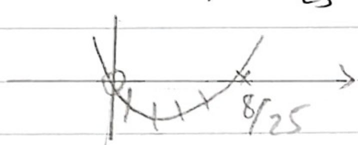
7 $b^2 - 4ac < 0.$

$(5p)^2 - 4(2 \times p) < 0.$

$25p^2 - 8p < 0$

$p(25p - 8) < 0$

$\cancel{p < 0}$ or $\cancel{p < \frac{8}{25}}$



$\therefore 0 < p < 8/25$

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8. Given $k > 3$ and

$$\int_3^k \left(2x + \frac{6}{x^2}\right) dx = 10k$$

show that $k^3 - 10k^2 - 7k - 6 = 0$

(5)

$\int \left(2x + \frac{6}{x^2}\right) dx$

$= \left[\frac{2x^2}{2} - \frac{6}{x}\right]_3^k$

$\left[k^2 - \frac{6}{k}\right] - \left[(3)^2 - \frac{6}{3}\right]$

$k \times \left(k^2 - \frac{6}{k} - 7\right) = 10k \cdot \times k$

$k^3 - 7k - 6 = 10k^2$

$k^3 - 10k^2 - 7k - 6 = 0$

as required

Leave blank

9. The circle C has equation

$$x^2 + y^2 + 10x - 6y + 9 = 0$$

(a) Find the coordinates of the centre of C.

(2)

(b) Find the radius of C.

(2)

The point $P(-2, 7)$ lies on C.

(c) Find an equation of the tangent to C at the point P.

Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(9) $(x+5)^2 - 5^2 + (y-3)^2 - 3^2 + 9 = 0$

$(x+5)^2 + (y-3)^2 = 25$

$\therefore$ centre $= (-5, 3)$

(b) radius $= 5$

(c) Grad between $(-2, 7)$ and $(-5, 3)$

$\frac{3-7}{-5+2} = \frac{-4}{-3} = \frac{4}{3}$

$\therefore$ grad of ~~t~~ line $= -\frac{3}{4}$

$y - y_0 = m(x - x_0)$

$y - 7 = -\frac{3}{4}(x+2)$

$y - 7 = -\frac{3}{4}x - \frac{3}{2}$

$4y - 28 = -3x - 6$

$4y + 3x - 22 = 0$

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10.

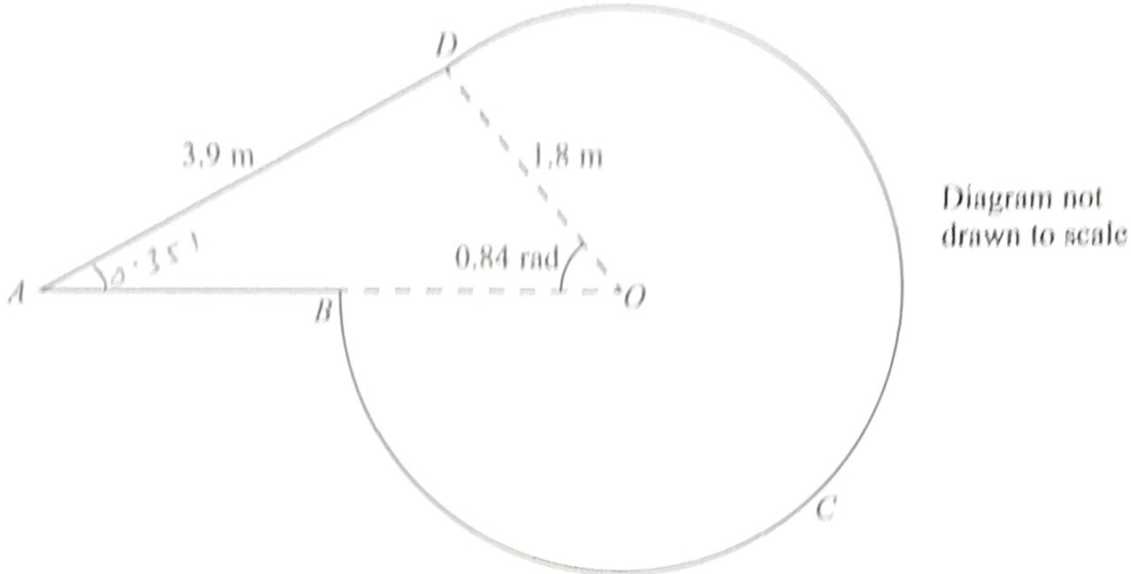


Figure 1

Figure 1 shows the design for a shop sign $ABCDA$.

