

(1 January 2010 (MA))

$$Q1) \quad y = x^4 + x^{1/3} + 3$$

$$\frac{dy}{dx} = 4x^3 + \frac{1}{3}x^{-2/3}$$

$$= 4x^3 + \frac{1}{3x^{2/3}}$$

$$= \boxed{4x^3 + \frac{1}{3(\sqrt[3]{x})^2}}$$

$$Q2a) \quad (7+\sqrt{5})(3-\sqrt{5}) = 21 - 7\sqrt{5} + 3\sqrt{5} - 5$$

$$= \boxed{16 - 4\sqrt{5}}$$

$$b) \quad \frac{7+\sqrt{5}}{3+\sqrt{5}} = \frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$$

$$= \frac{(7+\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

$$= \frac{16 - 4\sqrt{5}}{9 - 3\sqrt{5} + 3\sqrt{5} - 5}$$

$$= \frac{16 - 4\sqrt{5}}{4}$$

$$= \boxed{4 - \sqrt{5}}$$

$$3a) \quad 3x + 5y - 2 = 0$$

$$5y = -3x + 2$$

$$y = -\frac{3}{5}x + \frac{2}{5}$$

$$\boxed{\text{Gradient} = -\frac{3}{5}}$$

b) A perpendicular line would have a gradient of $\frac{5}{3}$.

Equation of perpendicular line:

$$y - y_1 = m(x - x_1)$$

Since line passes through (3,1), use 3 as x_1 and 1 as y_1 :

$$y - 1 = \frac{5}{3}(x - 3)$$

$$3(y - 1) = 5(x - 3)$$

$$3y - 3 = 5x - 15$$

$$3y = 5x - 12$$

$$\boxed{y = \frac{5}{3}x - 4}$$

$$\begin{aligned} \text{Q4)} \quad \frac{dy}{dx} &= 5x^{-\frac{1}{2}} + x\sqrt{x}, x > 0 \\ &= 5x^{-\frac{1}{2}} + x^{\frac{3}{2}} \end{aligned}$$

$$y = \int (5x^{-\frac{1}{2}} + x^{\frac{3}{2}}) dx$$

$$= \frac{5x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= 10x^{\frac{1}{2}} + \frac{2x^{\frac{5}{2}}}{5} + C$$

$$= 10\sqrt{x} + \frac{2x^2\sqrt{x}}{5} + C$$

Given $y=35$ at $x=4$,

$$35 = 10(\sqrt{4}) + \frac{2(4)^2(\sqrt{4})}{5} + C$$

$$35 = 10(2) + \frac{2(16)(2)}{5} + C$$

$$35 = 20 + \frac{64}{5} + C$$

$$\therefore C = \frac{11}{5}$$

$$\boxed{y = 10\sqrt{x} + \frac{2x^2\sqrt{x}}{5} + \frac{11}{5}}$$

$$5) \quad y - 3x + 2 = 0 \Rightarrow y = 3x - 2 \quad (1)$$

$$y^2 - x - 6x^2 = 0 \Rightarrow y^2 - x - 6x^2 = 0 \quad (2)$$

Substitute (1) into (2):

$$(3x - 2)^2 - x - 6x^2 = 0$$

$$(3x - 2)(3x - 2) - x - 6x^2 = 0$$

$$9x^2 - 12x + 4 - x - 6x^2 = 0$$

$$3x^2 - 13x + 4 = 0$$

$$(3x - 1)(x - 4) = 0$$

Either $x = \frac{1}{3}$ or $x = 4$

Substitute into (1) for y :

$$\text{When } x = \frac{1}{3}, y = 3\left(\frac{1}{3}\right) - 2 = 1 - 2 = \underline{-1}$$

$$\text{When } x = 4, y = 3(4) - 2 = 12 - 2 = \underline{10}$$

Solution set: $x = \frac{1}{3}, y = -1 \quad x = 4, y = 10$

$$Q6a) \quad y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

$$y = \frac{x^2 - 8x + 3x - 24}{x}$$

$$y = \frac{x^2 - 5x - 24}{x}$$

$$y = \frac{x^2}{x} - \frac{5x}{x} - \frac{24}{x}$$

$$y = x - 5 - 24x^{-1}$$

$$\frac{dy}{dx} = 1 + 24x^{-2}$$

$$= \boxed{1 + \frac{24}{x^2}}$$

$$b) \quad \text{When } x=2, \quad \frac{dy}{dx} = 1 + \frac{24}{2^2} = 1 + 6 = \underline{7}$$

