C1 January 2012 (MA)

Q1) $y = x^4 + 6x^{\frac{1}{2}}$

a) $\dfrac{dy}{dx} = 4x^3 + 3x^{-\frac{1}{2}}$

$= \boxed{4x^3 + \dfrac{3}{\sqrt{x}}}$

b) $\int (x^4 + 6x^{\frac{1}{2}})\,dx = \dfrac{x^5}{5} + \dfrac{6x^{3/2}}{\frac{3}{2}} + c$

$= \dfrac{x^5}{5} + 4x^{3/2} + c$

$= \boxed{\dfrac{x^5}{5} + 4x\sqrt{x} + c}$

Q2a) $\sqrt{32} + \sqrt{18} = \sqrt{8}\sqrt{4} + \sqrt{9}\sqrt{2}$

$= \sqrt{2}\sqrt{4}\sqrt{4} + \sqrt{9}\sqrt{2}$

$= 4\sqrt{2} + 3\sqrt{2}$

$\boxed{= 7\sqrt{2}}$

b) $\dfrac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}} = \dfrac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}} \times \dfrac{3 - \sqrt{2}}{3 - \sqrt{2}}$

$= \dfrac{(\sqrt{32} + \sqrt{18})(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$

$= \dfrac{7\sqrt{2}(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$

$$= \frac{21\sqrt{2} - 14}{9 - 3\sqrt{2} + 3\sqrt{2} - 2}$$

$$= \frac{21\sqrt{2} - 14}{7}$$

$$\boxed{= 3\sqrt{2} - 2}$$

Q3a) $4x - 5 > 15 - x$

$5x > 20$

$\boxed{x > 4}$

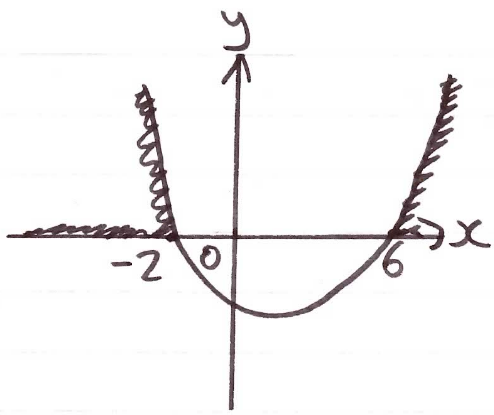
b) $x(x-4) > 12$

$x^2 - 4x > 12$

$x^2 - 4x - 12 > 0$

For $x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$



Either $x = 6$ or $x = -2$

Set of possibles values for x:

$\boxed{x < -2 \quad \text{or} \quad x > 6}$

Q4) $x_1 = 1$

$x_{n+1} = ax_n + 5, \quad n \geq 1$

a) $x_2 = ax_1 + 5$

$x_2 = a(1) + 5$

$\boxed{x_2 = a + 5}$

b) $x_3 = ax_2 + 5$

$x_3 = a(a+5) + 5$

$\boxed{x_3 = a^2 + 5a + 5}$

c) If $x_3 = 41$, $a^2 + 5a + 5 = 41$

$a^2 + 5a - 36 = 0$

$(a+9)(a-4) = 0$

$\boxed{\text{Either } a = -9 \text{ or } a = 4}$

Q5) curve $C: y = x(5-x)$. curve $L: 2y = 5x + 4$

$y = 5x - x^2$ ① $\times 2$ $2y = 10x - 2x^2$

$2y = 5x + 4$ ② $\times 1$ $2y = 5x + 4$

Substitute ① into ②:

$10x - 2x^2 = 5x + 4$

$2x^2 - 5x + 4 = 0$

$$b^2 - 4ac \equiv (-5)^2 - (4)(2)(4)$$

$$= 25 - 32$$

$$\underline{= -7}$$

Since $\underline{b^2 - 4ac < 0}$, there are no real solutions for the equation $C = L$ and thus the curves do not intersect.

