C1 January 2008 (MA)

Q1) $\int (3x^2+4x^5-7)\,dx = \frac{3x^3}{3} + \frac{4x^6}{6} - 7x + c$

$$= \boxed{x^3 + \frac{2x^6}{3} - 7x + c}$$

Q2a) $16^{1/4} = \sqrt[4]{16}$ $\boxed{=2}$

b) $(16x^{12})^{3/4} = 16^{3/4} \times (x^{12})^{3/4}$

$$= (\sqrt[4]{16})^3 \, (x^{12 \times 3/4})$$

$$= 2^3 \times x^9$$

$$\boxed{= 8x^9}$$

Q3) $\frac{5-\sqrt{3}}{2+\sqrt{3}} = \frac{5-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{(5-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{10 - 5\sqrt{3} - 2\sqrt{3} + 3}{4 - 2\sqrt{3} + 2\sqrt{3} - 3}$$

$$= \frac{13 - 7\sqrt{3}}{1}$$

$$= \boxed{13 - 7\sqrt{3}}$$

Q4) $A(-6,4)$ and $B(8,-3)$ lie on the line L.

a) Gradient $AB = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-3-4}{8-(-6)}$$

$$= \frac{-3-4}{14}$$

$$= \frac{-7}{14}$$

$$= -\frac{1}{2}$$

Equation : $y - y_1 = m(x - x_1)$

Using $A(-6,4)$ $\longrightarrow$ $y - 4 = -\frac{1}{2}(x - (-6))$

$$y - 4 = -\frac{1}{2}(x+6)$$

$$2(y-4) = -1(x+6)$$

$$2y - 8 = -x - 6$$

$$2y - 8 + x + 6 = 0$$

$$2y - 2 + x = 0$$

$$\boxed{x + 2y - 2 = 0}$$

b) Length $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-6-8)^2+(4-(-3))^2}$

