

C1 January 2011 (MA)

$$Q1a) 16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{16}} = \boxed{\frac{1}{2}}$$

$$\begin{aligned} b) x(2x^{-\frac{1}{4}})^4 &= x(2^4 \cdot x^{-\frac{1}{4} \times 4}) \\ &= x(16x^{-1}) \\ &= x\left(\frac{16}{x}\right) \end{aligned}$$

$$= \frac{16x}{x}$$

$$\boxed{= 16}$$

$$\begin{aligned} Q2) \int (12x^5 - 3x^2 + 4x^{\frac{1}{3}}) dx &= \frac{12x^6}{6} - \frac{3x^3}{3} + \frac{4x^{\frac{4}{3}}}{\frac{4}{3}} + C \\ &= \boxed{2x^6 - x^3 + 3x^{\frac{4}{3}} + C} \end{aligned}$$

$$\begin{aligned} Q3) \frac{5-2\sqrt{3}}{\sqrt{3}-1} &= \frac{5-2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\ &= \frac{(5-2\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \\ &= \frac{5\sqrt{3} + 5 - 6 - 2\sqrt{3}}{3 + \sqrt{3} - \sqrt{3} - 1} \\ &= \frac{3\sqrt{3}-1}{2} = \boxed{-\frac{1}{2} + \frac{3\sqrt{3}}{2}} \end{aligned}$$

Q4) $a_1 = 2$
 $a_{n+1} = 3a_n - c$

a) $a_2 = 3a_1 - c$

$$a_2 = 6 - c$$

b) $\sum_{i=1}^3 a_i = 0$

$$a_1 + a_2 + a_3 = 0$$

$$2 + (6 - c) + [3(6 - c) - c] = 0$$

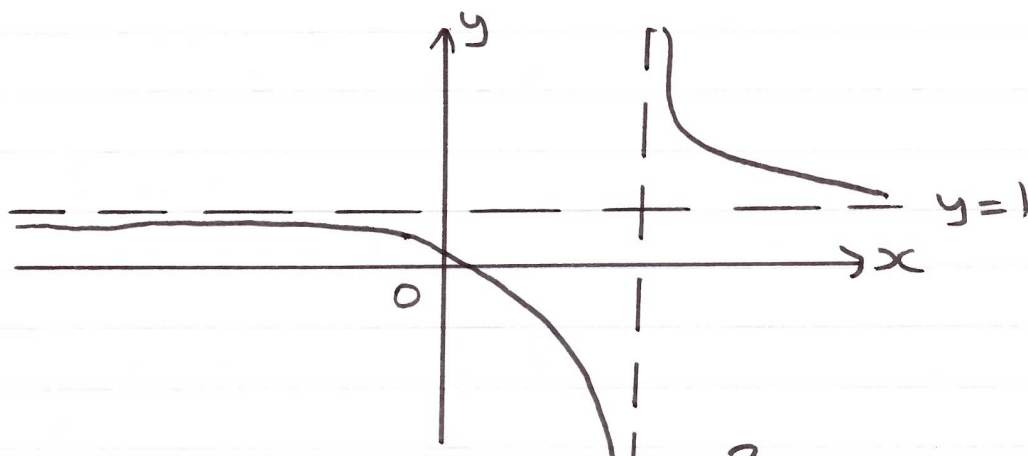
$$2 + 6 - c + [18 - 3c - c] = 0$$

$$2 + 6 - c + 18 - 4c = 0$$

$$26 = 5c$$

$$c = \frac{26}{5}$$

Q5a) $y = f(x-1)$ - transformation of $+1$ along the x -axis:



$$b) f(x) = \frac{x}{x-2}, \quad x \neq 2$$

$$f(x+1) = \frac{x+1}{(x+1)-2}$$

$$\text{When } x=0, f(x+1) = \frac{0+1}{(0+1)-2}$$

$$= \frac{1}{3}$$

Q6) First term = a . Common difference = d

$$S_{10} = 162$$

$$a) S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_{10} = \frac{10}{2} (2a + (10-1)d)$$

$$S_{10} = 5(2a + 9d)$$

$$162 = 10a + 45d$$

$$\therefore \boxed{10a + 45d = 162} \quad (1)$$

$$b) U_6 = 17$$

$$U_n = a + (n-1)d$$

$$U_6 = a + (6-1)d$$

$$17 = a + 5d$$

$$\therefore \boxed{a + 5d = 17} \quad (2)$$

$$c) \quad \begin{array}{l} 10a + 45d = 162 \quad (1) \\ a + 5d = 17 \quad (2) \end{array} \left| \begin{array}{l} \times 1 \\ \times 9 \end{array} \right| \begin{array}{l} 10a + 45d = 162 \\ 9a + 45d = 153 \end{array} \} -$$

$$\boxed{a = 9}$$

Substitute into (1) for d:

$$10a + 45d = 162$$

$$10(9) + 45d = 162$$

$$90 + 45d = 162$$

$$45d = 72$$

$$\boxed{d = \frac{8}{5}}$$

$$Q7) \quad f'(x) = 12x^2 - 8x + 1$$

$$f(x) = \frac{12x^3}{3} - \frac{8x^2}{2} + x + C$$

$$\underline{f(x) = 4x^3 - 4x^2 + x + C}$$

Since $f(x)$ passes through $(-1, 0)$,
substitute $x = -1$ and $f(x) = 0$

$$0 = 4(-1)^3 - 4(-1)^2 + (-1) + c$$

$$0 = 4(-1) - 4(1) - 1 + c$$

$$0 = -4 - 4 - 1 + c$$

$$\therefore c = 9$$

$$f(x) = 4x^3 - 4x^2 + x + 9$$

Q8) $x^2 + (k-3)x + (3-2k) = 0$ has two distinct real roots

a) If a quadratic has two distinct roots, then the discriminant is greater than

$$\therefore b^2 - 4ac > 0$$

$$(k-3)^2 - (4)(1)(3-2k) > 0$$

$$(k-3)(k-3) - 4(3-2k) > 0$$

$$k^2 - 6k + 9 - 12 + 8k > 0$$

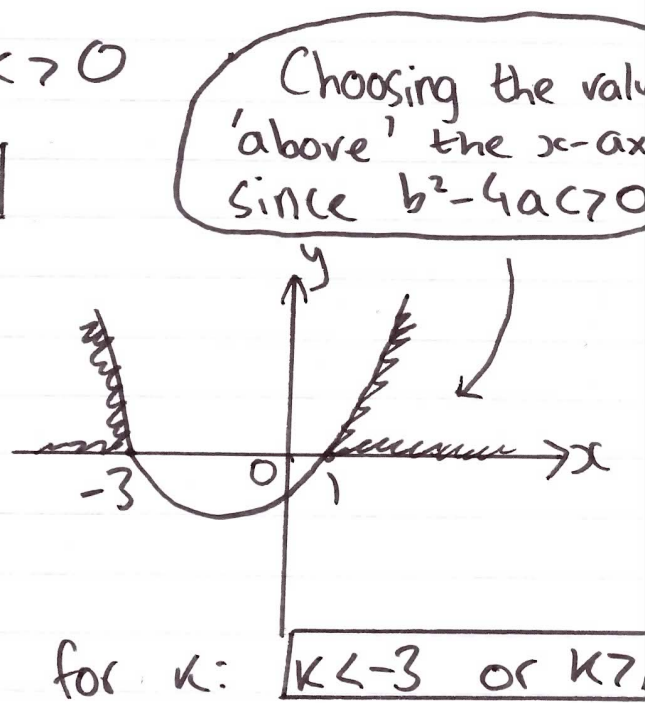
$$k^2 + 2k - 3 > 0$$

b) For $k^2 + 2k - 3 = 0$

$$(k+3)(k-1) = 0$$

Either $k = -3$ or $k = 1$

Set of possible values for k : $k < -3$ or $k > 1$



Q9) $2y - 3x - k = 0$

- a) Since $A(1,4)$ lies on the curve,
Substitute in $x=1$ and $y=4$:

$$2(4) - 3(1) - k = 0$$

$$8 - 3 - k = 0$$

$$5 - k = 0$$

$$\therefore \boxed{k = 5}$$

b) $2y - 3x - 5 = 0$

$$2y = 3x + 5$$

$$y = \frac{3}{2}x + \frac{5}{2}$$

so the $\boxed{\text{gradient is } \frac{3}{2}}$.

- c) The gradient of a perpendicular line would have a gradient of $-\frac{2}{3}$.

Equation of perpendicular line:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{2}{3}(x - 1)$$

Using $A(1,4)$
and $m = -\frac{2}{3}$

$$3(y-4) = -2(x-1)$$

$$3y - 12 = -2x + 2$$

$$3y - 12 + 2x - 2 = 0$$

$$\boxed{2x + 3y - 14 = 0}$$

d) When $2x + 3y - 14 = 0$ cuts the x -axis,
 $y = 0$

$$\text{So, } 2x + 3(0) - 14 = 0$$

$$2x = 14$$

$$\underline{x = 7}$$

$$\therefore \boxed{B \text{ is at } (7, 0)}$$

e) Length $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(7-1)^2 + (0-4)^2}$$

$$= \sqrt{6^2 + (-4)^2}$$

$$= \sqrt{52} = \sqrt{13 \times 4} = \sqrt{4} \sqrt{13}$$

$$= \boxed{2\sqrt{13} \text{ units}}$$

Q10a)

