C1 January 2009 (MA)

Q1a) $125^{1/3} = \sqrt[3]{125}$ $\boxed{= 5}$

b) $125^{-2/3} = \dfrac{1}{125^{2/3}} = \dfrac{1}{(\sqrt[3]{125})^2} = \dfrac{1}{5^2}$ $\boxed{= \dfrac{1}{25}}$

Q2) $\int (12x^5 - 8x^3 + 3)\,dx = \dfrac{12x^6}{6} - \dfrac{8x^4}{4} + 3x + C$

$= \boxed{2x^6 - 2x^4 + 3x + C}$

Q3) $(\sqrt{7} + 2)(\sqrt{7} - 2) = 7 - 2\sqrt{7} + 2\sqrt{7} - 4$

$\boxed{= 3}$

Q4) $f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7$

$f(x) = \int (3x^2 - 3x^{\frac{1}{2}} - 7)\,dx$

$= \dfrac{3x^3}{3} - \dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 7x + C$

$= x^3 - 2x^{\frac{3}{2}} - 7x + C$

$\underline{= x^3 - 2x\sqrt{x} - 7x + C}$

Since $(4, 22)$ lies on $f(x)$, substitute in $x = 4$ and $f(x) = 22$:

$22 = x^3 - 2x\sqrt{x} - 7x + C$

$22 = (4)^3 - 2(4)(\sqrt{4}) - 7(4) + C$

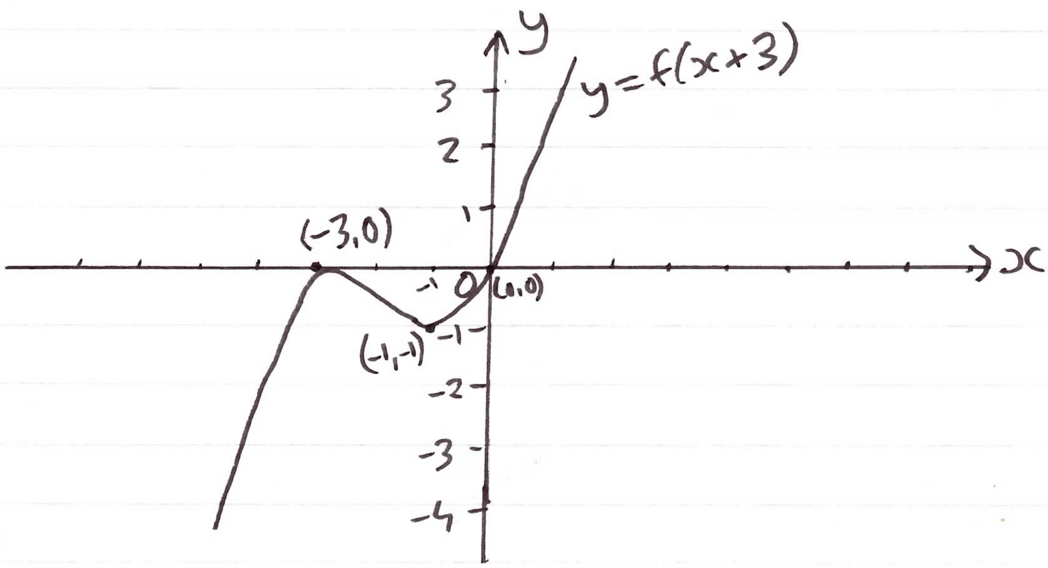
$22 = 64 - 16 - 28 + c$

$22 = 20 + c$

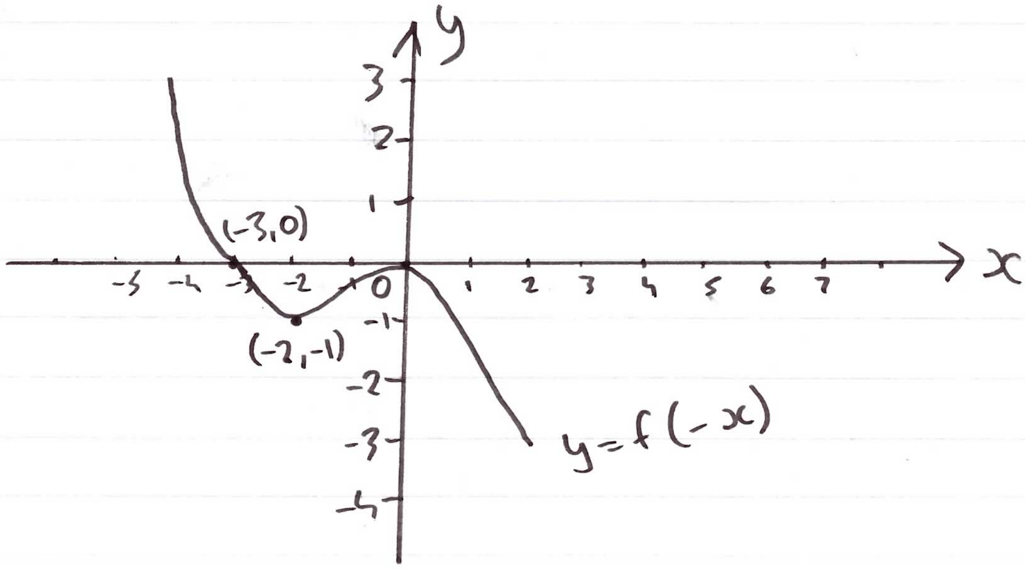
$\therefore \underline{c = 2}$

$$\boxed{f(x) = x^3 - 2x\sqrt{x} - 7x + 2}$$