$= \sqrt{(-14)^2+7^2}$

$= \sqrt{245} \quad = \sqrt{49\times5} \quad = \sqrt{49}\sqrt{5}$

$= \boxed{7\sqrt{5} \text{ units}}$

Q5a) $\dfrac{2\sqrt{x}+3}{x} = \dfrac{2x^{\frac{1}{2}}}{x} + \dfrac{3}{x}$

$= \boxed{2x^{-\frac{1}{2}} + 3x^{-1}}$

b) $y = 5x-7+\dfrac{2\sqrt{x}+3}{x}, \quad x>0$

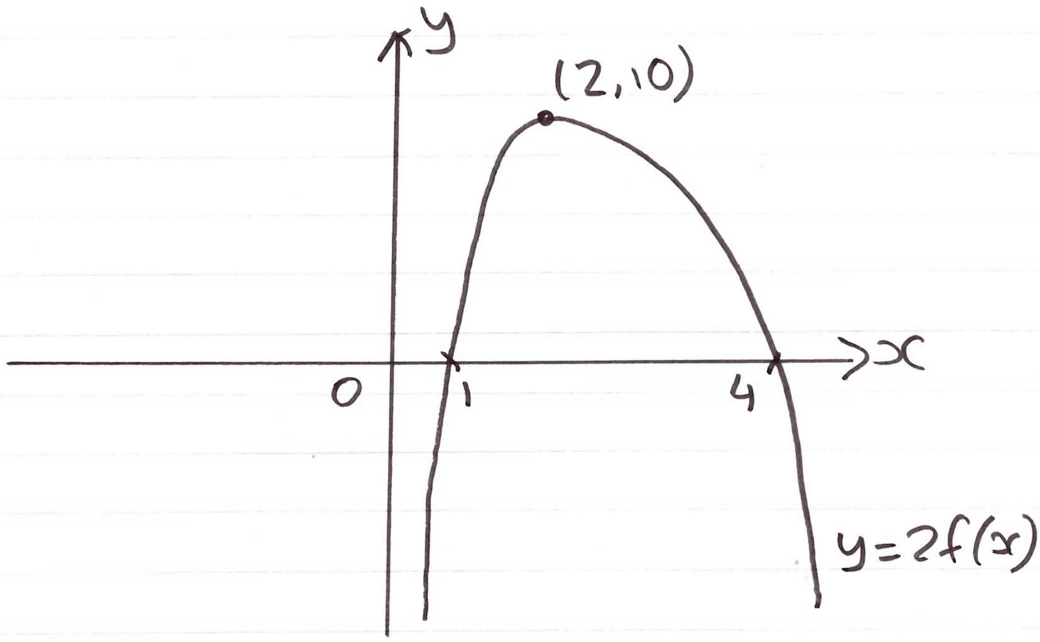
$y = 5x-7+2x^{-\frac{1}{2}}+3x^{-1}$

$\dfrac{dy}{dx} = 5 - x^{-\frac{3}{2}} - 3x^{-2}$

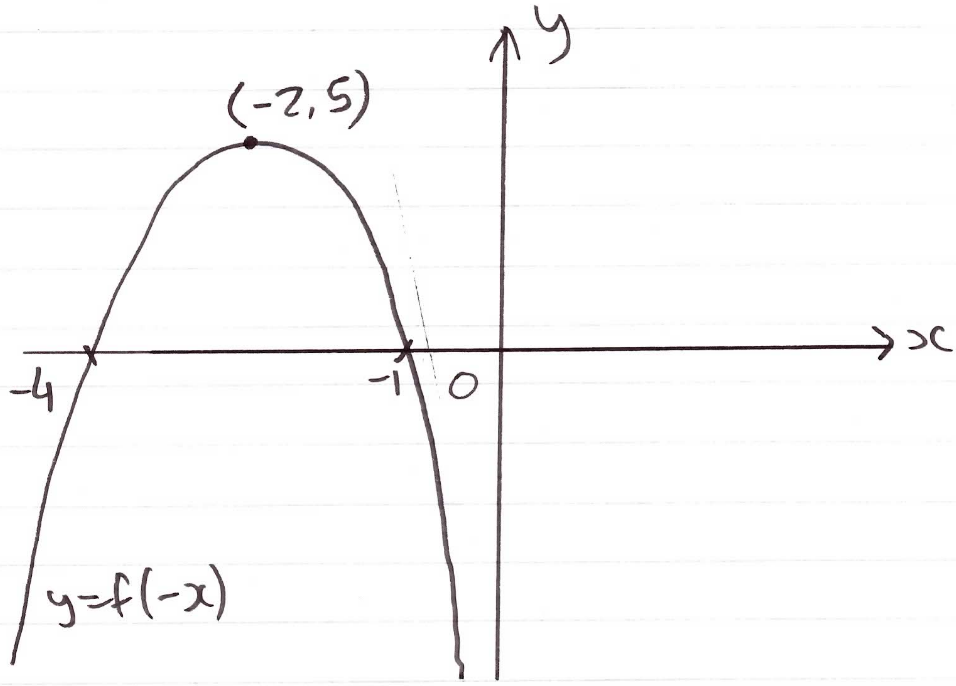
$= 5 - \dfrac{1}{x^{3/2}} - \dfrac{3}{x^2}$

$= \boxed{5 - \dfrac{1}{x\sqrt{x}} - \dfrac{3}{x^2}}$

Q6a) $y = 2f(x)$ - multiply y-coordinates by 2:



b) $y = f(-x)$ - Reflection in the y-axis:



c) The maximum point on the curve with equation $y = f(x+a)$ is on the y-axis.

