

C1 January 2013 (MA)

$$Q1) \quad x - 4x^3 = x(1 - 4x^2)$$

$$= \boxed{x(1 - 2x)(1 + 2x)}$$

$$Q2) \quad 8^{2x+3} = 2^{3(2x+3)}$$

$$= 2^{6x+9}$$

$$\therefore \boxed{y = 6x+9}$$

$$Q3i) \quad (5 - \sqrt{8})(1 + \sqrt{2}) = 5 + 5\sqrt{2} - \sqrt{8} - \sqrt{8}\sqrt{2}$$

$$= 5 + 5\sqrt{2} - \sqrt{4}\sqrt{2} - \sqrt{4}\sqrt{2}\sqrt{2}$$

$$= 5 + 5\sqrt{2} - 2\sqrt{2} - 4$$

$$= \boxed{1 + 3\sqrt{2}}$$

$$ii) \quad \sqrt{80} + \frac{30}{\sqrt{5}} = \sqrt{80} + \frac{30}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right)$$

$$= \sqrt{80} + \frac{30\sqrt{5}}{5}$$

$$= \sqrt{16}\sqrt{5} + 6\sqrt{5}$$

$$= 4\sqrt{5} + 6\sqrt{5}$$

$$= \boxed{10\sqrt{5}}$$

$$Q4) \quad u_{n+1} = 2u_n - 1, n \geq 1$$

$$a) \quad \text{Given } u_2 = 9,$$

$$u_3 = 2u_2 - 1$$

$$u_3 = 2(9) - 1$$

$$\boxed{u_3 = 17}$$

$$u_4 = 2u_3 - 1$$

$$u_4 = 2(17) - 1$$

$$\boxed{u_4 = 33}$$

$$\begin{aligned} b) \quad \sum_{r=1}^4 u_r &= u_1 + u_2 + u_3 + u_4 \\ &= u_1 + 9 + 17 + 33 \\ &= u_1 + 59 \end{aligned}$$

$$\text{Since } u_2 = 9, \quad u_2 = 2u_1 - 1$$

$$9 = 2u_1 - 1$$

$$u_1 = 5$$

$$\therefore \sum_{r=1}^4 u_r = 5 + 59$$

$$\boxed{= 64}$$

Q5) $l_1: y = -2x + 3$

l_2 : perpendicular to l_1 , passes through $(5, 6)$

Gradient of $l_1 = -2$

$\therefore$ Gradient, m of $l_2 = \underline{\underline{\frac{1}{2}}}$.

Equation of $l_2: y - y_1 = m(x - x_1)$

Using $(5, 6)$ and $m = \frac{1}{2}$ $\rightarrow y - 6 = \frac{1}{2}(x - 5)$

$$2(y - 6) = 1(x - 5)$$

$$2y - 12 = x - 5$$

$$\boxed{x - 2y + 7 = 0}$$

b) When l_2 crosses the x -axis, $y = 0$

$$\text{So, } x - 2(0) + 7 = 0$$

$$\underline{\underline{x = -7}}$$

$\therefore$ A is at $(-7, 0)$

When l_2 crosses the y -axis, $x = 0$

$$\text{So, } 0 - 2y + 7 = 0$$

$$\underline{\underline{y = \frac{7}{2}}}$$

$\therefore$ B is at $(0, \frac{7}{2})$

c) Length $OA = 7$ units
 Length $OB = \frac{7}{2}$ units

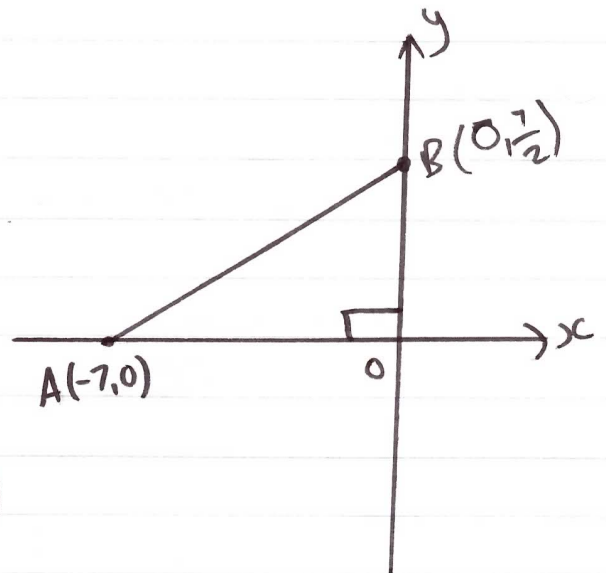
$$\text{Area } \triangle OAB = \frac{1}{2} bh$$