The sign consists of a triangle AOD joined to a sector of a circle $DOBCD$ with radius 1.8 m and centre O.

The points A, B and O lie on a straight line.

Given that $AD = 3.9$ m and angle BOD is 0.84 radians,

(a) calculate the size of angle DAO, giving your answer in radians to 3 decimal places. **(2)**

(b) Show that, to one decimal place, the length of AO is 4.9 m. **(3)**

(c) Find, in m^2, the area of the shop sign, giving your answer to one decimal place. **(3)**

(d) Find, in m, the perimeter of the shop sign, giving your answer to one decimal place. **(3)**

(a) $\frac{\sin 0.84}{3.9} = \frac{\sin\theta}{1.8}$

$\sin\theta = 0.34368144$

$\theta = 0.351 \text{ rad}$

Leave blank

Question 10 continued

(b) Angle $A\hat{D}O$

$= \pi - (0.351 + 0.84)$

$= 1.95$ radians.

$\frac{\sin 0.84}{3.9} = \frac{\sin 1.95}{x}$

$x = 4.86$

$x = \underline{4.9}$ (1dp)

(c) Area of sector.

$\frac{1}{2} \times 1.8^2 \times (2\pi - 0.84)$

$= 8.82$

$+$ Area of triangle.

$\frac{1}{2} \times 4.9 \times 3.9 \times \sin 0.351$

$= 3.3$

$= \underline{12.1 m^2}$

(d) $l = r\theta$

$1.8 \times (2\pi - 0.84)$

$= 9.8$

$9.8 + 3.9 + AB$

$AB = 4.9 - 1.8 = 3.1$

$9.8 + 3.9 + 3.1$

$= \underline{16.8 m}$

Leave blank

11. (i) Given that x is a positive real number, solve the equation

$$\log_x 324 = 4$$

writing your answer as a simplified surd.

(3)

(ii) Given that

$$\log_a(5y-4) - \log_a(2y) = 3 \qquad y > 0.8,\ 0 < a < 1$$

express y in terms of a.

(5)

(i) $x^4 = 324$

$x = \sqrt[4]{324}$

$x = \underline{\underline{+3\sqrt{2}}}$

(ii) $\log_a\left(\frac{5y-4}{2y}\right) = 3$

$a^3 = \frac{5y-4}{2y}$

$2ya^3 = 5y-4$

$2ya^3 - 5y = -4$

$y(2a^3-5) = -4$

$y = \frac{-4}{2a^3-5}$

$y = \frac{4}{5-2a^3}$

Leave blank

12. Karen is going to raise money for a charity.

She aims to cycle a **total** distance of 1000 km over a number of days.

On day one she cycles 25 km.

She increases the distance that she cycles each day by 10% of the distance cycled on the previous day, until she reaches the total distance of 1000 km.

She reaches the **total** distance of 1000 km on day N, where N is a positive integer.

(a) Find the value of N.

(4)

On day one, 50 people donated money to the charity. Each day, 20 more people donated to the charity than did so on the previous day, so that 70 people donated money on day two, 90 people donated money on day three, and so on.

(b) Find the number of people who donated to the charity on day fifteen.

(2)

Each day, the donation given by each person was £5

(c) Find the total amount of money donated by the end of day fifteen.