So, $y = f(x+2)$ would have a maximum point at $(0,5)$, so $\boxed{a=2}$

Q7) $x_1 = 1,$

$x_{n+1} = x_n(p + x_n)$

a) $x_2 = x_1(p + x_1)$

$x_2 = 1(p+1)$

$\boxed{x_2 = p+1}$

b) $x_3 = x_2(p + x_2)$

$x_3 = (p+1)(p+(p+1))$

$x_3 = (p+1)(p+p+1)$

$x_3 = (p+1)(2p+1)$

$x_3 = 2p^2 + p + 2p + 1$

$\therefore \boxed{x_3 = 1 + 3p + 2p^2}$

c) Given $x_3 = 1$, $1 = 1 + 3p + 2p^2$

$2p^2 + 3p = 0$

$p(2p+3) = 0$

Either $p=0$ or $p = -\frac{3}{2}$

Since $p \neq 0$, $\boxed{p = -\frac{3}{2}}$

d) $x_{2008} = x_{2007}\left(-\frac{3}{2} + x_{2007}\right)$

But, $x_1 = 1, x_2 = -\frac{1}{2}, x_3 = 1, x_4 = -\frac{1}{2}$

So, every odd term for x_n is 1, and every even term is $-\frac{1}{2}$

$\therefore \boxed{x_{2008} = -\frac{1}{2}}$

Q8a) $x^2 + kx + 8 = k$ has no real solutions for x

$x^2 + kx + 8 - k = 0$

$x^2 + kx + (8-k) = 0$

If the quadratic has no real solution, then the discriminant is less than 0.

$\therefore b^2 - 4ac < 0$

$k^2 - (4)(1)(8-k) < 0$

$k^2 - 4(8-k) < 0$

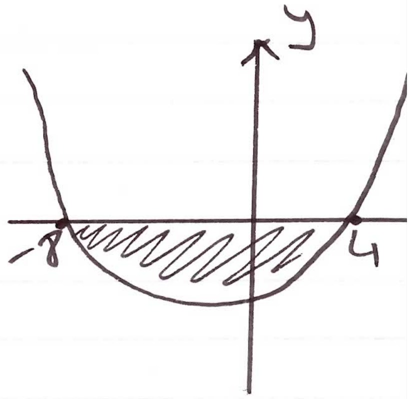
$k^2 - 32 + 4k < 0$

$\boxed{k^2 + 4k - 32 < 0}$

b) For $k^2 + 4k - 32 = 0$,

$$(k+8)(k-4) = 0$$

Either $k = -8$ or $k = 4$



Possible values of k:

$$\boxed{-8 < k < 4}$$

Chosing values 'under' the x-axi[s] since $b^2 - 4ac < 0$

Q 9a) $f'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$

$$f'(x) = 4x - 6x^{\frac{1}{2}} + 8x^{-2}$$

$$f(x) = \int (4x - 6x^{\frac{1}{2}} + 8x^{-2})\,dx$$

$$f(x) = \frac{4x^2}{2} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{8x^{-1}}{-1} + C$$

$$f(x) = 2x^2 - 4x^{3/2} - 8x^{-1} + C$$

$$f(x) = 2x^2 - 4x\sqrt{x} - \frac{8}{x} + C$$

Since the point $P(4,1)$ lies on $f(x)$, substitute in $x = 4$ and $y = 1$:

$$1 = 2(4)^2 - 4(4)(\sqrt{4}) - \frac{8}{4} + C$$

$1 = 2(16) - 32 - 2 + c$

$1 = 32 - 32 - 2 + c$

$1 = -2 + c$

$\therefore \underline{c = 3}$

$$\boxed{f(x) = 2x^2 - 4x\sqrt{x} - \frac{8}{x} + 3}$$

b) When $x=4$, $f'(x) = 4(4) - 6(\sqrt{4}) + \frac{8}{4^2}$

$= 16 - 12 + \frac{1}{2}$

$= \underline{\frac{9}{2}}$

$\therefore$ the gradient of the normal at $P = \underline{-\frac{2}{9}}$

Equation of normal: $y - y_1 = m(x - x_1)$

Using $P(4,1)$ $\rightarrow$ $y - 1 = -\frac{2}{9}(x-4)$

$9(y-1) = -2(x-4)$

$9y - 9 = -2x + 8$

$9y = -2x + 17$

$$\boxed{y = -\frac{2x}{9} + \frac{17}{9}}$$

Q10a) $y = (x+3)(x-1)^2$

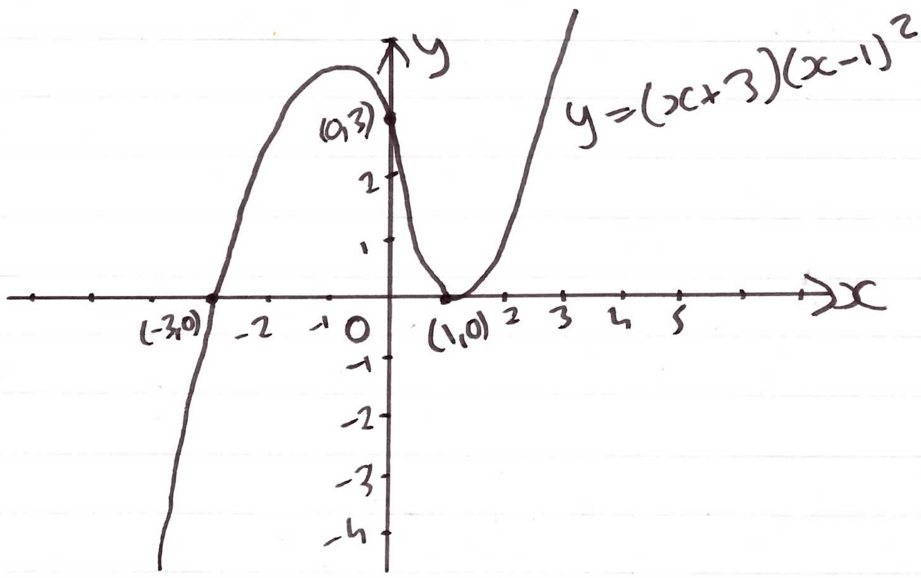
When $x = 0$, $y = (0+3)(0-1)^2$

$= (3)(-1)^2$

$\underline{= 3}$

When $y = 0$, $(x+3)(x-1)^2 = 0$

$\underline{\text{Either } x = -3 \text{ or } x = 1 \text{ or } x = 1}$



b) $y = (x+3)(x-1)^2$

$y = (x+3)(x-1)(x-1)$

$y = (x+3)(x^2 - 2x + 1)$

$y = x^3 - 2x^2 + x + 3x^2 - 6x + 3$

$\underline{y = x^3 + x^2 - 5x + 3}$

$\boxed{\therefore k = 3}$

c) $y = x^3 + x^2 - 5x + k$

To find the points where the gradient of the tangent is equal to 3, first differentiate with respect to x, the substitute in $\frac{dy}{dx} = 3$:

$$\frac{dy}{dx} = \underline{3x^2 + 2x - 5}$$

When $\frac{dy}{dx} = 3$, $3x^2 + 2x - 5 = 3$

$$3x^2 + 2x - 8 = 0$$

$$(3x - 4)(x + 2) = 0$$

$$\boxed{\text{Either } x = \frac{4}{3} \text{ or } x = -2}$$

Q11) $a = 30$, $d = -1.5$

a) $U_n = a + (n-1)d$

$U_{25} = 30 + (24)(-1.5) = 30 + (-36) \boxed{= -6}$

b) $r = U_n = 0 \Rightarrow 30 + (r-1)(-1.5) = 0$

$$30 - 1.5r + 1.5 = 0$$

$$1.5r = 31.5 \Rightarrow \boxed{r = 21}$$

c) $S_{21} = \frac{21}{2}(2(30) + (20)(-1.5)) \boxed{= 315}$