$$= \frac{1}{2} (OA)(OB)$$

$$= \frac{1}{2} (7) \left(\frac{7}{2}\right)$$

$$= \frac{1}{2} \left(\frac{49}{2}\right)$$

$$= \boxed{\frac{49}{4} \text{ units}^2}$$



Q6 a) Let $f(x) = \frac{2}{x}$

Then $\frac{2}{x} - 5$ is equal to $f(x) - 5$,

which is a transformation of -5 along the y -axis.

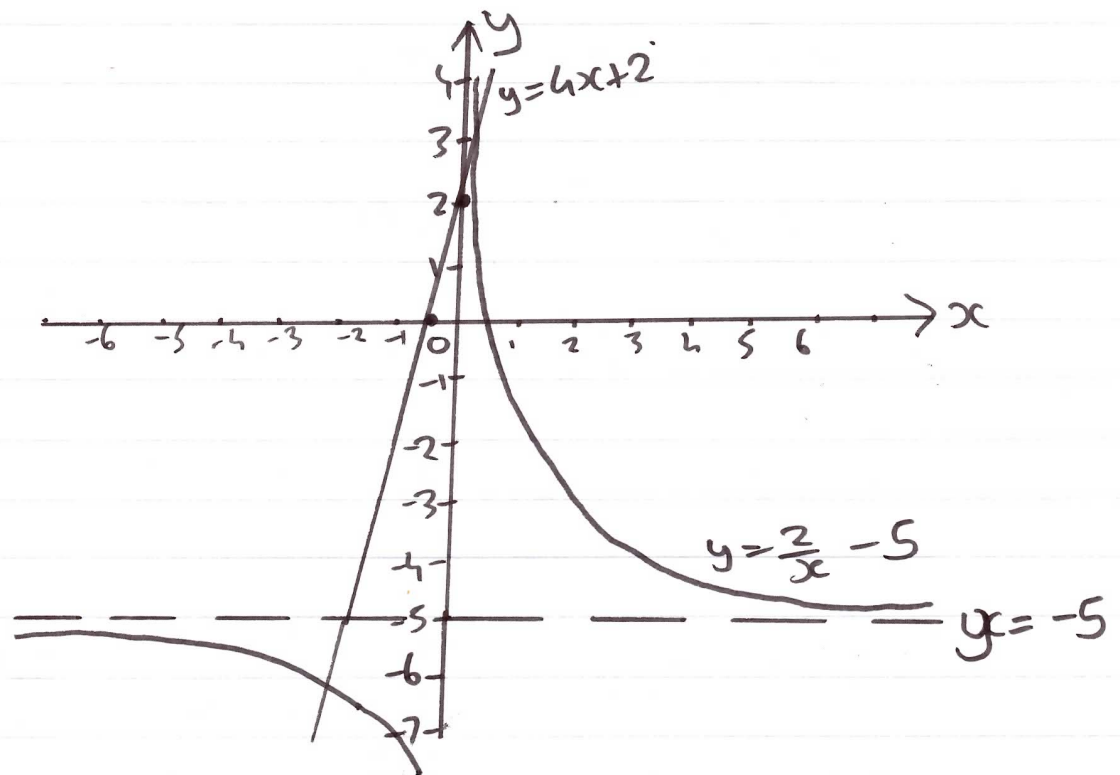
For line l , $y = 4x + 2$

$$\text{When } \underline{x=0, y=2}$$

$$\text{When } y=0, 4x+2=0$$

$$4x = -2$$

$$x = \underline{\underline{-\frac{1}{2}}}$$



b) $y = \frac{2}{x} - 5$ has asymptotes at :