Q5a) $y = f(x+3)$ - transformation of -3 along the x-axis:



b) $y = f(-x)$ - reflection in the y-axis:



Q6a) $\dfrac{2x^2 - x^{3/2}}{\sqrt{x}} = \dfrac{2x^2 - x^{3/2}}{x^{1/2}}$

$= \dfrac{2x^2}{x^{1/2}} \quad \dfrac{-x^{3/2}}{x^{1/2}}$

$= \boxed{2x^{3/2} \quad -x^1}$

b) $y = 5x^4 - 3 \quad + \dfrac{2x^2 - x^{3/2}}{\sqrt{x}}$

$y = 5x^4 - 3 + 2x^{\frac{3}{2}} - x$

$\dfrac{dy}{dx} = 20x^3 + 3x^{\frac{1}{2}} - 1$

$= \boxed{20x^3 + 3\sqrt{x} - 1}$

Q7) $Kx^2 + 4x + (5-K) = 0$ has 2 distinct real solutions.

a) If a quadratic has 2 distinct real root then the discriminant is greater than 0.

$\therefore b^2 - 4ac > 0$

$4^2 - (4)(K)(5-K) > 0$

$16 - 20K + 4K^2 > 0$

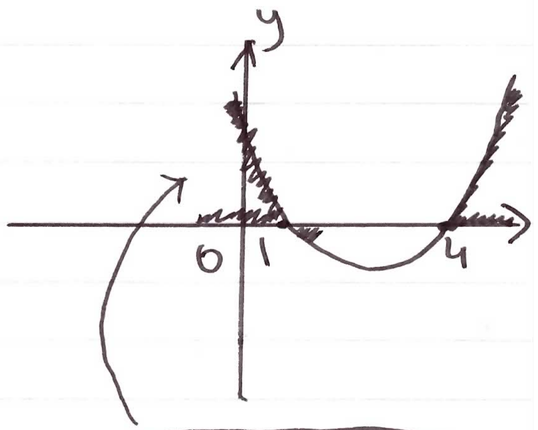
$4K^2 - 20K + 16 > 0$

$\therefore \boxed{K^2 - 5K + 4 > 0}$

b) For $k^2 - 5k + 4 = 0$

$(k-4)(k-1) = 0$

Either $k=4$ or $k=1$



Set of possibles values for k:

$\boxed{k<1 \quad \text{or} \quad k>4}$

Chosing values 'above' the x-ax[is] since $b^2-4ac>$[0]

Q8a) $y = (x+1)^2(2-x)$

when $x=1$, $y = (1+1)^2(2-1) = (2)^2(1) =$[4]