i) $y = x(x+2)(3-x)$

When $x=0, y = 0(0+2)(3-0) = \underline{0}$

When $y=0, x(x+2)(3-x) = 0$

Either $x=0$ or $x=-2$ or $x=3$

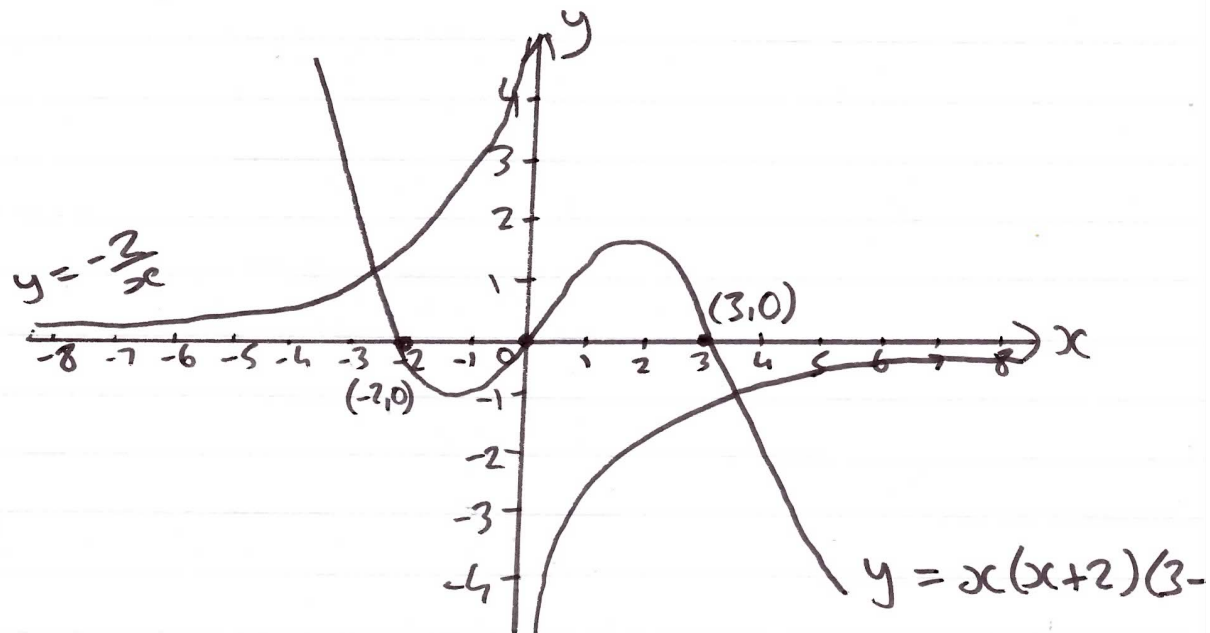
∴ ii) $y = -\frac{2}{x}$

When $x \rightarrow 0, y \rightarrow \infty$

When $x \rightarrow -0, y \rightarrow \infty$

When $y \rightarrow 0, x \rightarrow \infty$

When $y \rightarrow -0, x \rightarrow \infty$



b) For the equation $x(x+2)(3-x) + \frac{2}{x} = 0$,

there would be 2 distinct real solutions, since there are 2 points of intersection between the two curves.

$$\text{Q11a)} \quad y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \quad x > 0$$

$$y = \frac{1}{2}x^3 - 9x^{3/2} + 8x^{-1} + 30$$

$$\frac{dy}{dx} = \frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$$

$$= \boxed{\frac{3x^2}{2} - \frac{27\sqrt{x}}{2} - \frac{8}{x^2}}$$

b) Substitute in $x=4$ and $y=-8$:

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30$$

$$-8 = \frac{1}{2}(4)^3 - 9(4)^{3/2} + \frac{8}{4} + 30$$

$$-8 = \frac{1}{2}(64) - 9(4)(\sqrt{4}) + 2 + 30$$

$$-8 = 32 - 72 + 2 + 30$$

$$\underline{-8 = -8}$$

$\therefore$ the point $P(4, -8)$ lies on the curve

$$\begin{aligned}
 \text{c) When } x=4, \frac{dy}{dx} &= \frac{3(4)^2}{2} - \frac{27(\sqrt{4})}{2} - \frac{8}{4^2} \\
 &= \frac{3(16)}{2} - \frac{27(2)}{2} - \frac{8}{16} \\
 &= \frac{48}{2} - \frac{54}{2} - \frac{1}{2} \\
 &= \underline{\underline{-\frac{7}{2}}}
 \end{aligned}$$

$\therefore$ the gradient of the normal at P is $\underline{\underline{\frac{2}{7}}}$.

Equation of normal: $y - y_1 = m(x - x_1)$

Using $P(4, -8)$ and $m = \frac{2}{7}$

$$y - (-8) = \frac{2}{7}(x - 4)$$

$$7(y + 8) = 2(x - 4)$$

$$7y + 56 = 2x - 8$$

$$7y + 56 + 8 - 2x = 0$$

$$\boxed{-2x + 7y + 64 = 0}$$