$$\boxed{x=0 \text{ and } y=-5}$$

c) To find points of intersection,
Solve simultaneous equations:

$$y = \frac{2}{x} - 5 \quad \textcircled{1}$$

$$y = 4x + 2 \quad \textcircled{2}$$

Substitute $\textcircled{1}$ into $\textcircled{2}$:

$$\frac{2}{x} - 5 = 4x + 2$$

$$\frac{2}{x} = 4x + 7$$

$$4x + 7 - \frac{2}{x} = 0$$

$$\frac{4x^2 + 7x - 2}{x} = 0$$

$$4x^2 + 7x - 2 = 0$$

$$(4x - 1)(x + 2) = 0$$

$$\text{Either } x = \frac{1}{4} \text{ or } x = -2$$

Substitute into ② for y:

$$\text{When } x = \frac{1}{4}, y = 4\left(\frac{1}{4}\right) + 2 = \underline{3}$$

$$\text{When } x = -2, y = 4(-2) + 2 = \underline{-6}$$

The coordinates of the points of intersection are:

$$\left(\frac{1}{4}, 3\right) \text{ and } (-2, -6)$$

Q7) First term, $a = 140$
Common difference, $d = 20$

$$a) \quad U_n = a + (n-1)d$$

$$U_{20} = 140 + (20-1)20$$

$$U_{20} = 140 + (19)20$$

$$\boxed{U_{20} = 520}$$

$$b) S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{20} = \frac{20}{2} (2(140) + (20-1)20)$$

$$S_{20} = 10(280 + (19)20)$$

$$S_{20} = 10(660)$$

$$\boxed{S_{20} = 6600}$$

c) For S_{10} , first term, $a = 300$
common difference, $d = d$

$$U_n = a + (n-1)d$$

$$700 = 300 + (n-1)d$$

$$\underline{400 = (n-1)d}$$

$$\text{Also, } S_n = \frac{n}{2} (2a + (n-1)d)$$

$$8500 = \frac{n}{2} (2(300) + (n-1)d)$$

$$17000 = n(600 + (n-1)d)$$

$$\underline{17000 = 600n + n(n-1)d}$$

$$\text{But } (n-1)d = 400$$

$$\therefore 17000 = 600n + n(400)$$

$$17000 = 600n + 400n$$

$$1000n = 17000$$

$$\boxed{n = 17}$$

$$\text{Q8)} \quad \frac{dy}{dx} = -x^3 + \frac{4x-5}{2x^3}, \quad x \neq 0$$

$$\frac{dy}{dx} = -x^3 + \frac{4x}{2x^3} - \frac{5}{2x^3}$$

$$\frac{dy}{dx} = -x^3 + 2x^{-2} - \frac{5}{2}x^{-3}$$

$$y = \int \left(-x^3 + 2x^{-2} - \frac{5}{2}x^{-3} \right) dx$$

$$y = -\frac{x^4}{4} + \frac{2x^{-1}}{-1} - \frac{5}{2} \cdot \frac{x^{-2}}{-2} + C$$

$$y = -\frac{x^4}{4} - 2x^{-1} - \frac{5x^{-2}}{-4} + C$$

$$y = -\frac{x^4}{4} - \frac{2}{x} + \frac{5}{4x^2} + C$$

Since $y=7$ at $x=1$, substitute in $x=1$ and $y=7$:

$$7 = \frac{-1^4}{4} - \frac{2}{1} + \frac{5}{4(1)^2} + C$$

$$7 = \frac{-1}{4} - 2 + \frac{5}{4} + C$$

$$C = 7 + \frac{1}{4} + 2 - \frac{5}{4}$$

$$\therefore C = 8$$

$$y = \frac{-x^4}{4} - \frac{2}{x} + \frac{5}{4x^2} + 8$$

Q9) $(k+3)x^2 + 6x + k = 5$ has two distinct real solutions for x .

a) If a quadratic has two distinct real solutions, then the discriminant is greater than 0.