$\therefore \boxed{a=4}$

bi) $y=(x+1)^2(2-x)$

When $x=0, y=(0+1)^2(2-0) = (1)^2(2) \underline{= 2}$

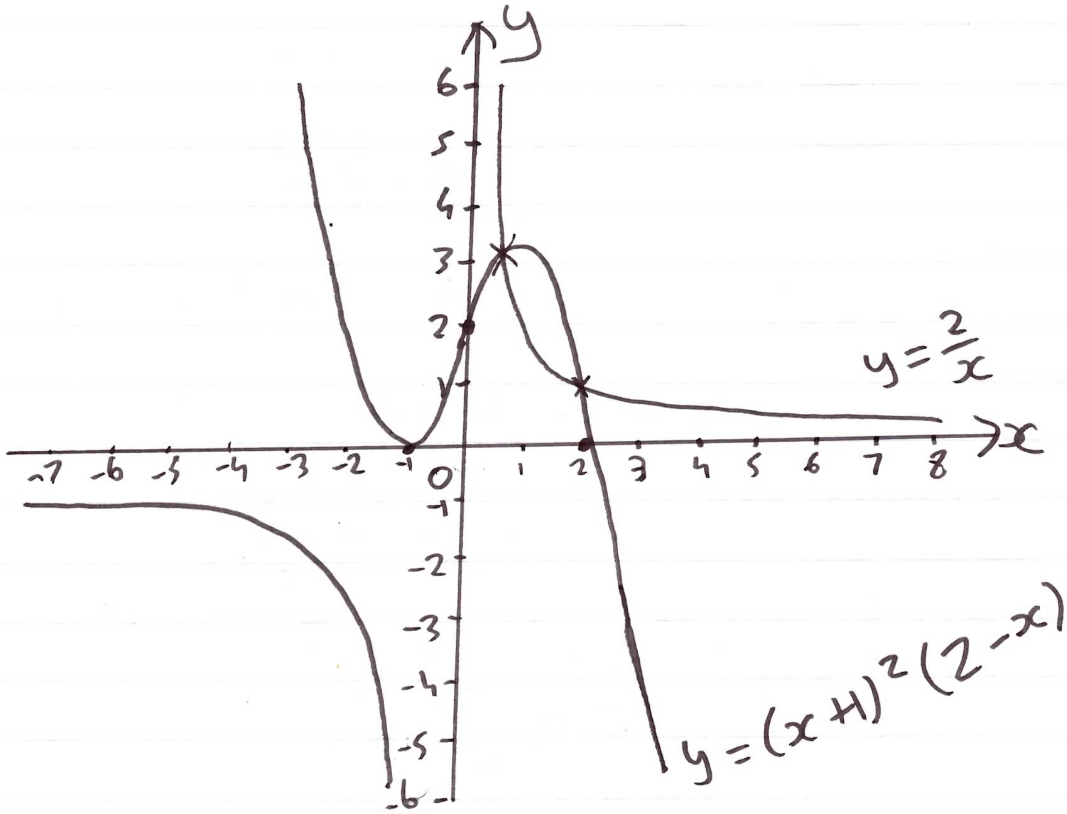
When $y=0$, $(x+1)^2(2-x) = 0$

Either $x=-1$ or $x=-1$ or $x=2$

ii) $y = \dfrac{2}{x}$

When $x=0$, $y \to \infty$
When $x=-0$, $y \to -\infty$

When $y=0$, $x \to \infty$
When $y=-0$, $x \to -\infty$



c) For the equation $(x+1)^2(2-x) = \frac{2}{x}$,

there are 2 real solutions, since there are 2 intersections between the curves

Q9) First term $= a$. common difference $= d$

$U_n = a + (n-1)d$

a) $U_{18} = a + (18-1)d$

$\boxed{25 = a + 17d}$ ①

$U_{21} = a + (21-1)d$

$\boxed{32.5 = a + 20d}$ ②

b) ② − ①: $7.5 = 3d$

$$\boxed{\therefore d = 2.5}$$

Substitute into ① for a:

$$25 = a + 17d$$

$$25 = a + 17(2.5)$$

$$25 = a + 42.5$$

$$\therefore \boxed{a = -17.5}$$

c) $S_n = 2750$

Since $S_n = \frac{n}{2}(2a + (n-1)d)$

$$S_n = \frac{n}{2}(2(-17.5) + (n-1)2.5)$$

$$S_n = \frac{n}{2}(-35 + 2.5n - 2.5)$$

$$S_n = \frac{n}{2}(2.5n - 37.5)$$

$$S_n = \frac{2.5n^2}{2} - \frac{37.5n}{2}$$

$$2S_n = 2.5n^2 - 37.5n$$

$$4S_n = 5n^2 - 75n$$

$$4S_n = n(5n - 75)$$

Substitute in $S_n = 2750$

$$4 \times 2750 = n(5n - 75)$$

$$4 \times 550 = n(n-15)$$

$$\boxed{\therefore n^2 - 15n = 55 \times 40}$$

d) $n^2 - 15n = 55 \times 40$

$$n^2 - 15n = 2200$$

$$n^2 - 15n - 2200 = 0$$

$$(n-55)(n+40) = 0$$

Since $n > 0$, $\boxed{n = 55}$

Q10a) The line l_1 passes through $A(2,5)$ and has gradient $-\frac{1}{2}$.

Equation of l_1 : $y - y_1 = m(x - x_1)$

Using $A(2,5)$ and $m = -\frac{1}{2}$

$$y - 5 = -\frac{1}{2}(x-2)$$

$$2(y-5) = -1(x-2)$$

$$2y - 10 = -x + 2$$

$$2y = -x + 12$$

$$\boxed{y = -\frac{x}{2} + 6}$$

b) Using $x=-2,\ y=7$:

$$y = -\frac{x}{2} + 6$$

$$7 = -\frac{(-2)}{2} + 6$$

$$7 = 1 + 6$$

$$\underline{7 = 7}$$

$\therefore$ point $B(-2,7)$ lies on l_1

c) Length $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$= \sqrt{(-2-2)^2+(7-5)^2}$$

$$= \sqrt{(-4)^2+2^2}$$

$$= \sqrt{20} = \sqrt{4\times5} = \sqrt{4}\sqrt{5}$$

$$\boxed{= 2\sqrt{5} \text{ units}}$$

d) point C lies on l_1 and has x-coordinate equal to p. length AC is 5 units.

$$\therefore \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = 5$$

$$\sqrt{(2-p)^2+(5-(-\tfrac{1}{2}p+6))^2} = 5$$

$$(2-p)^2+(5-(-\tfrac{1}{2}p+6))^2 = 25$$

$$(2-p)(2-p)+(5+\tfrac{1}{2}p-6)(5+\tfrac{1}{2}p-6) = 25$$

$$(2-p)(2-p)+(-1+\tfrac{1}{2}p)(-1+\tfrac{1}{2}p)=25$$

$$(2-p)(2-p)+(\tfrac{1}{2}p-1)(\tfrac{1}{2}p-1)=25$$

$$4-4p+p^2+\tfrac{1}{4}p^2-p+1=25$$

$$p^2-4p+4+\tfrac{1}{4}p^2-p+1=25$$

$$p^2-4p+4+\tfrac{1}{4}p^2-p+1-25=0$$

$$\frac{5}{4}p^2-5p-20=0$$

$$5p^2-20p-80=0$$

$$\therefore \boxed{p^2-4p-16=0}$$

Q11) $y=9-4x-\dfrac{8}{x}$, $\quad x>0$

a) $y=9-4x-8x^{-1}$

$$\frac{dy}{dx}=-4+8x^{-2}$$

$$=-4+\frac{8}{x^2} \qquad \underline{=\frac{8}{x^2}-4}$$

When $x=2$, $\dfrac{dy}{dx}=\dfrac{8}{2^2}-4 \quad \underline{=-2}$

$\therefore$ the gradient of the tangent at $x=$
is -2

Also, when $x=2$, $y = 9 - 4(2) - \frac{8}{2}$

$$y = 9 - 8 - 4$$

$$\underline{y = -3}$$

Equation of normal: $y - y_1 = m(x - x_1)$

Using $P(2, -3)$ and $m = -2$ $\rightarrow$ $y - (-3) = -2(x - 2)$

$$y + 3 = -2x + 4$$

$$y = -2x + 1$$

$$\boxed{y = 1 - 2x}$$

b) Gradient of tangent at P is -2, so gradient of normal at P is $\frac{1}{2}$.