b) Curve C: $y = x(5-x)$

When $x = 0$, $y = 0(5-0)$ $\underline{= 0}$

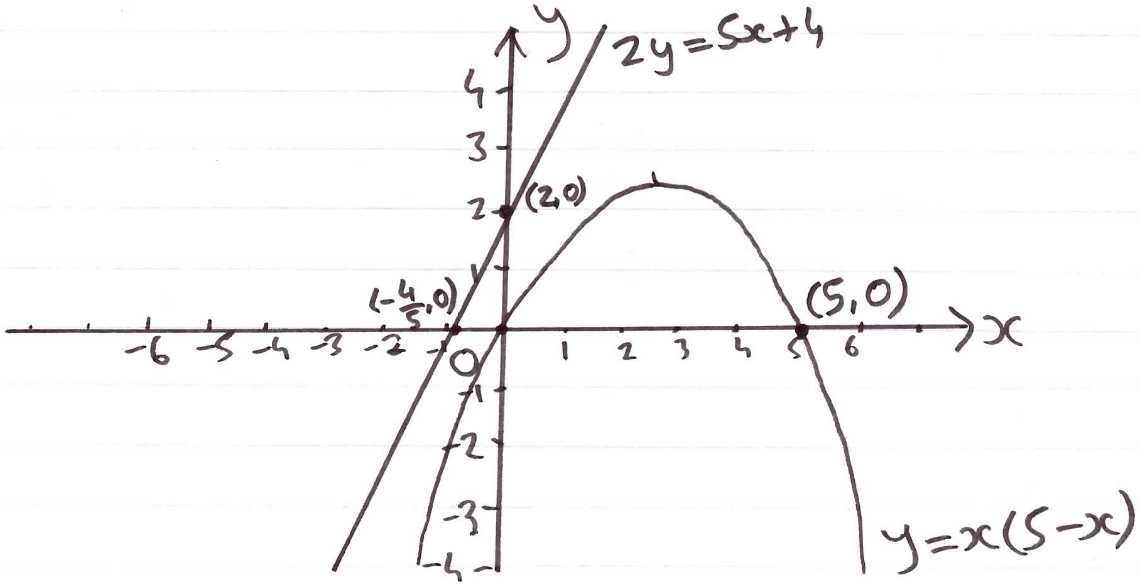
When $y = 0$, $x(5-x) = 0$

$\underline{\text{Either } x = 0 \text{ or } x = 5}$

Line L: $2y = 5x + 4$

When $x = 0$, $2y = 4 \Rightarrow \underline{y = 2}$

When $y = 0$, $0 = 5x + 4 \Rightarrow \underline{x = -\frac{4}{5}}$



Q6a) $2x - 3y + 12 = 0$

$$3y = 2x + 12$$

$$y = \frac{2}{3}x + 4$$

$$\boxed{\text{Gradient} = \frac{2}{3}}$$

b) when $2x - 3y + 12 = 0$ cuts the y-axis, $x = 0$

$$\therefore -3y + 12 = 0$$

$$3y = 12$$

$$y = 4$$

$\therefore$ B is at $(0, 4)$

A perpendicular line would have a gradient of $-\frac{3}{2}$,

(since $\frac{2}{3} \times -\frac{3}{2} = -1$)

Since the perpendicular line passes through B, use x_1 as 0 and y_1 as 4, and m as $-\frac{3}{2}$:

Equation of perpendicular line:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{-3}{2}(x - 0)$$

$$2(y-4) = -3(x-0)$$

$$2y - 8 = -3x$$

$$\boxed{2y + 3x - 8 = 0}$$

c) When $2x - 3y + 12 = 0$ cuts the x-axis, $y=0$

$$\therefore \quad 2x + 12 = 0$$

$$x = -6$$

$\therefore$ <u>A is at $(-6, 0)$</u>

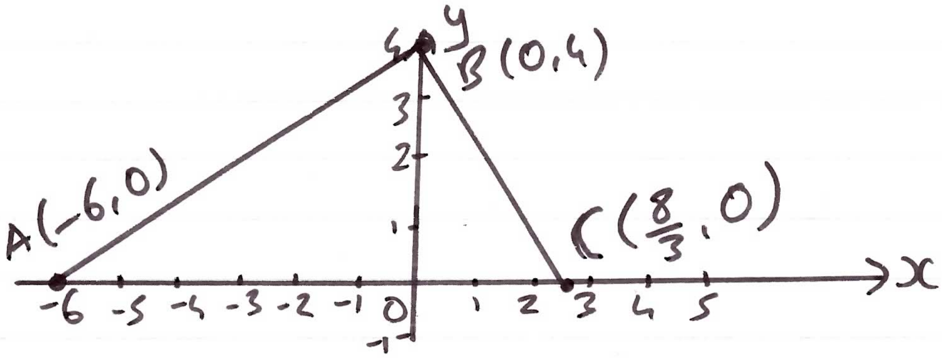
When $2y + 3x - 8 = 0$ crosses the x-axis, $y = 0$

$$\therefore \quad 3x - 8 = 0$$

$$3x = 8$$

$$x = \frac{8}{3}$$

$\therefore$ <u>C is at $\left(\frac{8}{3}, 0\right)$</u>



Area $\Delta ABC = \frac{1}{2}bh = \frac{1}{2}(AC)(OB)$

$= \frac{1}{2}\left(\frac{26}{3}\right)(4)$

$= \frac{1}{2}\left(\frac{104}{3}\right)$

$= \boxed{\frac{52}{3} \text{ units}^2}$

Q7) $f'(x) = 3x^2 - 3x + 5$

$f(x) = \int (3x^2 - 3x + 5)\,dx$

$f(x) = x^3 - \frac{3x^2}{2} + 5x + c$

Since $f(x)$ passes through $(2, 10)$, substitute in $x=2$ and $f(x)=10$:

$10 = 2^3 - \frac{3(2)^2}{2} + 5(2) + c$

$10 = 8 - 6 + 10 + c$

$10 = 12 + c$

$\therefore\ c = -2$

$$f(x) = x^3 - \frac{3x^2}{2} + 5x - 2$$

$$f(1) = 1^3 - \frac{3(1)^2}{2} + 5(1) - 2$$

$$= 1 - \frac{3}{2} + 5 - 2$$

$$= 4 - \frac{3}{2}$$

$$\boxed{= \frac{5}{2}}$$

Q8) $y = x^2(x+2) \quad = x^3 + 2x^2$

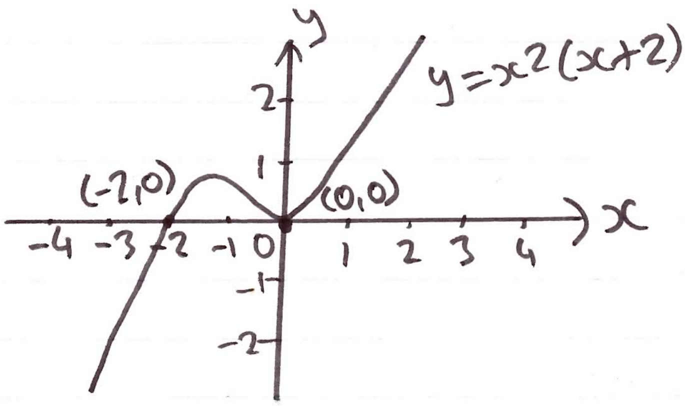
a) $\boxed{\frac{dy}{dx} = 3x^2 + 4x}$

b) $y = x^2(x+2)$

When $x = 0$, $y = 0^2(0+2)$ $\underline{=0}$

When $y = 0$, $x^2(x+2) = 0$

$\underline{\text{Either } x=0,\ x=0, \text{ or } x=-2}$



c) At $x=0$, $\frac{dy}{dx} = 3(0)^2 + 4(0) \underline{= 0}$

At $x=-2$, $\frac{dy}{dx} = 3(-2)^2 + 4(-2)$

$= 3(4) - 8$

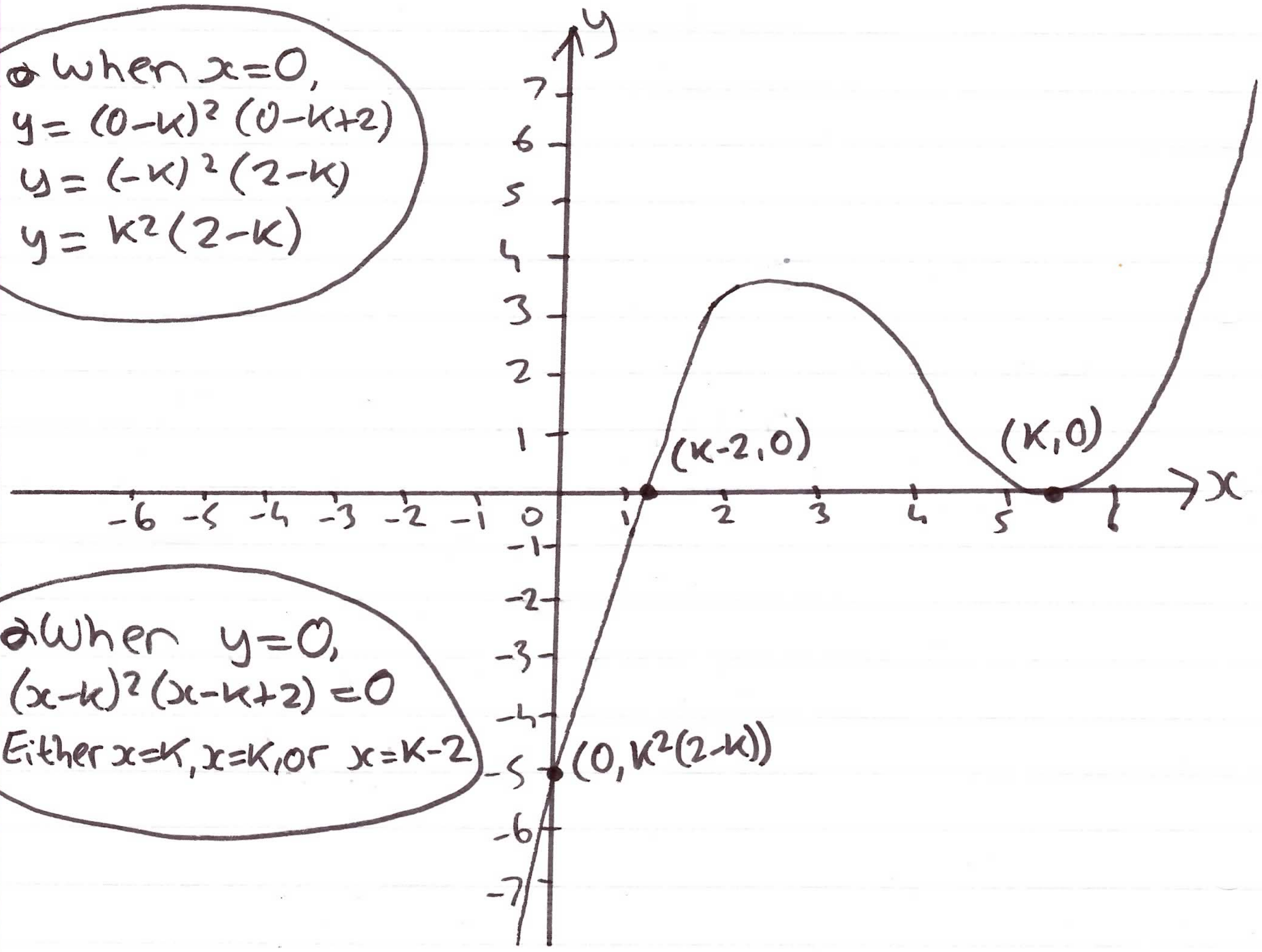
$= 12 - 8$

$\underline{= 4}$

d) $y = (x-k)^2(x-k+2)$, $k > 2$

This is a horizontal translation (along the x-axis) of $+k$.