$$\therefore b^2 - 4ac > 0$$

The equation $(k+3)x^2 + 6x + k = 5$, is also equal to $(k+3)x^2 + 6x + (k-5) = 0$, with $b = 6$, $a = (k+3)$, and $c = (k-5)$

$$b^2 - 4ac = 6^2 - (4)(k+3)(k-5)$$

$$\therefore 6^2 - 4(k+3)(k-5) > 0$$

$$36 - 4(k^2 - 2k - 15) > 0$$

$$36 - 4k^2 + 8k + 60 > 0$$

$$4k^2 - 8k - 96 < 0$$

$$\therefore \boxed{k^2 - 2k - 24 < 0}$$

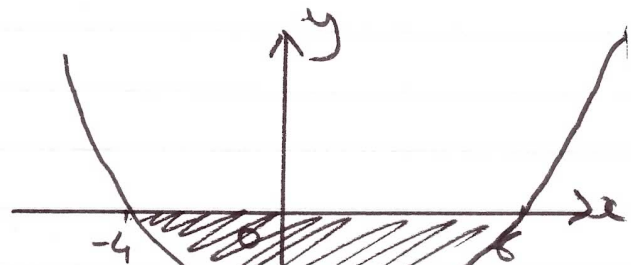
b) For $k^2 - 2k - 24 = 0$

$$(k-6)(k+4) = 0$$

Either $k=6$ or $k=-4$

Set of possible values of k :

$$\boxed{-4 < k < 6}$$



Choosing the values 'under' the x-axis, since $b^2 - 4ac < 0$

$$\begin{aligned} \text{Q10a)} \quad 4x^2 + 8x + 3 &= 4\left(x^2 + 2x + \frac{3}{4}\right) \\ &= 4\left((x+1)^2 - 1^2 + \frac{3}{4}\right) \\ &= 4\left((x+1)^2 - 1 + \frac{3}{4}\right) \\ &= 4(x+1)^2 - 4 + 3 \\ &= \underline{4(x+1)^2 - 1} \end{aligned}$$