(3)

(a) $\text{sum} = \frac{a(1-r^n)}{1-r}$

$a = 25$

$r = 1.1$

$\frac{25(1-1.1^n)}{1-1.1} = 1000$

$1 - 1.1^n = -4$

$5 = 1.1^n$

$\log 5 = n \log 1.1$

$n = 16.88$

$\therefore N = 17$

(b) $a = 50$

$d = 20$

$a + d(n-1)$

$50 + 20(15-14)$

$= 330$ people

(c) $\text{Sum} = \frac{15}{2}[2a + d(n-1)]$

$= \frac{15}{2}[2(50) + 20(14)]$

$= 2850$ people

$2850 \times 5 = £14,250$

Leave blank

13. $$f(x) = 3x^3 + 3x^2 + cx + 12, \text{ where } c \text{ is a constant}$$

Given that $(x + 3)$ is a factor of $f(x)$,

(a) show that $c = -14$

(2)

(b) Write $f(x)$ in the form

$$f(x) = (x + 3)\,Q(x)$$

where $Q(x)$ is a quadratic function.

(2)

(c) Use the answer to part (b) to prove that the equation $f(x) = 0$ has only one real solution.

(2)

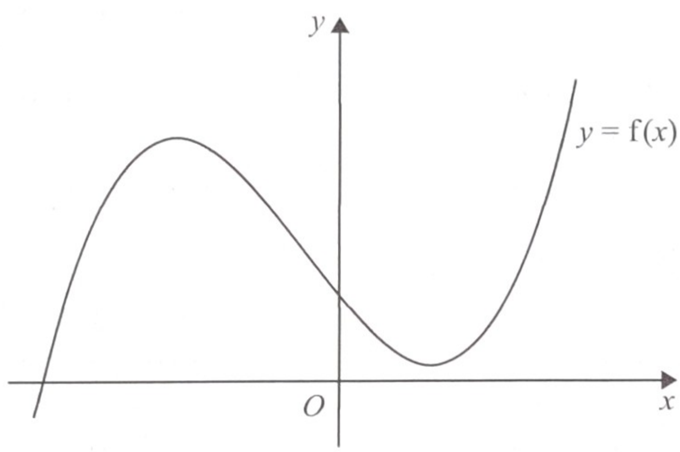


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

On **separate** diagrams sketch the curve with equation

(d) (i) $y = f(3x)$

(ii) $y = -f(x)$

On each diagram show clearly the coordinates of the points where the curve crosses the coordinate axes.

(4)

(a) $f(-3) = 0$

$3(-3)^3 + 3(-3)^2 + c(-3) + 12 = 0$

$C = -14$

(b)
$$\begin{array}{r} 3x^2 - 6x + 4 \\ (x+3)\overline{\big)\,3x^3 + 3x^2 - 14x + 12} \\ -\;\underline{3x^3 + 9x^2 \qquad\qquad} \\ -6x^2 - 14x \qquad \\ -\;\underline{-6x^2 - 18x \qquad} \\ 4x + 12 \\ -\;\underline{4x + 12} \\ 0 \end{array}$$

$\Rightarrow (x+3)(3x^2 - 6x + 4)$.