Equation of normal: $y - y_1 = m(x - x_1)$

Using $P(2, -3)$ and $m = \frac{1}{2}$ $\rightarrow$ $y - (-3) = \frac{1}{2}(x - 2)$

$$2(y + 3) = 1(x - 2)$$

$$2y + 6 = x - 2$$

$$2y = x - 8$$

$$\boxed{y = \frac{x}{2} - 4}$$

c) For the tangent at P, when this meets the x-axis at A, $y=0$

$$\therefore \quad y = 1-2x$$

$$0 = 1-2x$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$\therefore$ A is at $\left(\frac{1}{2}, 0\right)$

For the normal at P, when this meet the x-axis at B, $y=0$

$$\therefore \quad y = \frac{x}{2} - 4$$

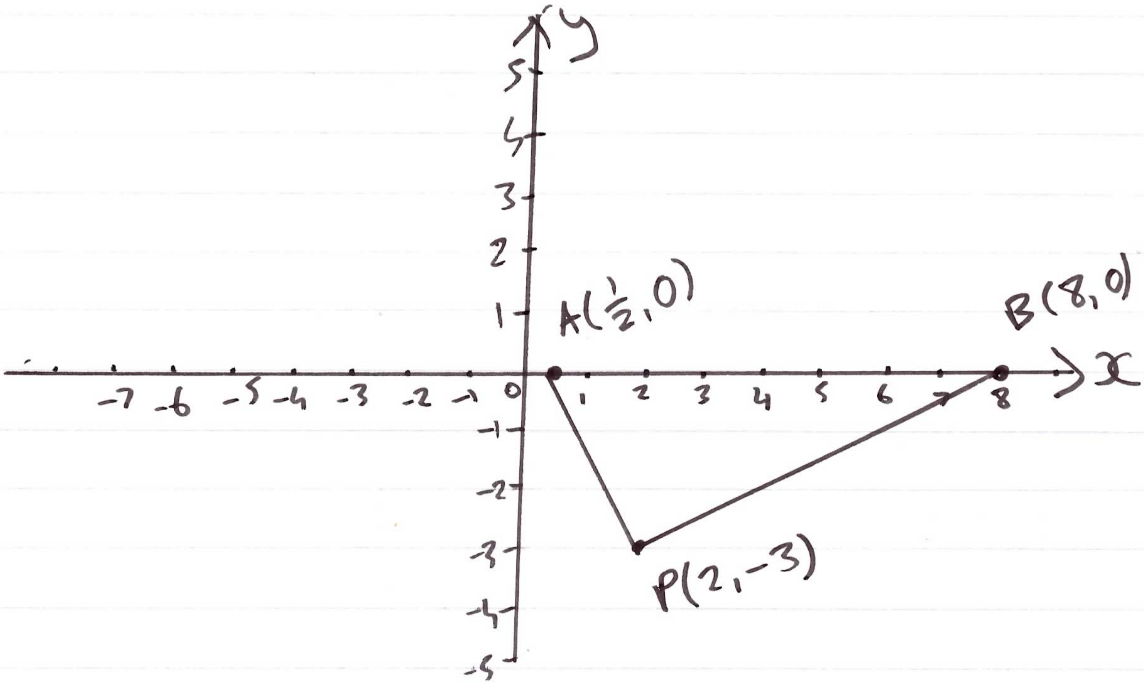
$$0 = \frac{x}{2} - 4$$

$$\frac{x}{2} = 4$$

$$x = 8$$

$\therefore$ B is at $(8, 0)$

Find the area of the triangle APB



Length $AP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - \frac{1}{2})^2 + (-3 - 0)^2}$

$= \sqrt{(\frac{3}{2})^2 + (-3)^2}$

$= \sqrt{\frac{9}{4} + 9}$

$= \sqrt{\frac{45}{4}}$

$= \frac{\sqrt{45}}{\sqrt{4}} = \frac{\sqrt{9 \times 5}}{2} = \frac{\sqrt{9}\sqrt{5}}{2}$

$= \frac{3\sqrt{5}}{2}$ units

Length $PB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-8)^2+(-3-0)^2}$

$= \sqrt{(-6)^2+(-3)^2}$

$= \sqrt{36+9}$

$= \sqrt{45} \quad = \sqrt{9\times 5} \quad = \sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$ units

Area $\Delta APB = \frac{1}{2}bh = \frac{1}{2}(PB)(AP)$

$= \frac{1}{2}(3\sqrt{5})\left(\frac{3\sqrt{5}}{2}\right)$

$= \frac{1}{2}\left(\frac{45}{2}\right)$

$= \frac{45}{4}$

$\boxed{= 11.25 \text{ units}^2}$