$$\text{Also, when } x=2, \quad y = 2 - 5 - \frac{24}{2} = \underline{-15}$$

Equation of tangent: $y - y_1 = m(x - x_1)$

Using $(2, -15)$
and $m = 7$

$$y - (-15) = 7(x - 2)$$

$$y + 15 = 7x - 14$$

$$\boxed{y = 7x - 29}$$

Q7) First term, $a = £150$.
Common difference, $d = £10$

a) $U_n = a + (n-1)d$

$$U_{10} = 150 + (10-1)10$$

$$U_{10} = 150 + 90$$

$$\boxed{U_{10} = £240}$$

b) $S_n = \frac{n}{2} (2a + (n-1)d)$

$$S_{20} = \frac{20}{2} (2(150) + (20-1)10)$$

$$S_{20} = 10(300 + 190)$$

$$S_{20} = 10 \times 490$$

$$\boxed{S_{20} = £4900}$$

c) For Kevin, $S_{20} = £9800$. Given $d = £30$:

$$9800 = 10(2a + (19)(30))$$

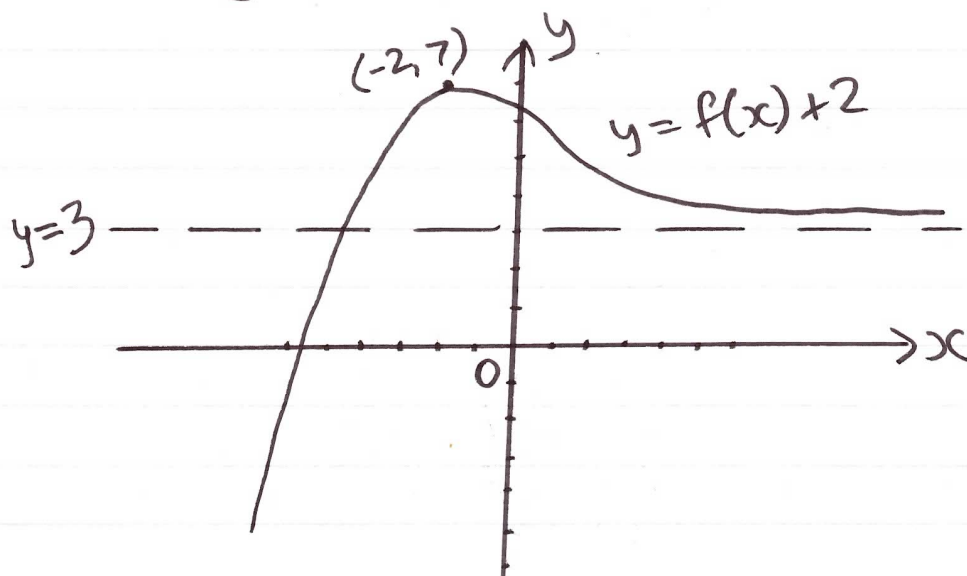
$$9800 = 10(2a + 570)$$

$$9800 = 20a + 5700$$

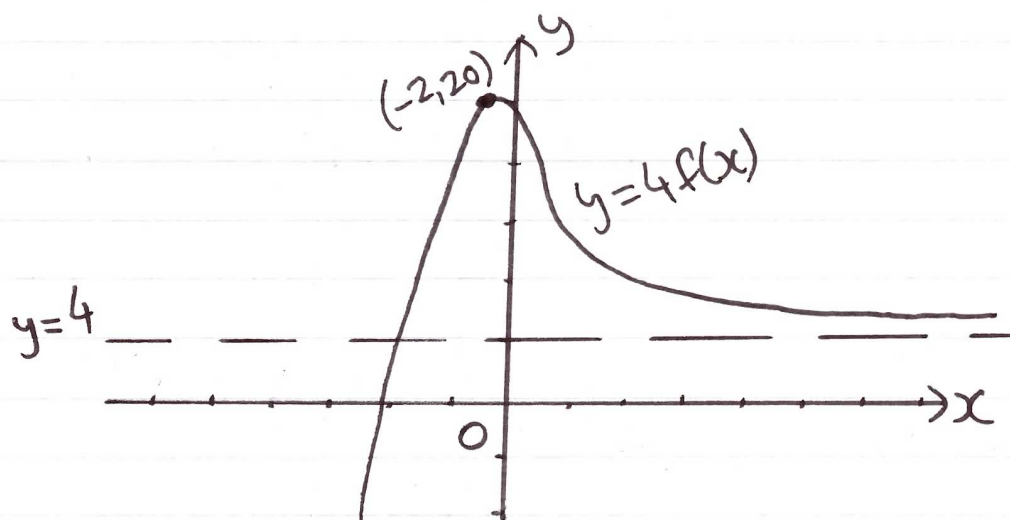
$$20a = 41200$$

$$\therefore \boxed{a = £205}$$

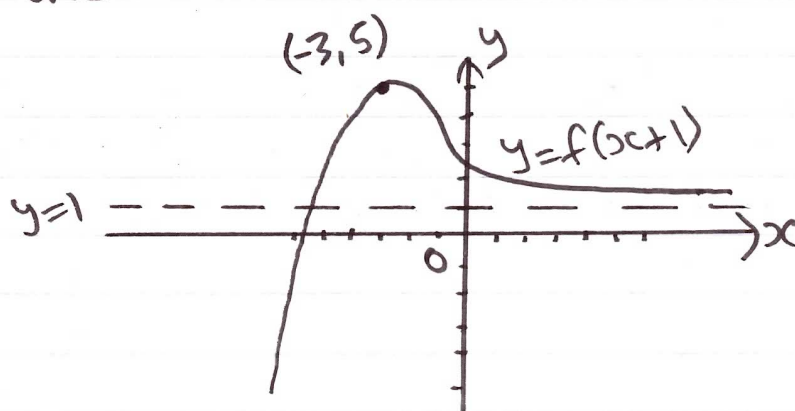