Leave blank

Question 13 continued

$f(x) = (x+3)(3x^2 - 6x + 4)$

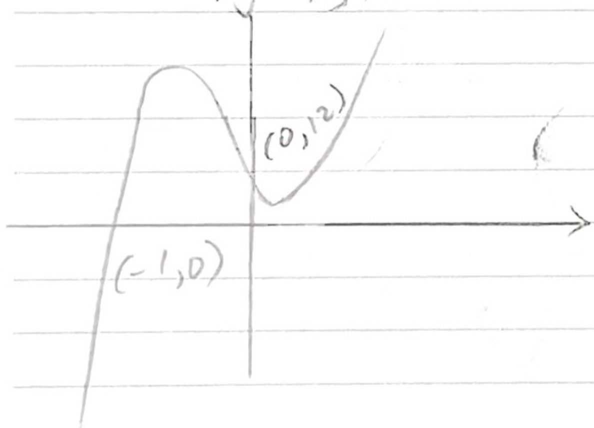
$b^2 - 4ac$ of quadratic

$= 6^2 - 4(3 \times 4)$

$= -12 < 0$

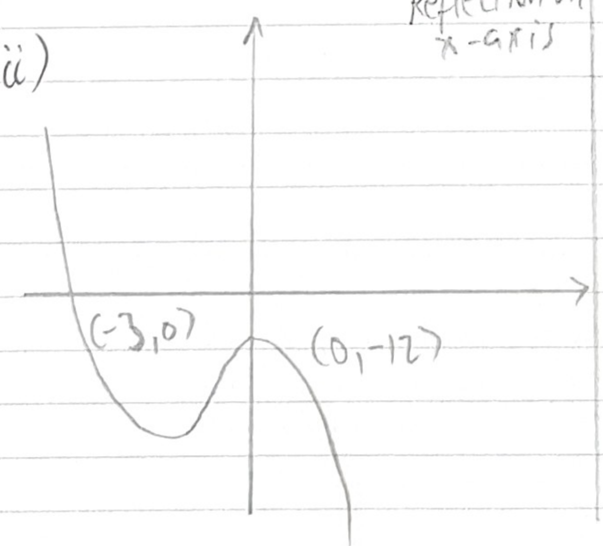
$\therefore$ only one real soln. to $f(x)$. as $Q(x)$ has no real solns.

(d)(i) (Multiply all x-co by $1/3$)



(ii)

Reflection on x-axis



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14. *In this question solutions based entirely on graphical or numerical methods are not acceptable.*

(i) Solve, for $-180° \leqslant x < 180°$, the equation

$$\sin(x + 60°) = -0.4$$

giving your answers, in degrees, to one decimal place.

(4)

(ii) (a) Show that the equation

$$2\sin\theta\tan\theta - 3 = \cos\theta$$

can be written in the form

$$3\cos^2\theta + 3\cos\theta - 2 = 0$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$2\sin\theta\tan\theta - 3 = \cos\theta$$

showing each stage of your working and giving your answers, in degrees, to one decimal place.

(4)

$\sin^{-1}(-0.4)$

$\theta = -23.6,$
$\theta = 203.6$
$\theta = -156.4$

$x + 60 = -23.6$
$x = -83.6°$ ✓

$x + 60 = 203.6$
$x = 143.6°$ ✓

$x + 60 = -156.4$
$x = -216.4$ (Not in range)

$\therefore\ x = -83.6°, 143.6°$

(ii) $2\sin\theta \cdot \dfrac{\sin\theta}{\cos\theta} - 3 = \cos\theta$

$2\sin^2\theta - 3\cos\theta = \cos^2\theta$

$2(1 - \cos^2\theta) - 3\cos\theta = \cos^2\theta$

$2 - 2\cos^2\theta - 3\cos\theta = \cos^2\theta$

$3\cos^2\theta + 3\cos\theta - 2 = 0$ as required

(b) $\dfrac{-3 \pm \sqrt{3^2 - 4(3 \times -2)}}{2 \times 3}$

$\cos\theta = -1.46$ or 0.46

not in range for $\cos\theta$

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Question 14 continued

$\therefore \cos\theta = 0.46$

$\theta = 62.8^\circ$ or 297.2°

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15.

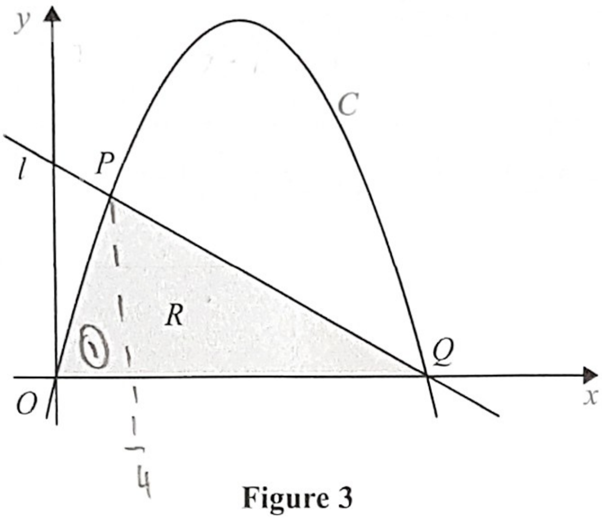
Figure 3

The straight line l with equation $y = 5 - 3x$ cuts the curve C, with equation $y = 20x - 12x^2$, at the points P and Q, as shown in Figure 3.

(a) Use algebra to find the exact coordinates of the points P and Q.
(5)

The finite region R, shown shaded in Figure 3, is bounded by the line l, the x-axis and the curve C.