$$\therefore \boxed{a=4, b=1, c=-1}$$

$$b) y = 4x^2 + 8x + 3$$

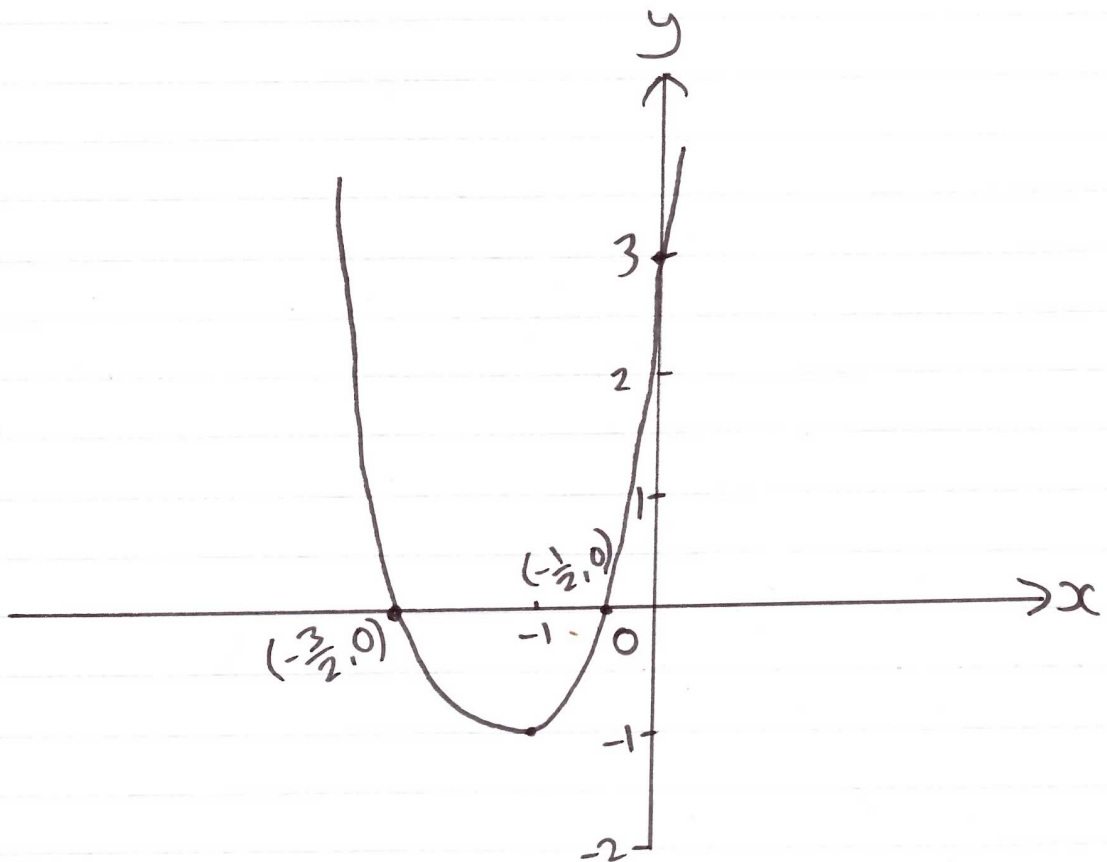
$$\text{When } \underline{x=0, y=3}$$

$$\text{When } y=0, 4x^2 + 8x + 3 = 0$$

$$(2x+3)(2x+1) = 0$$

$$\text{Either } x = -\frac{3}{2} \text{ or } x = -\frac{1}{2}$$

Turning point from (a) is $(-1, -1)$



Q11a) $y = 2x - 8\sqrt{x} + 5, x \geq 0$

$$y = 2x - 8x^{\frac{1}{2}} + 5$$

$$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}}$$

$$= \boxed{2 - \frac{4}{\sqrt{x}}}$$

b) When $x = \frac{1}{4}$, $y = 2\left(\frac{1}{4}\right) - 8\left(\sqrt{\frac{1}{4}}\right) + 5$

$$= \frac{1}{2} - 8\left(\frac{1}{2}\right) + 5$$

$$= \frac{1}{2} - 4 + 5$$

$$= \underline{\underline{\frac{3}{2}}}$$

$\therefore P$ has coordinates $\left(\frac{1}{4}, \frac{3}{2}\right)$

When $x = \frac{1}{4}$, $\frac{dy}{dx} = 2 - \frac{4}{\sqrt{\frac{1}{4}}}$

$$= 2 - \frac{4}{\frac{1}{2}}$$

$$= 2 - 8$$

$$= \underline{\underline{-6}}$$

$\therefore$ the gradient of the tangent at P is -6.

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $P(\frac{1}{4}, \frac{3}{2})$,
and $m = -6$

$$\rightarrow y - \frac{3}{2} = -6(x - \frac{1}{4})$$

$$y - \frac{3}{2} = -6x + \frac{6}{4}$$

$$y = -6x + \frac{6}{4} + \frac{3}{2}$$

$$y = -6x + \frac{3}{2} + \frac{3}{2}$$

$$\boxed{y = -6x + 3}$$

c) $2x - 3y + 18 = 0$

$$3y = 2x + 18$$

$$y = \frac{2}{3}x + \frac{18}{3}$$

$$y = \frac{2}{3}x + 6$$

If the tangent to the curve is parallel to $2x - 3y + 18 = 0$ at point Q, then its gradient is also $\frac{2}{3}$.

$$\therefore \frac{dy}{dx} = \frac{2}{3} = 2 - \frac{4}{\sqrt{x}}$$

$$\frac{2}{3} = 2 - \frac{4}{\sqrt{x}}$$

$$-\frac{4}{3} = -\frac{4}{\sqrt{x}}$$

$$\frac{4}{\sqrt{x}} = \frac{4}{3}$$

$$4 = \left(\frac{4}{3}\right)\sqrt{x}$$

$$\frac{4}{\left(\frac{4}{3}\right)} = \sqrt{x}$$

$$4 \times \frac{3}{4} = \sqrt{x}$$

$$3 = \sqrt{x}$$

$$\therefore \underline{x = 9}$$

$$\text{When } x=9, y = 2(9) - 8(\sqrt{9}) + 5$$

$$= 18 - 24 + 5$$

$$\underline{= -1}$$

$\therefore$ the coordinates of Q are (9, -1)