Q8a) $y = f(x) + 2$ - transformation of +2 along the y-axis:



b) $y = 4f(x)$ - multiply y-coordinates by 4:



c) $y = f(x+1)$ - transformation of -1 along the x-axis:



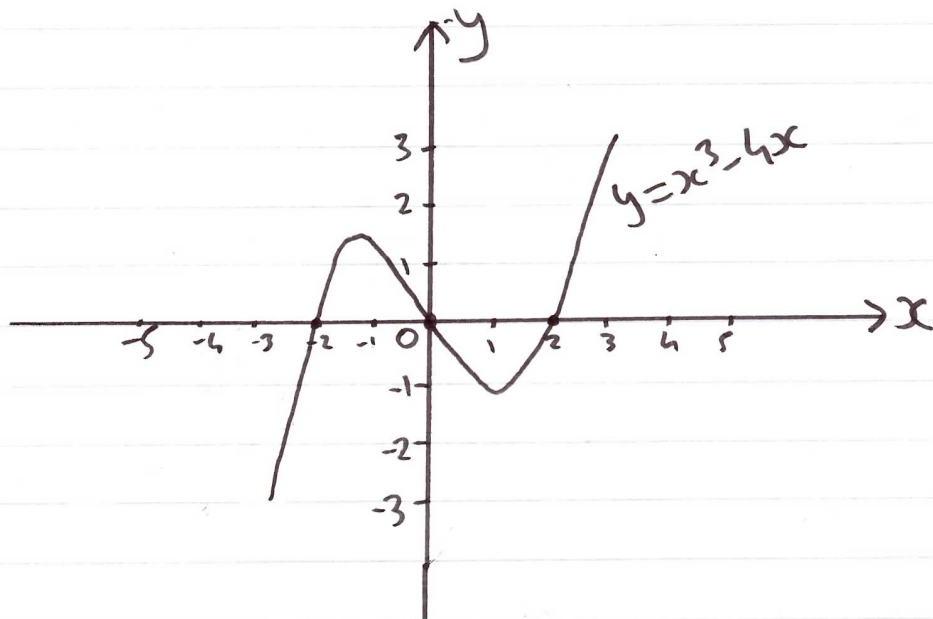
$$\begin{aligned} \text{Q9a) } x^3 - 4x &= x(x^2 - 4) \\ &= \boxed{x(x+2)(x-2)} \end{aligned}$$

$$\text{b) } y = x^3 - 4x$$

$$\text{When } \underline{x=0, y=0}$$

$$\begin{aligned} \text{When } y=0, \quad x^3 - 4x &= 0 \\ x(x+2)(x-2) &= 0 \end{aligned}$$

$$\underline{\text{Either } x=0 \text{ or } x=-2 \text{ or } x=2}$$



$$\text{c) When } x=-1, y = (-1)^3 - 4(-1) = -1 + 4 = 3$$

$$\therefore \underline{A \text{ is at } (-1, 3)}$$

$$\text{When } x=3, y = 3^3 - 4(3) = 27 - 12 = 15$$

$$\therefore \underline{B \text{ is at } (3, 15)}$$

$$\text{Gradient } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{15 - 3}{3 - (-1)}$$