① When $x=0$,
$y = (0-k)^2(0-k+2)$
$y = (-k)^2(2-k)$
$y = k^2(2-k)$



② When $y=0$,
$(x-k)^2(x-k+2) = 0$
Either $x=k$, $x=k$, or $x=k-2$

Q9a) Scheme 1: First term, $a = £P$

Commen differererence, $d = £(2T)$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2P + (10-1)(2T))$$

$$S_{10} = 5(2P + 18T)$$

$$\boxed{S_{10} = £(10P + 90T)}$$

b) Scheme 2: First term, $a = £(P + 1800)$

common difference, $d = £T$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2(P+1800) + (10-1)T)$$

$$S_{10} = 5(2P + 3600 + 10T - T)$$

$$S_{10} = 5(2P + 3600 + 9T)$$

$$S_{10} = £(10P + 45T + 18000)$$

But, $10P + 45T + 18000 = 10P + 90T$

$$18000 = 45T$$

$$\boxed{T = 400}$$

c) $U_n = a + (n-1)d$

For Scheme 2, $U_{10} = (P+1800) + (10-1)T$

$U_{10} = (P+1800) + 9(400)$

$29850 = P + 1800 + 3600$

$29850 = P + 5400$

$\boxed{P = 24450}$

Q10a) $y = 2 - \frac{1}{x}$, $x \neq 0$

When the curve crosses the x-axis, $y=0$

So, $0 = 2 - \frac{1}{x}$

$2 = \frac{1}{x}$

$2x = 1$

$x = \frac{1}{2}$

$\therefore$ $\boxed{A \text{ is at } \left(\frac{1}{2}, 0\right)}$

b) $y = 2 - \frac{1}{x}$

$y = 2 - x^{-1}$

$\frac{dy}{dx} = x^{-2}$

$= \frac{1}{x^2}$

When $x = \frac{1}{2}$, $\frac{dy}{dx} = \frac{1}{(\frac{1}{2})^2} = \frac{1}{\frac{1}{4}} = 4$

The gradient of the normal at A would be $-\frac{1}{4}$

Equation of normal: $y - y_1 = m(x - x_1)$

Using $A(\frac{1}{2}, 0)$ and $m = -\frac{1}{4}$:

$y - 0 = -\frac{1}{4}(x - \frac{1}{2})$

$4(y - 0) = -1(x - \frac{1}{2})$

$4y = -x + \frac{1}{2}$

$4y + x - \frac{1}{2} = 0 \Rightarrow \boxed{2x + 8y - 1 = 0}$

c) To find points of intersection, solve simultaneous equations:

$$y = 2 - \frac{1}{x} \quad ①$$
$$2x + 8y - 1 = 0 \quad ②$$

Substitute ① into ②:

$$2x + 8\left(2 - \frac{1}{x}\right) - 1 = 0$$

$$2x + 16 - \frac{8}{x} - 1 = 0$$

$$2x + 15 - \frac{8}{x} = 0$$

$$\frac{2x^2 + 15x - 8}{x} = 0$$

$$2x^2 + 15x - 8 = 0$$

$$(2x - 1)(x + 8) = 0$$

Either $x = \frac{1}{2}$ or $x = -8$

We already know that $(\frac{1}{2}, 0)$ is a point of intersection, so find y when $x = -8$

Substitute into ① for y:

$$y = 2 - \frac{1}{-8} = 2 + \frac{1}{8} = \frac{17}{8}$$

$\therefore$ $\boxed{B \text{ is at } \left(-8, \frac{17}{8}\right)}$