(b) Use calculus to find the exact area of R.
(6)

(a) $5 - 3x = 20x - 12x^2$

$12x^2 - 23x + 5 = 0$

$\dfrac{23 \pm \sqrt{23^2 - 4(12 \times 5)}}{2 \times 12}$

$x = 5/3$ or $\frac{1}{4}$

$y = 0$ or $\frac{17}{4}$

$(5/3, 0)$ and $\left(\frac{1}{4}, \frac{17}{4}\right)$

(b) $\int_0^{1/4} 20x - 12x^2 \, dx$ -- 1

$\left[10x^2 - \frac{12x^3}{3}\right]_0^{1/4}$

$= \frac{9}{16}$

$+ \frac{1}{2} \times \frac{17}{4} \times \left(\frac{5}{3} - \frac{1}{4}\right)$

$= \frac{289}{96}$

$= \frac{343}{96}$

Turn over

16.

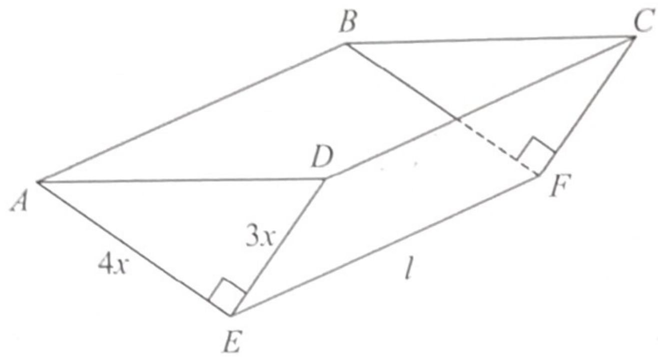

Figure 4

Figure 4 shows the design for a container in the shape of a hollow triangular prism.

The container **is open at the top**, which is labelled *ABCD*.

The sides of the container, *ABFE* and *DCFE*, are rectangles.

The ends of the container, *ADE* and *BCF*, are congruent right-angled triangles, as shown in Figure 4.

The ends of the container are vertical and the edge *EF* is horizontal.

The edges *AE*, *DE* and *EF* have lengths $4x$ metres, $3x$ metres and l metres respectively.

Given that the container has a capacity of $0.75\,\text{m}^3$ and is made of material of negligible thickness,

(a) show that the internal surface area of the container, $S\,\text{m}^2$, is given by

$$S = 12x^2 + \frac{7}{8x}$$

(5)

(b) Use calculus to find the value of x, for which S is a minimum. Give your answer to 3 significant figures.

(5)

(c) Justify that the value of x found in part (b) gives a minimum value for S.

(2)

Using the value of x found in part (b), find to 2 decimal places,

(d) (i) the length of the edge *AD*,

(ii) the length of the edge *CD*.

(4)

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Question 16 continued

(a) Volume

$= \frac{1}{2} \times CSA \times \text{length}$

$= \frac{1}{2} \times \overset{2}{\cancel{4}}x \times 3x \times l.$

$= 6x^2 l.$

$6x^2 l = 0.75$

$l = \frac{1}{8x^2}$

Surface area

$\frac{1}{2} \times 4x \times 3x \times 2$

$= 12x^2$ -- ①

$l \times 3x = 3xl$ -- ②.

$4x \times l = 4xl.$

$12x^2 + 7xl = SA.$

$l = \frac{1}{8x^2}$

$\therefore SA = 12x^2 + \frac{7}{8x}$ as req.

(b) $\frac{dS}{dx} = 24x - \frac{7}{8x^2}$

$24x - \frac{7}{8x^2} = 0$

$192x^3 = 7$

$x^3 = \frac{7}{192}$

$x = \underline{\underline{0.332}}$

(c) $\frac{d^2S}{dx^2} = 24 + \frac{7}{4x^3}$

when $x = 0.332$

$\frac{d^2S}{dx^2} > 0 \therefore$ minimum value of x.

di) AD =

$\sqrt{(4x)^2 + (3x)^2} = 5x$

$5(0.332) = \underline{\underline{1.66}}$

$\underline{\underline{CD}}$

$l = \frac{1}{8x^2} = \frac{1}{8(0.332)^2}$

$= \underline{\underline{1.13}}$