$$= \frac{12}{4}$$

$$= \underline{3}$$

$$\text{Equation: } y - y_1 = m(x - x_1)$$

$$\text{Using } A(-1, 3) \text{ and } m=3 \longrightarrow y - 3 = 3(x - (-1))$$

$$y - 3 = 3(x + 1)$$

$$y - 3 = 3x + 3$$

$$\boxed{y = 3x + 6}$$

$$\text{d) Length } AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-1))^2 + (15 - 3)^2}$$

$$= \sqrt{4^2 + 12^2}$$

$$= \sqrt{160} = \sqrt{16 \times 10} = \sqrt{16} \sqrt{10}$$

$$= \boxed{4\sqrt{10} \text{ units}}$$

$$\begin{aligned}
 \text{Q10a)} \quad f(x) &= x^2 + 4kx + (3 + 11k) \\
 &= (x + 2k)^2 - (2k)^2 + (3 + 11k) \\
 &= (x + 2k)^2 - 4k^2 + (3 + 11k) \\
 &= \boxed{(x + 2k)^2 - 4k^2 + (3 + 11k)}
 \end{aligned}$$

b) If $f(x) = 0$ has no real roots, then the discriminant is less than 0.

$$\therefore b^2 - 4ac < 0$$

$$(4k)^2 - (4)(1)(3 + 11k) < 0$$

$$16k^2 - 4(3 + 11k) < 0$$

$$16k^2 - 12 - 44k < 0$$

$$4k^2 - 3 - 11k < 0$$

$$4k^2 - 11k - 3 < 0$$

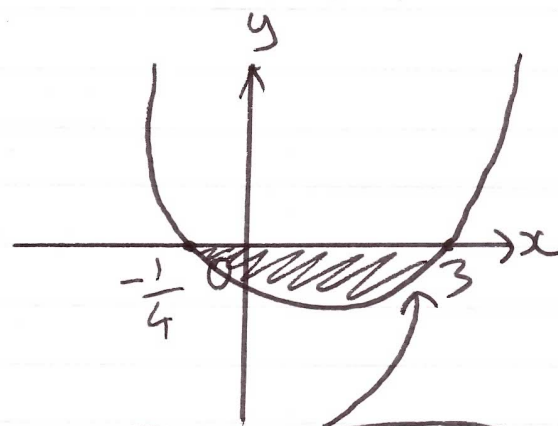
For $4k^2 - 11k - 3 = 0$,

$$(4k + 1)(k - 3) = 0$$

Either $k = -\frac{1}{4}$ or $k = 3$

Set of possible values of k :

$$\boxed{-\frac{1}{4} < k < 3}$$



Choosing values 'under' the x-axis since $b^2 - 4ac < 0$

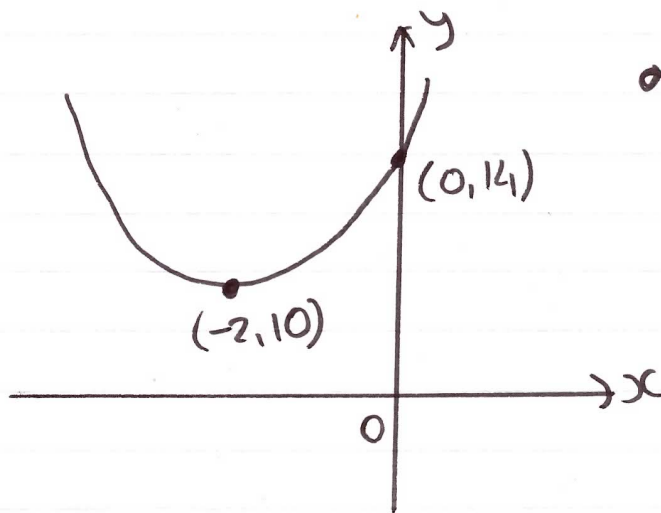
c) Given $k=1$

$$f(x) = x^2 + 4x + 14$$

When $x=0$, $f(x) = 14$

When $f(x)=0$, $x^2 + 4x + 14 = 0$

• No real roots, so curve does not cross x -axis



• Turnig point at $-(2k)$, $-4k^2 + (3+11k)$
 when $k=1$,
Turning point is
at $(-2, 10)$